Phys 221.

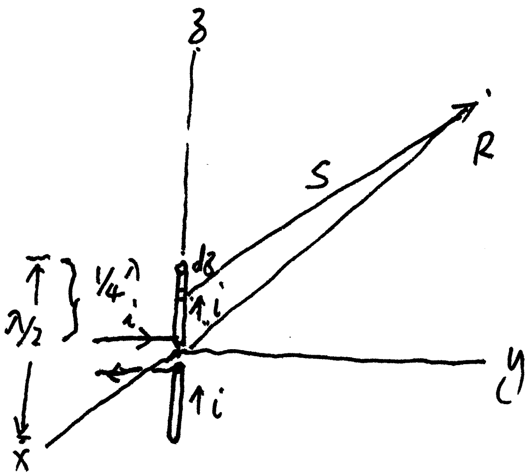
Lecture 34.

• Half-wave dipole antenna

— Idea: $i = I_0 \cos(\omega t)\cos(kz)$

$= \mathrm{Re}\left[I_0 \cos(kz) e^{j\omega t}\right]$

i.e. $\tilde{I}(z) = I_0 \cos kz$.



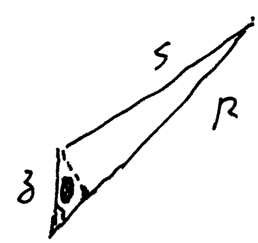
why?

No current at the end: $\tilde{I}(\frac{\lambda}{4}) = \tilde{I}(-\frac{\lambda}{4}) = 0$.

— Analysis: use the Hertzian dipole as elemental current source

$\left[l I_0 \longrightarrow \hat{I}(z)\,dz \right]$ $\quad (\tilde{I}(z)\,dz)$

— Far field: $R \gg \lambda$, $S \approx R - z\cos\theta$



$$d\tilde{E}_\theta(z) = \frac{jk\eta_0}{4\pi} \tilde{I}(z)\,dz\, \frac{e^{-jks}}{s}$$

$$d\tilde{H}_\phi(z) = \frac{d\tilde{E}_\theta(z)}{\eta_0}.$$

Note: why $\frac{1}{R} \approx \frac{1}{S}$, but $e^{-jks} \neq e^{-jkR}$?

$\because$ $z\cos\theta$ can be $> 1\%\,\lambda$! significant! phase!

e.g. $\dfrac{1}{\underline{5000} + \frac{\pi}{2}} \approx \dfrac{1}{5000}$.

the larger this term, the better the approximation.

but $\cos\left(\underline{5000} + \frac{\pi}{2}\right) \neq \cos(5000)$

phase change can be signifirant

No matter how large this term is.

i.e. $e^{jks} = e^{j(kR - kz\cos\theta)} \neq e^{jkR}$

as long as $z\cos\theta > 1\%\ \lambda$.

$1\%\ \lambda$ means $360^\circ \times 1\% = 3.6^\circ$ of phase.

NOT too small!

$$\tilde{E}_\theta = \int_{-\frac{\lambda}{4}}^{\lambda/4} j\,\frac{k\eta_0}{4\pi}\,\tilde{I}(z)\,dz\,\frac{e^{-jkR}}{R}\,\sin\theta \cdot e^{jkz\cos\theta}.$$

$I_0 \cos(kz)$, $\eta_0 = 120\pi$

$$= j\,60\,I_0\,\frac{e^{-jkR}}{R}\left\{\frac{\cos\left[\left(\frac{\pi}{2}\right)\cdot\cos\theta\right]}{\sin\theta}\right\}$$

$$\tilde{H}_\phi = \frac{\tilde{E}_\theta}{\eta_0}$$

No ϕ-dependence!

