

Phys 221

Lecture 30

Assign # 11.

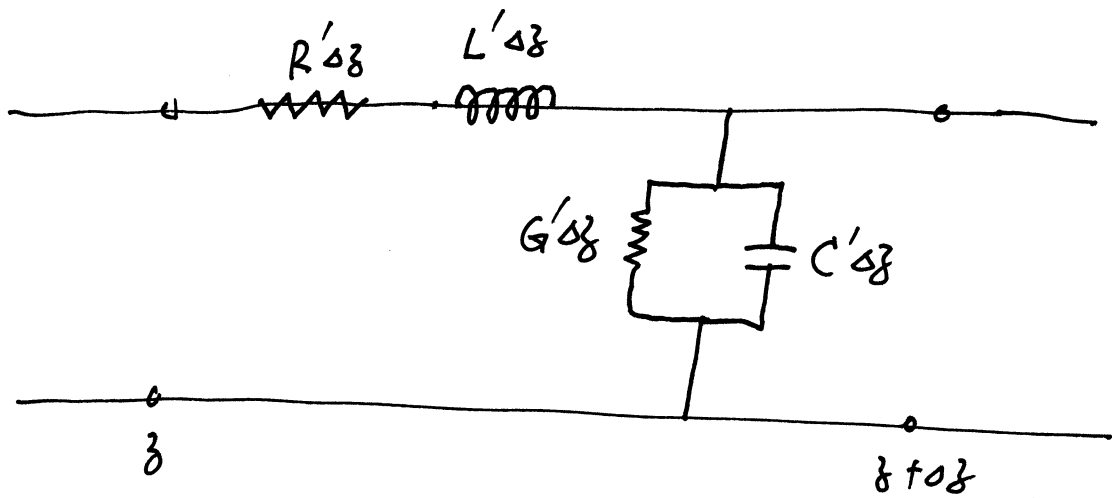
Ref. 8-3, 8-4, 8-5*

p. 302, 8.1, 2, 3, 5.

• Last lecture

Transmission line equations

(using the lumped-element model)



phasors:

$$-\frac{d\tilde{V}(z)}{dz} = (R' + j\omega L') \tilde{I}(z) \quad (8.18a)$$

$$-\frac{d\tilde{I}(z)}{dz} = (G' + j\omega C') \tilde{V}(z) \quad (8.18b)$$

$\{R', L', G', C'\}$ determines the behavior of the transmission line.

differentiate 8.18 a:

$$- \frac{d^2 \tilde{V}(z)}{dz^2} = (R' + j\omega L) \frac{d \tilde{I}(z)}{dz}$$

$$\begin{aligned} - \frac{d^2 \tilde{V}(z)}{dz^2} &= - (R' + j\omega L)(G' + j\omega C) \tilde{V}(z) \\ &= - \gamma^2 \tilde{V}(z) \end{aligned}$$

i.e. $\boxed{\frac{d^2 \tilde{V}(z)}{dz^2} - \gamma^2 \tilde{V}(z) = 0}$ (8.21)

$$\gamma = \sqrt{(R' + j\omega L)(G' + j\omega C)}$$
 (8.22)

wave eq !

Similarly. $\boxed{\frac{d^2 \tilde{I}(z)}{dz^2} - \gamma^2 \tilde{I}(z) = 0}$ (8.23)

$$\gamma = \alpha + j\beta$$

α - attenuation constant

β - phase constant

Solution of 8-21 and 8-23:

$$\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}.$$

$$\tilde{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}.$$

$V_0^+ e^{-\gamma z}$ — propagating along $+z$ direction.

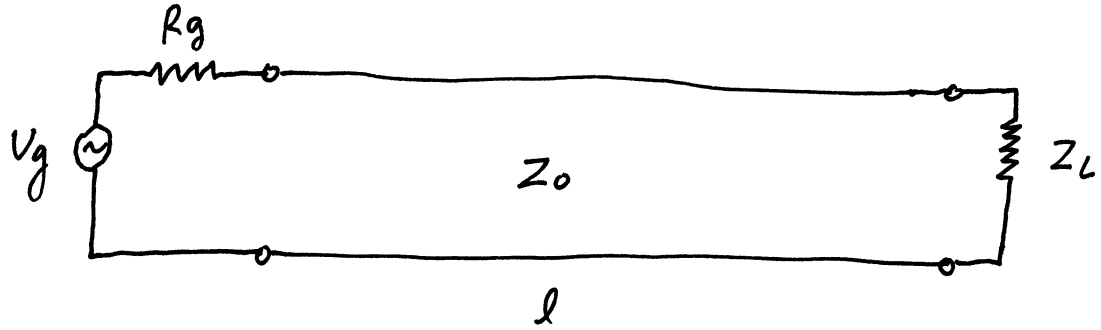
$V_0^- e^{\gamma z}$ — " " " " $-z$ —
(reflection)

