Phys 221

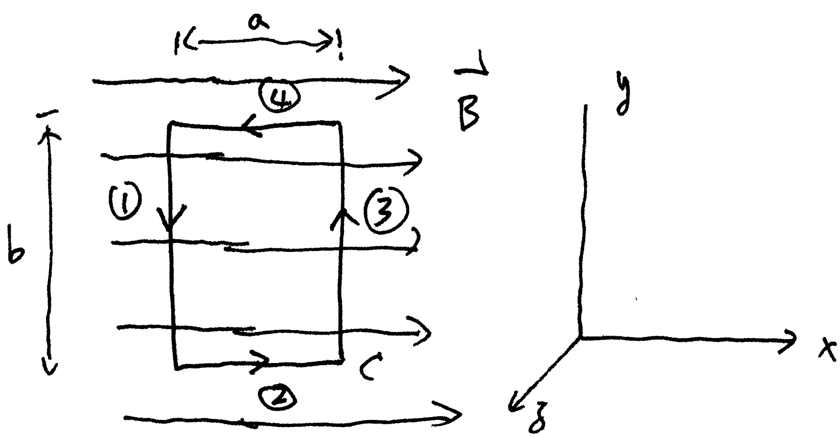
Lecture 12

Ref: 5-1, 5-2.

e.g. Magnetic force on a rectangular current loop

Given magnetic field: $\vec{B} = \hat{x} B_0$ (constant).

$$\vec{F}_m = I \cdot \oint_C d\vec{l} \times \vec{B}$$



on ② and ④

$I d\vec{l} \parallel \vec{B}$.

$I d\vec{l} \times \vec{B} = 0$.

$\therefore \vec{F}_2 = 0, \quad \vec{F}_4 = 0$. $\quad (\vec{A} \times \vec{B} = 0$ if $\vec{A} \parallel \vec{B}$, $\because \sin 0° = 0)$

$$\vec{F}_1 = I \int_{①} d\vec{l} \times \vec{B} = \hat{z}\, I b B_0 .$$

$$\vec{F}_2 = -\hat{z}\, I b B_0 .$$

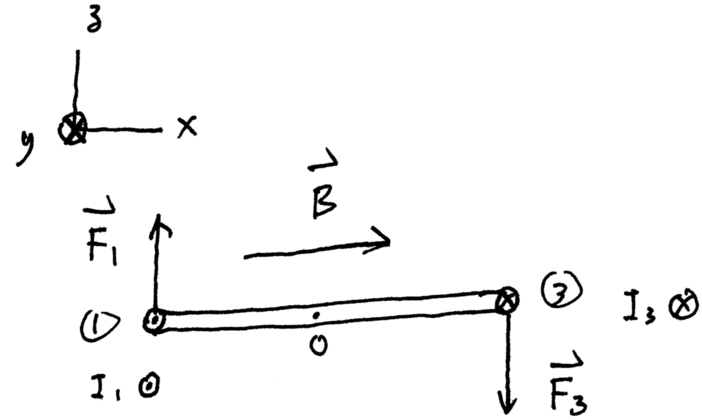
Total force: $\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 0$.

Centre of mass doesn't move.

But, Torque $\neq 0$ $\quad (T = \vec{r} \times \vec{F})$

So it can rotate.

view from bottom:



Torques about O:

$$\vec{T_1} = \left(-\hat{x}\frac{a}{2}\right) \times \hat{z}\, I b B_0$$

$$= \hat{y}\,\frac{ab}{2}\, I B_0$$

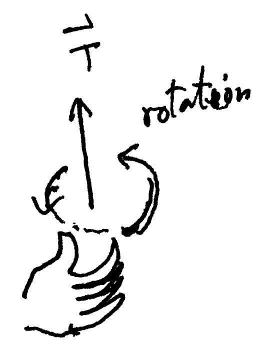
$$\vec{T_3} = \left(\hat{x}\frac{a}{2}\right) \times \left(-\hat{z}\, I b B_0\right)$$

$$= \hat{y}\,\frac{ab}{2}\, I B_0 = \vec{T_1}$$

Total torque: $\vec{T} = \vec{T_1} + \vec{T_2} = \hat{y}\,\underbrace{ab}_{\text{area of loop}} \cdot I \cdot B_0 = \hat{y}\, A \cdot I \cdot B_0$

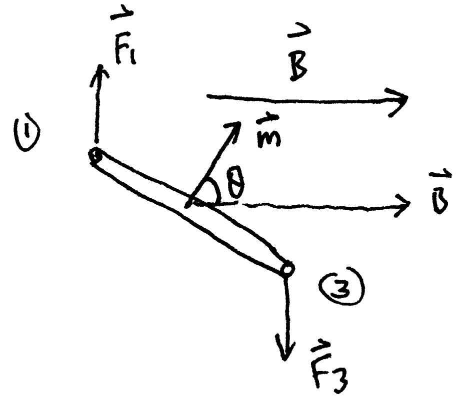
- Define magnetic moment: $\vec{m} \equiv I \cdot \vec{A}$ $\quad (\hat{A} - \text{normal})$

In general: $\boxed{\vec{T} = \vec{m} \times \vec{B}}$ $\quad (5.20)$



The loop will rotate about $\underline{y\text{-axis}}$

axis of rotation.



