

• Gauss' Law.

$$\nabla \cdot \vec{D} = \rho_v \quad (4.26)$$

$$\oint_S \vec{D} \cdot d\vec{s} = Q \quad (4.29)$$

$$\vec{D} = \epsilon \vec{E} = \epsilon_0 \epsilon_r \vec{E} \quad \text{for isotropic and uniform medium}$$

ρ_v — free charge per unit volume.

e.g.: A long straight wire with a uniform linear charge density ρ_ℓ .

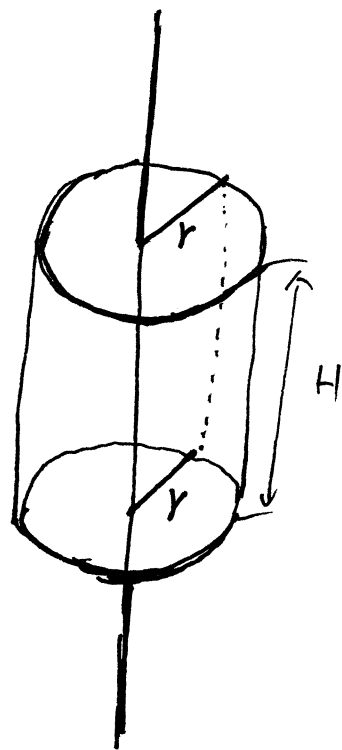
Find $\vec{E}$ field.

Gaussian surface.

by symmetry: $\vec{E} = \hat{r} E_r$.

$\vec{D} \cdot d\vec{s} = 0$ on top and bottom.

$E_r = \text{const}$ on lateral surface $\Rightarrow E$



$$Q = \oint_S \vec{D} \cdot d\vec{s} = \oint_S \epsilon_0 \vec{E} \cdot d\vec{s}$$

$$= \epsilon_0 E_r \cdot 2\pi r \cdot H$$

$$E_r = \frac{Q}{2\pi \epsilon_0 H} = \frac{\rho_\ell}{2\pi \epsilon_0}$$

use cylindrical coord.

(same as previous example)

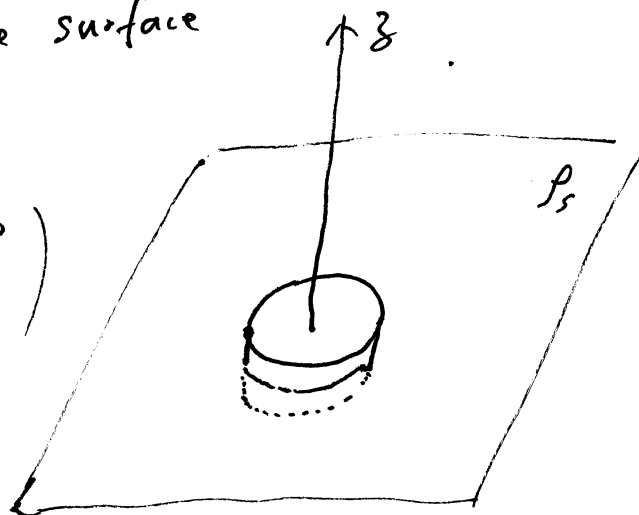
e.g. For a large flat sheet of charge with a uniform area density of charge ρ_s .

Find $\vec{E}$ -field near the surface

by symmetry.

$$\vec{E}_x = 0, \quad E_y = 0 \quad \left(\begin{array}{l} E_r = 0 \\ E_\phi = 0 \end{array} \right)$$

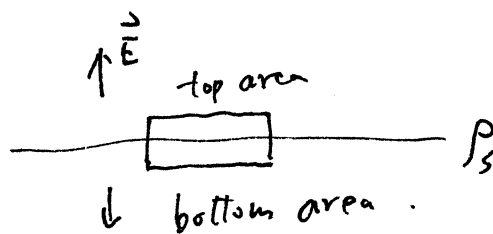
Thin drum-shaped Gaussian surface.



$$\oint_S \vec{D} \cdot d\vec{S} = \epsilon_0 \oint_S \vec{E} \cdot d\vec{S} = Q.$$

Since $E_r = 0$

$\vec{E} \cdot d\vec{S} = 0$ on lateral area. $\vec{E}$



on top and bottom: $\vec{E} \parallel d\vec{S}$ also, $E = \text{const.}$

$$\begin{aligned} \therefore Q &= \epsilon_0 \oint_S \vec{E} \cdot d\vec{S} = \epsilon_0 (E A_{\text{Top}} + E A_{\text{bottom}}) \\ &= 2 \epsilon_0 E A. \end{aligned}$$

$$E = \frac{Q}{2 \epsilon_0 A} = \frac{\rho_s}{2 \epsilon_0} \quad \left(\text{same as previous example} \right)$$

3(5/15). A very long solid cylinder with a radius R is uniformly charged. The charge density (charge per unit volume) is ρ . Figure A below depicts its cross section.