- Time-average power density

$$S(R,\theta) = \left| \frac{1}{2} \mathrm{Re}\left[\tilde{\vec{E}} \times \tilde{\vec{H}}^* \right] \right|$$

$$= \frac{1}{2} \mathrm{Re}\left[\tilde{E}_\theta \cdot \tilde{H}_\phi^* \right]$$

$$= \frac{\tilde{E}_\theta^2}{2\eta_0}$$

$$= \frac{15 I_0^2}{\pi R^2} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$$

$$= S_0 \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$$

$$S_{max} = S_0 = \frac{15 I_0^2}{\pi R^2}.$$

$$F(\theta,\phi) = F(\theta) = \frac{S(R,\theta)}{S_0} = \left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right]^2$$

- Directivity.

$$D = \frac{4\pi R^2 S_{max}}{P_{rad}}.$$

$$P_{rad} = R^2 \iint S(R,\theta)\, d\Omega = \frac{15 I_0^2}{\pi} \int_0^{2\pi} d\phi \underbrace{\int_0^{\pi} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta} \cdot d\theta}_{\text{Need to be done numerically.}}.$$

$$= 36.6\, I_0^2$$

Then: $D = \frac{4\pi R^2}{36.6 I_0^2} \cdot \frac{15 I_0^2}{\pi R^2} = 1.64$.

$= 10 \cdot \log 1.64 = 2.15\ dB$.

- Radiation resistance.

$$R_{rad} = \frac{P_{rad}}{\frac{1}{2} I_0^2} = \frac{36.6 I_0^2}{0.5 I_0^2} = 73\ \Omega.$$

e.g. In the previous example, if we make the length longer ~~so that~~ $l = 2m$, then it's a $\frac{\lambda}{2}$ dipole. ($\lambda = 4m$).

$R_{loss} = 1.8\ \Omega$ (50 times longer than the short dipole)

Then, radiation efficiency.

$$\xi = \frac{R_{rad}}{R_{loss} + R_{rad}} = \frac{73}{1.8 + 73} \approx 98\%$$

considerably better than the Hertzian dipole (69%).

More important: it's much easier to match the impedance.

$\lambda/2$: $R_{rad} + R_{loss} \approx 75\ \Omega$.

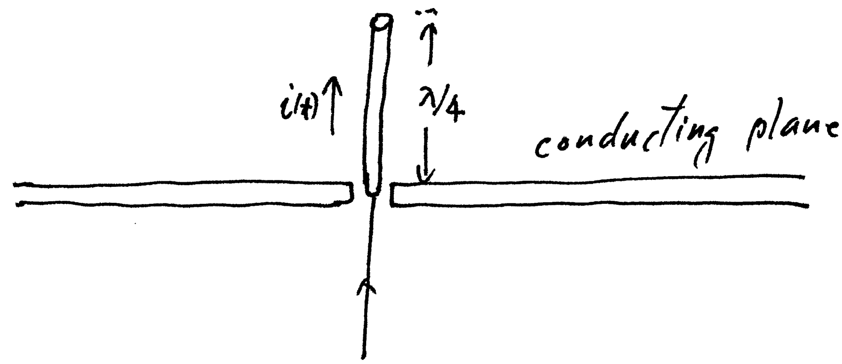
Hertzian: $R_{rad} + R_{loss} \approx 0.1\ \Omega$. (very difficult).

(too small)

- Quarter-Wave Monopole Antenna .

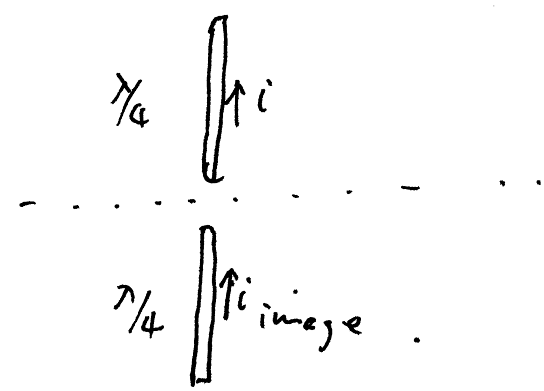
Idea: use the image method .

monopole
$\lambda/4$ antenna
in half-space .



$\Downarrow$ same radiation pattern as : (in the upper half-plane)

$\frac{\lambda}{2}$ antenna
in total space .



but: the radiation is
limited : $0 \leq \theta \leq \frac{\pi}{2}$. (half-space)

(Total power is half as a $\frac{\lambda}{2}$ antenna) .

e.g.,

$P_{rad} = 18.3\, I_0^2$.

$R_{rad} = 36.5\, \Omega$.

Image method can be used for other ~~ant~~ monopole antennas