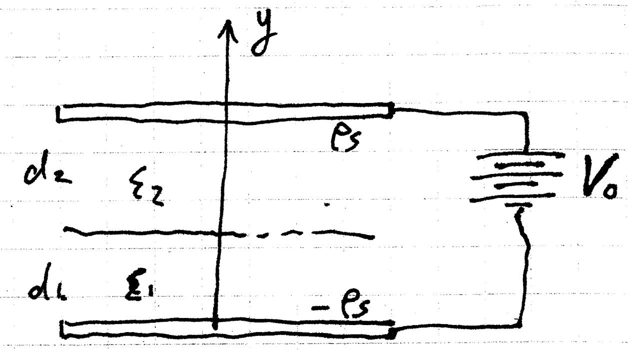
Phys 221 Midterm 2.

Solutions. Summer 2009



1.

(a) . From Gauss' law
(OR Boundary conditions for $\vec{D}$)

$\vec{D}_1 = \vec{D}_2 = -\hat{y}\,\rho_s$.

$$\vec{E}_1 = \frac{\vec{D}_1}{\varepsilon_1} = -\hat{y}\frac{\rho_s}{\varepsilon_1} \quad , \quad \vec{E}_2 = \frac{\vec{D}_2}{\varepsilon_2} = -\hat{y}\frac{\rho_s}{\varepsilon_2}$$

$$V_0 = -\int_0^{d_1+d_2} \vec{E}\cdot d\vec{l} = \frac{\rho_s}{\varepsilon_1}d_1 + \frac{\rho_s}{\varepsilon_2}d_2 = \rho_s\left(\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2}\right)$$

$$\rho_s = \frac{V_0}{\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2}} = \frac{V_0\,\varepsilon_1\varepsilon_2}{d_1\varepsilon_2 + d_2\varepsilon_1}$$

$$\therefore\ \vec{E}_1 = -\hat{y}\frac{V_0\varepsilon_2}{d_1\varepsilon_2 + d_2\varepsilon_1} \quad ; \quad \vec{E}_2 = -\hat{y}\frac{V_0\varepsilon_1}{d_1\varepsilon_2 + d_2\varepsilon_1}$$

(b)
$$C = \frac{Q}{V} = \frac{\rho_s S}{V_0} = \frac{\varepsilon_1\varepsilon_2 S}{d_1\varepsilon_2 + d_2\varepsilon_1}$$

(c) . $J_{1n} = J_{2n}$: $\vec{J}_1 = \vec{J}_2 = -\hat{y}\,J_0$.

$$\vec{E}_1 = \frac{\vec{J}_1}{\sigma_1} = -\hat{y}\frac{J_0}{\sigma_1} \quad , \quad \vec{E}_2 = -\hat{y}\frac{J_0}{\sigma_2}$$

$$V_0 = -\int \vec{E}\cdot d\vec{l} = \frac{J_0}{\sigma_1}d_1 + \frac{J_0}{\sigma_2}d_2 = J_0\left(\frac{d_1}{\sigma_1} + \frac{d_2}{\sigma_2}\right)$$

$$J_0 = \frac{V_0}{\frac{d_1}{\sigma_1} + \frac{d_2}{\sigma_2}} = \frac{V_0\sigma_1\sigma_2}{d_1\sigma_2 + d_2\sigma_1}$$

$$\vec{D}_1 = \varepsilon_1 \vec{E}_1 = -\hat{y}\,\frac{J_0 \varepsilon_1}{\sigma_1}, \qquad \vec{D}_2 = \varepsilon_2 \vec{E}_2 = -\hat{y}\,\frac{J_0 \varepsilon_2}{\sigma_2}$$

$$\vec{D}_1 = -\hat{y}\,\frac{V_0 \varepsilon_1 \sigma_2}{d_1\sigma_2 + d_2\sigma_1}, \qquad \vec{D}_2 = -\hat{y}\,\frac{V_0 \varepsilon_2 \sigma_1}{d_1\sigma_2 + d_2\sigma_1}$$