(a) Determine the electric field E at point P. The distance from the center of the rod to point P is $2R/3$.

(b) If the rod has a cylindrical hole along the rod, with a radius $R/2$, as shown in figure B below, determine the electric field at point P ($2R/3, 0$). Note that the centre of the rod, the centre of the hole and point P are all on the x-axis.

(c) If the cylindrical hole inside the rod has a radius w and the centre of the hole is located at (a, b) , find the electric field at an arbitrary point (x, y) inside the hole.

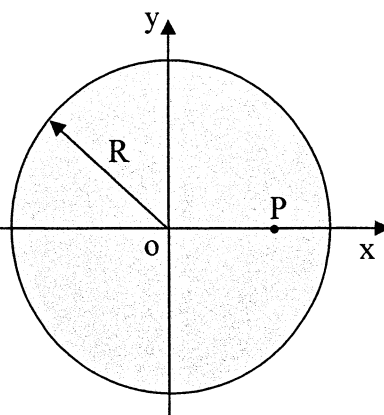
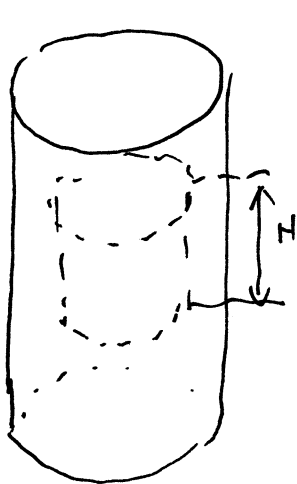


Figure A

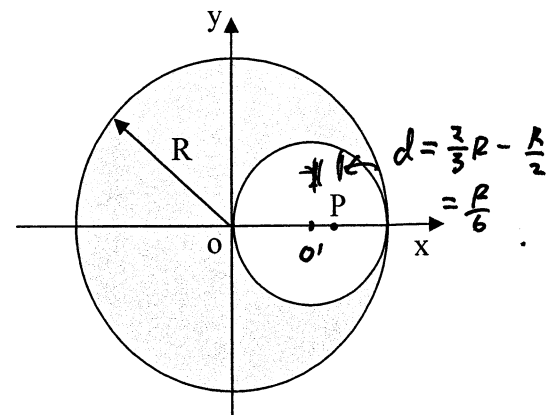


Figure B

(a) Use Gauss' law.

$$\frac{Q}{\epsilon_0} = \oint \vec{E} \cdot d\vec{S} = \int_{\text{lateral}} \vec{E} \cdot d\vec{S} = E_r \cdot 2\pi r H = \frac{\rho \pi r^2 H}{\epsilon_0}$$

$$\therefore \vec{E} = \hat{r} E_r = \hat{r} \frac{\rho r}{2\epsilon_0} = \frac{\rho \vec{r}}{2\epsilon_0} \quad \left(\begin{array}{l} E_z = 0, E_\phi = 0 \\ \text{from symmetry} \end{array} \right)$$

$$= \hat{r} \frac{\rho R}{3\epsilon_0} \quad (r = \frac{2R}{3})$$

~~(b)~~

$$(b) \quad \vec{E} = \vec{E}_+ + \vec{E}_- \quad \left(\text{hole} = \text{positive charge} + \text{negative charge} \right)$$

$$\text{at P, } \hat{r} = \hat{x}$$

$$\vec{E} = \hat{x} \left(\frac{\rho R}{3\epsilon_0} - \frac{\rho}{2\epsilon_0} \cdot \frac{R}{6} \right) = \hat{x} \frac{\rho R}{\epsilon_0} \left(\frac{1}{3} - \frac{1}{12} \right) = \hat{x} \frac{\rho R}{4\epsilon_0}$$

$$(c) \quad \vec{E} = \vec{E}_+ + \vec{E}_- = \left(\frac{\rho \vec{r}_+}{2\epsilon_0} - \frac{\rho \vec{r}_-}{2\epsilon_0} \right)$$

$$= \frac{\rho}{2\epsilon_0} (\vec{r}_+ - \vec{r}_-) = \frac{\rho}{2\epsilon_0} \vec{r}_0 = \frac{\rho}{2\epsilon_0} (\hat{x}a + \hat{y}b)$$

$\vec{r}_0$ — center of hole.

