Phys221. Electromagnetics

Lecture 1. Introduction, vectors.

- Course Introduction

  Course website: www.sfu.ca/~mxchen/ → phys 221.

  Instructor: Michael CHEN. mxchen@sfu.ca.

  Office hours: (P9442) Tue. 2:30 – 5:20

  (+ by appointment).

  TA: Kamran Kaveh-Maryan. Kaveh@sfu.ca.

  Textbook: Required.

  Grading: 5% attendance of Tutorials.

  5% homework.

  20% × 2. midterms.

  50% final exam (or more)

- New in this year:

  Midterm 1. covers more.

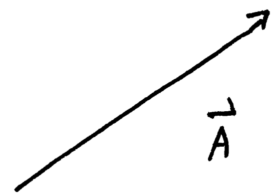
  more on ch.10. (Radiation and Antennas).

- Before coming to lectures:

  Read the ref. at least briefly. (see course calendar).

Review of vectors

- Vector $\vec{A}$



magnitude : $A = |\vec{A}|$ (length)

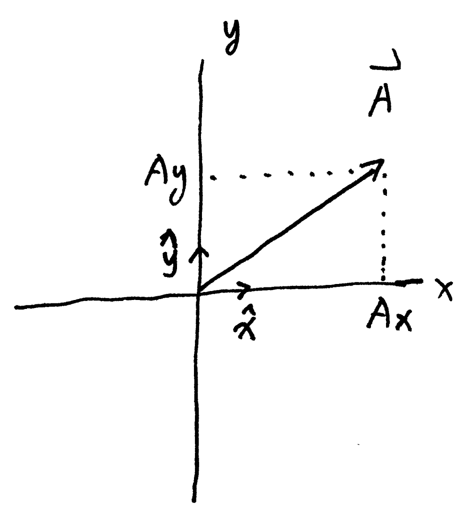
direction : $\hat{a}$ (unit vector along $\vec{A}$)

└ length = 1.

$$\therefore \quad \hat{a} = \frac{\vec{A}}{A}$$

$$\vec{A} = A\,\hat{a}.$$

- Cartesian components



$$\vec{A} = A_x\,\hat{x} + A_y\,\hat{y}$$

$\hat{x}$, $\hat{y}$ — base vectors

($\vec{i}$, $\vec{j}$).

A_x, A_y — components.

3-D: $\vec{A} = A_x\,\hat{x} + A_y\,\hat{y} + A_z\,\hat{z}$.

- Addition and subtraction of vectors

Method 1 : Tip-to-tail

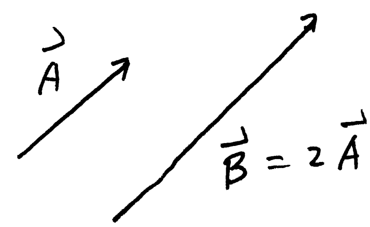
Method 2 : components — most useful.

why? { physics : { components are Independent
Math : easier . (No geometric drawing)

- Multiplications .

① Simple product

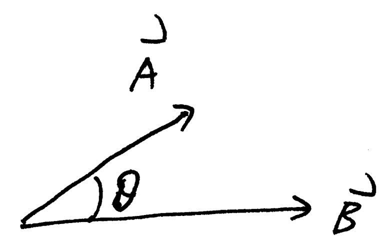
$$\vec{B} = \underbrace{k}_{\text{scalar}} \vec{A}$$



② Dot product . (scalar product) .

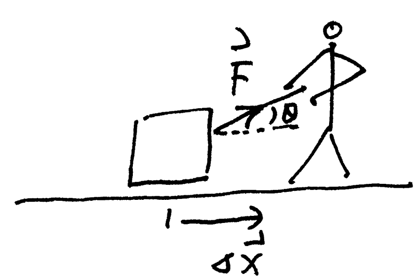
$$\vec{A} \cdot \vec{B} = \underline{A B \cos\theta}$$

↑ Result is a scalar .



e.g. work $= \vec{F} \cdot \Delta\vec{x}$

$= F \cdot |\Delta x| \cdot \cos\theta$



Component form :

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z .$$

Note : $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$ (commutative)

- Cross product (vector product)

$$\vec{A} \times \vec{B} = \hat{n}\, A B \sin\theta$$

$\hat{n}$ — normal to the plane containing $\vec{A}$ and $\vec{B}$

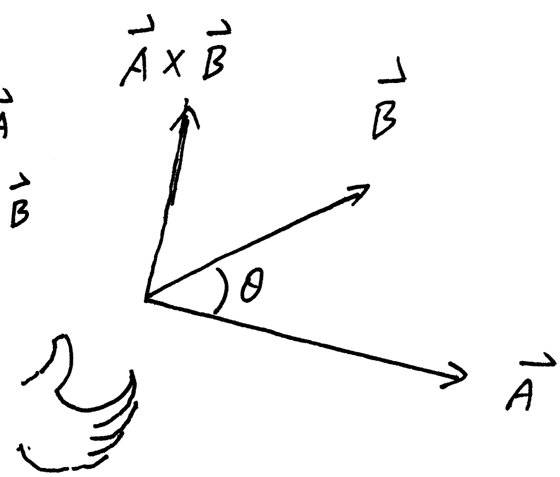
$\vec{A} \times \vec{B}$: The result is a vector

Direction obeys the right-hand rule

magnitude $= A \cdot B \cdot \sin\theta$.

