Phys 221

Lecture 10.

- Ohmic media. — conductor, but not perfect.

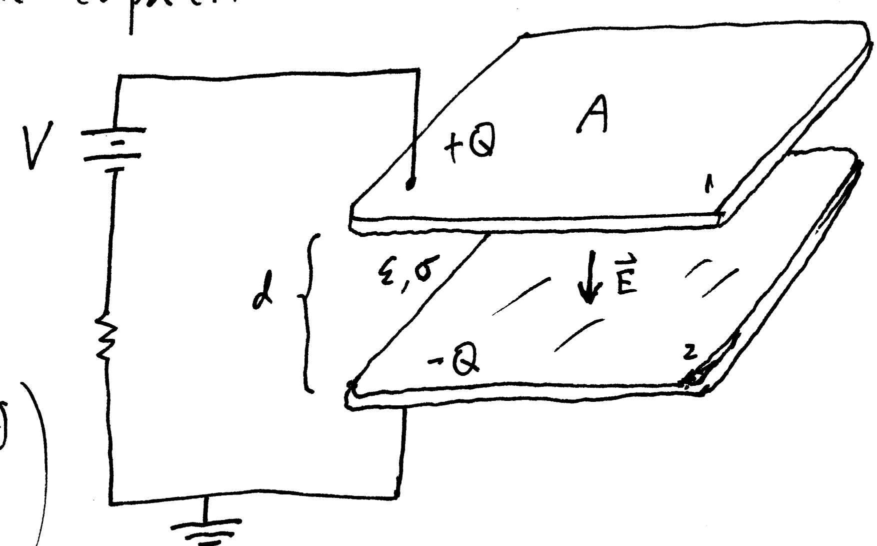
$\sigma \neq 0$, $\sigma \neq \infty$: $\vec{E} \neq 0$, $\vec{J} \neq 0$, current can flow.

We can have a steady state, but not static equilibrium.

i.e, charge can move, but $\vec{J}, \vec{E}, \vec{D}$, are t-independent.

- e.g. Parallel plate capacitor

— capacitance $C = ?$



$$C \equiv \frac{Q}{V}.$$

(cabability of holding charge Q at the expense of V)

Q: Gauss' law: $Q = \int_A \varepsilon \vec{E} \cdot d\vec{s} = \varepsilon E A$. (large A)

$$E = \frac{Q}{\varepsilon A} = \frac{\rho_s}{\varepsilon}$$

V: $V = V_1 - V_2 = \int_1^2 \vec{E} \cdot d\vec{l} = E d.$

$[-(V_2 - V_1)]$

$$\therefore C = \frac{Q}{V} = \frac{\varepsilon A}{d}.$$

why not $\frac{\rho_s}{2\varepsilon}$?

- How to increase C : Increase A

  decrease d

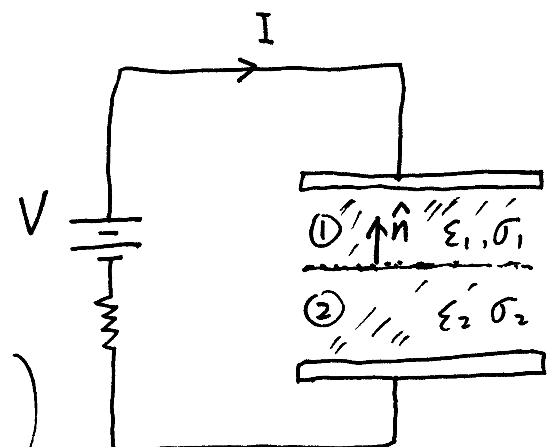
  increase ε — use medium.

— Resistance $R = \rho \frac{d}{A}$ ρ – resistivity $\rho = \frac{1}{\sigma}$

$$= \frac{d}{\sigma A}$$

$$\therefore \quad \boxed{R \cdot C = \frac{\varepsilon}{\sigma}} \qquad 4.111$$

- e.g. 2 layers of media in the capacitor



In media:

$\vec{J} \neq 0$, $\vec{J} = \sigma \vec{E}$

E_t – continuous:

$$\frac{J_{1t}}{\sigma_1} = \frac{J_{2t}}{\sigma_2} \qquad \left(\frac{\vec{J}}{\sigma} = \vec{E}\right)$$

D_n – B.C.

$$\varepsilon_1 \frac{J_{1n}}{\sigma_1} - \varepsilon_2 \frac{J_{2n}}{\sigma_2} = \rho_s$$

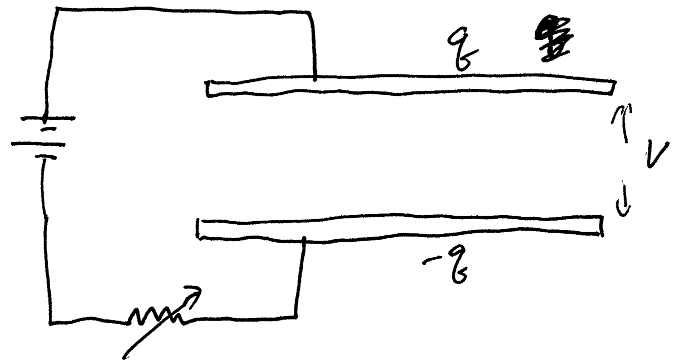
ρ_s – free charge surface density at boundary

Continuity conditions for steady current (e.g. D.C.)

$$J_{1n} = J_{2n} = J_n$$

$$\therefore \quad J_n \left(\frac{\varepsilon_1}{\sigma_1} - \frac{\varepsilon_2}{\sigma_2}\right) = \rho_s .$$

- Electric potential Energy (4-11).
  stored in a capacitor.



$$W_e = \frac{1}{2} QV$$
$$= \frac{1}{2} CV^2$$
$$= \frac{1}{2} \frac{Q^2}{C}.$$

Question: why not $W_e = QV$? $\qquad C = \frac{Q}{V}$

V — P.E. per unit charge !

where does the "$\frac{1}{2}$" come from ?

Reason: for a given C, V varies with q. $\left(V = \frac{q}{C}\right)$

As we charge the capacitor, q increase from 0 to Q.

v also increase from 0 to V.

$$\therefore \quad W_e = \int_0^Q v\, dq = \int_0^Q \frac{q}{C}\, dq = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2} QV = \frac{1}{2} CQ^2 = \frac{1}{2} CV^2.$$

- electric field: $\quad E \cdot d = V, \quad E = \frac{V}{d}$

$$W_e = \frac{1}{2} CV^2 = \frac{1}{2} \frac{\varepsilon A}{d} \cdot E^2 d^2 = \frac{1}{2} \varepsilon \underbrace{Ad}_{\text{volume}} \cdot E^2.$$

Electric field energy density:

$$\boxed{w_e = \frac{1}{2} \varepsilon E^2} \qquad 4\text{-}123.$$