ρ_V : $\quad \rho_v = \nabla \cdot \vec{D} = 0$

ρ_s : upper plate : $\quad \rho_{su} = 0 - D_{2y} = \dfrac{V_0 \varepsilon_2 \sigma_1}{d_1\sigma_2 + d_2\sigma_1}$

lower plate : $\quad \rho_{sl} = D_{1y} - 0 = \dfrac{-V_0 \varepsilon_1 \sigma_2}{d_1\sigma_2 + d_2\sigma_1}$

at boundary between σ_1 and σ_2 :

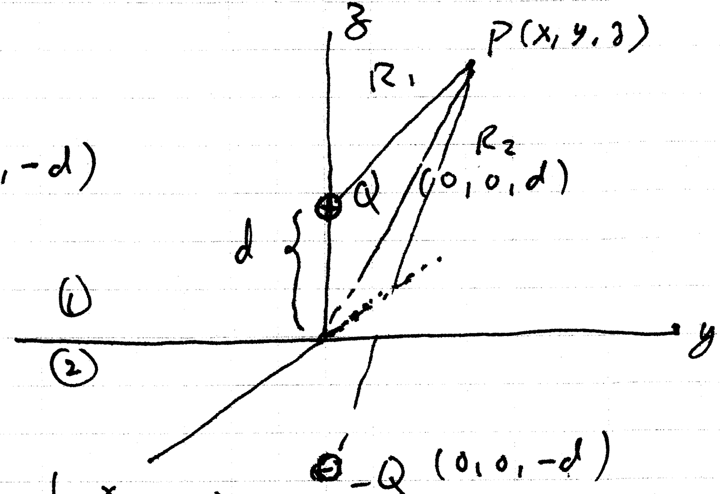
$$\rho_{12} = D_{2y} - D_{1y}$$

$$= -\frac{V_0 \varepsilon_2 \sigma_1}{d_1\sigma_2 + d_2\sigma_1} + \frac{V_0 \varepsilon_1 \sigma_2}{d_1\sigma_2 + d_2\sigma_1}$$

$$= \frac{V_0 (\varepsilon_1\sigma_2 - \varepsilon_2\sigma_1)}{d_1\sigma_2 + d_2\sigma_1}$$

2.

Image charge : $-Q$ at $(0,0,-d)$



a) $V(x,y,z) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R_1} - \frac{Q}{R_2}\right)$

$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{x^2+y^2+(z-d)^2}} - \frac{1}{\sqrt{x^2+y^2+(z+d)^2}}\right)$

b) $\vec{E}(x,y,z) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{\hat{x}x+\hat{y}y+\hat{z}(z-d)}{[x^2+y^2+(z-d)^2]^{3/2}} - \frac{\hat{x}x+\hat{y}y+\hat{z}(z+d)}{[x^2+y^2+(z+d)^2]^{3/2}}\right)$

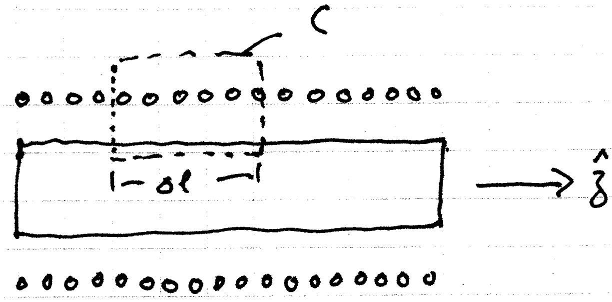
c) $\rho_s = D_{1n} - D_{2n} = D_{1z} = E_z(x,y,0)$

$= \frac{Q}{4\pi\varepsilon_0}\left\{\frac{z-d}{[x^2+y^2+(z-d)^2]^{3/2}} - \frac{z+d}{[x^2+y^2+(z+d)^2]^{3/2}}\right\}$

$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{-d}{(x^2+y^2+d^2)^{3/2}} - \frac{d}{(x^2+y^2+d^2)^{3/2}}\right)$

$= \frac{-Qd}{2\pi\varepsilon_0}\frac{1}{(x^2+y^2+d^2)^{3/2}}$

3.



a). $\vec{H} = \hat{z} H$ inside solenoid.

$\vec{H} = 0$ outside solenoid.