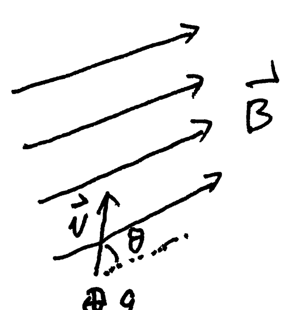
$\hat{n} \perp \vec{A}$: $(\vec{A} \times \vec{B}) \perp \vec{A}$

$\hat{n} \perp \vec{B}$: $(\vec{A} \times \vec{B}) \perp \vec{B}$



[Then. $(\vec{A} \times \vec{B}) \cdot \vec{A} = 0$].

e.g. of cross product : A charge q moves in a magnetic field $\vec{B}$. The force on q is:



$$\vec{F} = q\, \vec{v} \times \vec{B}$$

$\vec{F}$: into the page .

$$F = q v B \sin\theta$$

Note : $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

In fact, from the right-hand rule:

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A} \quad \text{(anticommutative)}$$

Component form: $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

- Scalar triple product

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

- Vector triple product.

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}\,(\vec{A} \cdot \vec{C}) - \vec{C}\,(\vec{A} \cdot \vec{B})$$

(can be verified using components)

Note: $\vec{A} \times (\vec{B} \times \vec{C}) \neq (\vec{A} \times \vec{B}) \times \vec{C}$

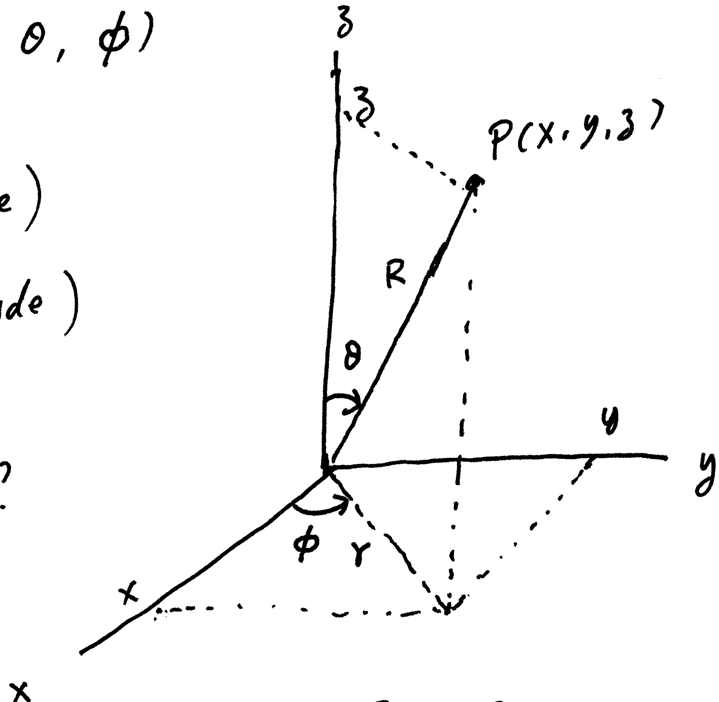
(Doesn't obey the associative law).

- 3 commonly used coordinate systems

  Cartesian (x, y, z)

  cylindrical (r, ϕ, z)

  spherical (R, θ, ϕ)



θ — zenith (latitude)

ϕ — azimuth (longitude)

$r = R\sin\theta$.

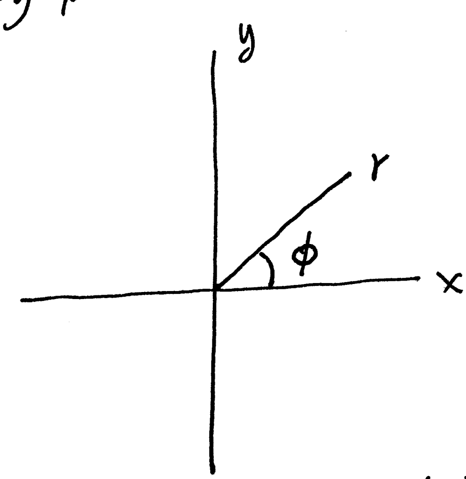
- why different coordinates?

  simplify the description of problems by choosing a coordinate system that best fits the geometry of the problem.

  e.g: coaxil cable : use cylindrical system.

  e.g: charged sphere : use spherical system.