• characteristic impedance of a transmission line.

$$Z_0 \triangleq \frac{V_0^+}{I_0^+} = \frac{\text{V of right-going wave } V_0^+ e^{-\gamma z}}{\text{I of right-going wave } I_0^+ e^{-\gamma z}}.$$

$$\begin{aligned} (8.18a) \Rightarrow I_0^+ e^{-\gamma z} &= \frac{-1}{R' + j\omega L} \frac{d}{dz} (V_0^+ e^{-\gamma z}) \\ Z_0 &= \frac{R' + j\omega L}{\gamma} \\ &= \sqrt{\frac{R' + j\omega L}{G' + j\omega C}}. \end{aligned}$$

• Meaning of Z_0



$$\textcircled{1} \quad Z_0 = \frac{V_+}{I_+} = \frac{V_-}{I_-}$$

but in general $Z_0 \neq \frac{V}{I}$

It's the $\frac{V}{I}$ ratio of individual wave, $\rightarrow$ or $\leftarrow$, NOT the total wave!

$$\textcircled{2} \quad Z_0 \neq \text{input impedance, (in general) i.e., } Z_0 \neq \frac{V(z=0)}{I(z=0)}$$

but $Z_0 = \text{input impedance}$ if the line was extended to infinity (i.e., no reflection)

$\textcircled{3}$ Impedance matching

can be shown: If $Z_L = Z_0$, No reflection.
(8-5)

Then, the transmission line can be equivalent to $l \rightarrow \infty$.

That's what you want when you try to send a signal through.

e.g. A parallel-plate transmission line

made of two brass strips : $\sigma_c = 1.6 \times 10^7 \text{ S/m}$.

$$W = 20 \text{ mm} = 0.02 \text{ m} = 2 \times 10^{-2} \text{ m}$$

separated by a lossy
dielectric slab:

$$\begin{cases} d = 0.0025 \text{ m} & (2.5 \text{ mm}) = 2.5 \times 10^{-3} \\ \mu = \mu_0 \\ \epsilon_r = 3 \\ \sigma = 10^{-3} \text{ (S/m)} \end{cases}$$

The operating frequency is $f = 500 \text{ MHz} = 5 \times 10^8 \text{ Hz}$.

a). calculate R' , L' , G' and C'

b). Find γ and Z_0 .

[solution] use the last column of Table 8-1. on page 251.

a). $R_s = \sqrt{\pi f \mu_c / \sigma_c}$ (μ_c, σ_c — properties of conductor)

$$= \left(\frac{\pi \times 5 \times 10^8 \times 4\pi \times 10^{-7}}{1.6 \times 10^7} \right)^{1/2} = 1.11 \times 10^{-2} \Omega$$

$$R' = \frac{2R_s}{W} = \frac{2 \times 1.11 \times 10^{-2}}{0.02} = 1.11 \Omega/\text{m}$$

$$L' = \frac{\mu d}{W} = \frac{4\pi \times 10^{-7} \times 2.5 \times 10^{-3}}{2 \times 10^{-2}} = 1.57 \times 10^{-7} \text{ H/m} = 0.157 \mu\text{H/m}$$

$$G' = \frac{\sigma W}{d} = \frac{(10^{-3}) \times 2 \times 10^{-2}}{2.5 \times 10^{-3}} = 0.008 \text{ S/m}$$

$$C' = \frac{\epsilon W}{d} = \frac{3 \times 8.854 \times 10^{-12} \times 2 \times 10^{-2}}{2.5 \times 10^{-3}} = 2.12 \times 10^{-10} \text{ F/m} = 0.212 \text{ nF/m}$$

b). γ and Z_0 .

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$$

$$\left(\omega = 2\pi f = 2\pi \times (5 \times 10^8) \text{ Hz} \right. \\ \left. = \pi \times 10^9 \text{ rad/s} \right.$$

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

$$\omega L' = \pi \times 10^9 \times 1.57 \times 10^{-7} = 493$$

$$\omega C' = \pi \times 10^9 \times 2.12 \times 10^{-10} = 0.666$$

$$R' + j\omega L' = 1.11 + j493 = 493 e^{j89.87^\circ}$$

$$G' + j\omega C' = 0.008 + j0.666 = 0.666 e^{j89.31^\circ}$$

$$\gamma = [493 e^{j89.87^\circ} \cdot 0.666 e^{j89.31^\circ}]^{1/2}$$

$$= 18.12 e^{j89.59^\circ}$$

$$= 0.130 + j18.12$$

$$Z_0 = \left[\frac{493 e^{j89.87^\circ}}{0.666 e^{j89.31^\circ}} \right]^{1/2}$$

$$= 27.207 e^{j0.28^\circ}$$

$$= 27.21 + j0.13$$