$\vec{T}$: direction: $\otimes$ — meaning: RHR.

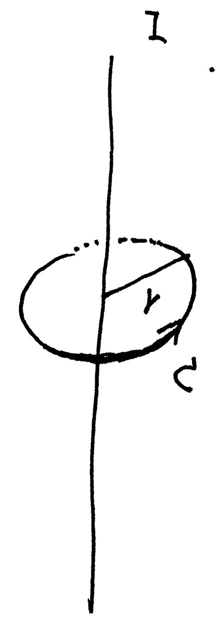
magnitude: $mB\sin\theta = T$.

· Magneto statics

Ampere's law : $\oint_C \vec{H} \cdot d\vec{l} = I_{free}$.

$(\vec{B} = \mu \vec{H})$

e.g. $\vec{B}$ field of a current of a long straight line .



Along C : $\vec{B} \parallel \hat{\phi}$.

$B = \text{constant}$.

Then:

$\cancel{B \cdot 2\pi r\ I}$

$$\oint_C \vec{B} \cdot d\vec{l} = \mu I .$$

$$B \int_C dl = \mu I .$$

$$B \, 2\pi r = \mu I .$$

$$\therefore \quad \vec{B} = \hat{\phi} \frac{\mu I}{2\pi r} .$$

just like Gauss' law in electrostatics .

You need symmetry !

(for $\oint_C \vec{B} \cdot d\vec{l}$) to make $B = \text{const}$.)

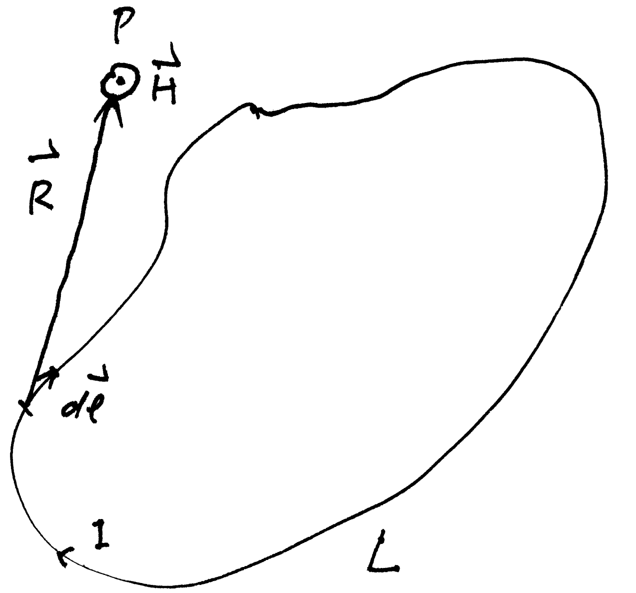
- The Biot-Savart law.

$$d\vec{H} = \frac{I}{4\pi} \frac{\vec{dl} \times \vec{R}}{R^2}$$

(due to $I\vec{dl}$)

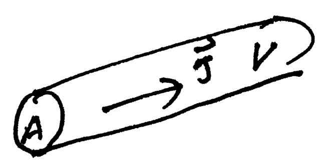
$$\vec{H} = \frac{I}{4\pi} \int_L \frac{\vec{dl} \times \hat{R}}{R^2}$$

(Similar to Coulomb's law in electro statics)



$\vec{R}$ — source to field.

- For a current distribution $\vec{J}$



$$I = \int_A \vec{J} \cdot d\vec{A}$$

note: $I\vec{dl} = \vec{J}\, dV$

$\because$ $\vec{J} dV = (\vec{J} \cdot d\vec{A}) \cdot \vec{dl}$

$= I \cdot \vec{dl}$.

(Note: $\vec{J} \parallel \vec{dl}$)

$$\therefore \boxed{\vec{H} = \frac{1}{4\pi} \int \frac{\vec{J} \times \hat{R}}{R^2} dV} \quad (5.24)$$

For surface current:

$$\vec{H} = \frac{1}{4\pi} \int \frac{\vec{J}_s \times \hat{R}}{R^2} dS \quad (5.34)$$