On x-y plane.



(r, ϕ) — polar coordinate.

(spherical system : $(R\sin\theta, \phi)$)

- Table 2-1 on page 32. (Will be provided in exams)

Note 1: Base vectors : $\hat{x}$, $\hat{y}$, $\hat{z}$ cartesian

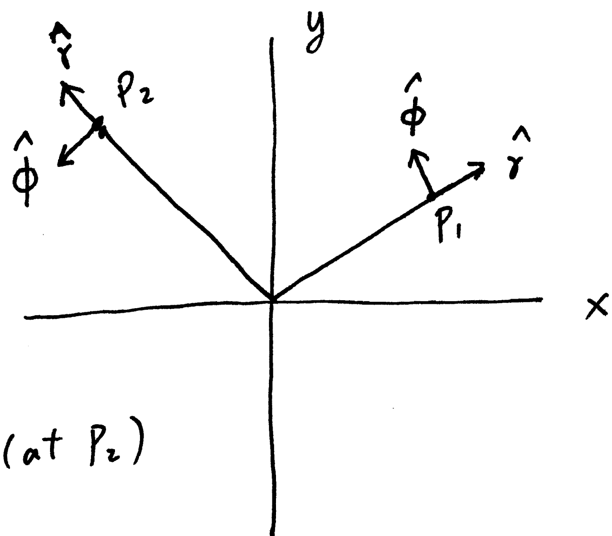
$\hat{r}$, $\hat{\phi}$, $\hat{z}$ cylindrical

$\hat{R}$, $\hat{\theta}$, $\hat{\phi}$ spherical

$\hat{r}$, $\hat{\phi}$, $\hat{R}$, $\hat{\theta}$ — NOT constant.

i.e. direction varies with location.

e.g. $\hat{r}$, $\hat{\phi}$ (viewed on x-y plane : polar coord.)



$\hat{r}$ (at P_1) $\neq$ $\hat{r}$ (at P_2)

$\hat{\phi}$ (at P_1) $\neq$ $\hat{\phi}$ (at P_2).

Note 2. Differential length $d\vec{l}$

$\hat{x}\,dx + \hat{y}\,dy + \hat{z}\,dz$ — cartesian

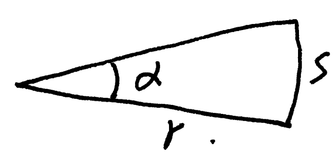
$\hat{r}\,dr + \hat{\phi}\,r\,d\phi + \hat{z}\,dz$ — cylindrical

$\hat{R}\,dR + \hat{\theta}\,R\,d\theta + \hat{\phi}\,R\sin\theta \cdot d\phi$ — spherical.

Watch out : $d\phi$, $d\theta$ are angles, NOT length !

but $r\,d\phi$, $R\sin\theta\,d\phi$ and $R\,d\theta$ are lengths.

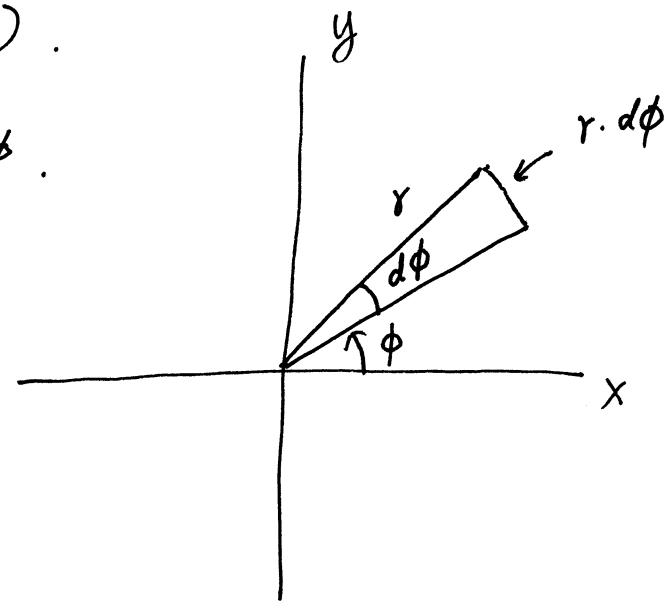
why ? arc length



$S = r \cdot \alpha$ if α is in radians.

e.g: 2-D (polar coord.).

small arc length: $r \cdot d\phi$.



Note 3 : conversion between different coordinate systems

Table 2-2 on page 39 — will be provided in Exams.