Ampere's law: $\oint \vec{H} \cdot d\vec{l} = nI \cdot \delta l = H \cdot \delta l$.

$\therefore \quad \vec{H} = \hat{z} nI \quad$ (for $r < b$)

(including $r < a$ and $a < r < b$)

(b). $\vec{B} = \mu \vec{H} = \hat{z} \mu n I \quad (r < a)$

$\vec{B} = \mu_0 \vec{H} = \hat{z} \mu_0 n I \quad (a < r < b)$

$\vec{M} = \dfrac{\vec{B}}{\mu_0} - \vec{H}$: $\quad r < a$: $\quad \vec{M} = \hat{z} \dfrac{\mu n I}{\mu_0} - \hat{z} n I = \hat{z} n I \left(\dfrac{\mu}{\mu_0} - 1\right)$

$a < r < b$: $\quad \vec{M} = \hat{z} \dfrac{\mu_0 n I}{\mu_0} - \hat{z} n I = 0$

(c) Boundary condition: $B_{1n} = B_{2n}$. $\quad (\hat{n} \parallel \hat{z})$

$\therefore \quad B_{inside} = B_{outside}$

$\therefore \quad \vec{B} = \hat{z} \mu n I$.

$\vec{H} = \dfrac{\vec{B}}{\mu_0} = \hat{z} \dfrac{\mu}{\mu_0} n I$.

4.

(a). $\Phi = \int \vec{B} \cdot d\vec{s}$

$= A \cdot B \cdot \cos\theta$

$= (0.2)^2 \cdot (0.5) \cos 60°$

$= 0.01$ (web).

(b). $V_{emf} = -N \cdot \frac{d\Phi}{dt}$ $(N=1)$

$|V_{emf}| = \left|\frac{\Delta\Phi}{\Delta t}\right| = \frac{0.01-0}{0.2} = 0.05$ V.

(c). C.C.W.

(Since B is decreasing, the induced current should ~~lead to a~~ produce a $\vec{B}$ field in the same direction as the original $\vec{B}$.

i.e. against the decrease of Φ)

5. $\vec{E} = \hat{x}\, 10 \cos(10^9 t + 5z)$

(a). $-\hat{z}$

(b). $\omega = 2\pi f = 10^9, \quad f = \dfrac{10^9}{2\pi} = 1.6 \times 10^8\ Hz.$

(c). $k = \dfrac{2\pi}{\lambda} = 5, \quad \lambda = \dfrac{2\pi}{5} = 1.26\ m$

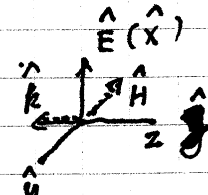
(d). $u_p = \dfrac{\omega}{k} = \dfrac{10^9}{5} = 2\times 10^8 = \dfrac{1}{\sqrt{\mu\varepsilon}} = \dfrac{1}{\sqrt{\mu_0\varepsilon_0\mu_r\varepsilon_r}}$

$$2\times 10^8 = \frac{c}{\sqrt{\mu_r\varepsilon_r}} = \frac{3\times 10^8}{\sqrt{2}\cdot\sqrt{\varepsilon_r}}.$$

$$\sqrt{\varepsilon_r} = \frac{3\times 10^8}{\sqrt{2}\times 2\times 10^8}$$

$$\varepsilon = \frac{9}{8} = 1.125$$

(e). $\tilde{\vec{H}} = \dfrac{1}{\eta}\, \hat{k} \times \tilde{\vec{E}}, \quad \left(\hat{k} = -\hat{z}, \quad \hat{E} = \hat{x} \quad \therefore \hat{H} = -\hat{y} \right.$



$$= -\hat{y}\sqrt{\frac{\varepsilon}{\mu}}\, 10\, e^{j5z}$$

$$= -\hat{y}\sqrt{\frac{\varepsilon_0\varepsilon_r}{\mu_0\mu_r}}\, 10\, e^{j5z}.$$

$$= -\hat{y}\sqrt{\frac{8.854\times 10^{-12}\times 1.125}{4\pi\times 10^{-7}\times 2}}\, 10\, e^{j5z}$$

$$= -\hat{y}\, 0.02\, e^{j5z}.$$