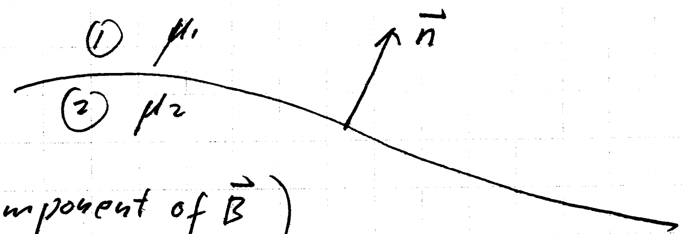
Phys 221

Lecture 17. Ref. 5-7.8.9.

- Magnetic Boundary conditions.

(normal: $\hat{n}$ · from ② to ①.)



$B_{1n} = B_{2n}$ (Normal component of $\vec{B}$ is continuous).

$H_{2t} - H_{1t} = J_s$.

└ free surface current.

Vector form: $\hat{n} \cdot (\vec{B}_1 - \vec{B}_2) = 0$ 5.79.

$\hat{n} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s$. 5.84

- why?

$\oint_S \vec{B} \cdot d\vec{S} = 0. \Longrightarrow B_{1n} = B_{2n}.$

(No: electrostatics: $\oint_S \vec{D} \cdot d\vec{S} = Q \Longrightarrow D_{1n} - D_{2n} = \rho_s$

Since there is no "magnetic charge", $B_{1n} - B_{2n} = 0$)

Also, $\oint_C \vec{E}\cdot d\vec{l} = 0 \implies E_{1t} = E_{2t}$.

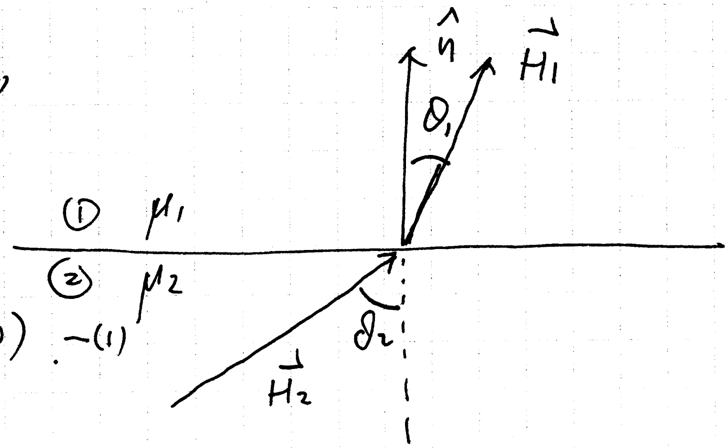
Now $\oint_C \vec{H}\cdot d\vec{l} = I$. — tangential component of $\vec{H}$ is NOT continuous if there is free current at the boundary.

$$H_{2t} - H_{1t} = J_s.$$

If $\vec{J_s} = 0$, $H_{2t} = H_{1t}$.

e.g. when there is no surface free current at Boundary. $\vec{J_s} = 0$

what's the relationship between θ_1 and θ_2?



$H_{1t} = H_{2t}$. $(J_s = 0)$ —(1)

$B_{1n} = B_{2n}$.

$\mu_1 H_{1n} = \mu_2 H_{2n}$. —(2)

$$\tan\theta_2 = \frac{H_{2t}}{H_{2n}}, \quad \tan\theta_1 = \frac{H_{1t}}{H_{1n}}.$$

$\frac{(1)}{(2)}$: $$\frac{H_{1t}}{\mu_1 H_{1n}} = \frac{H_{2t}}{\mu_2 H_{2n}} \implies \frac{1}{\mu_1}\tan\theta_1 = \frac{1}{\mu_2}\tan\theta_2.$$

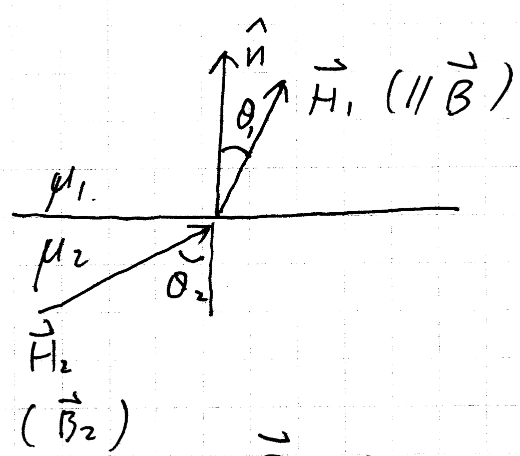
$$\therefore \frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}.$$

~~Also, last lecture~~

Boundary conditions: $B_{1n} = B_{2n}$

$H_{1t} = H_{2t}$ (if $J_s = 0$)

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}.$$



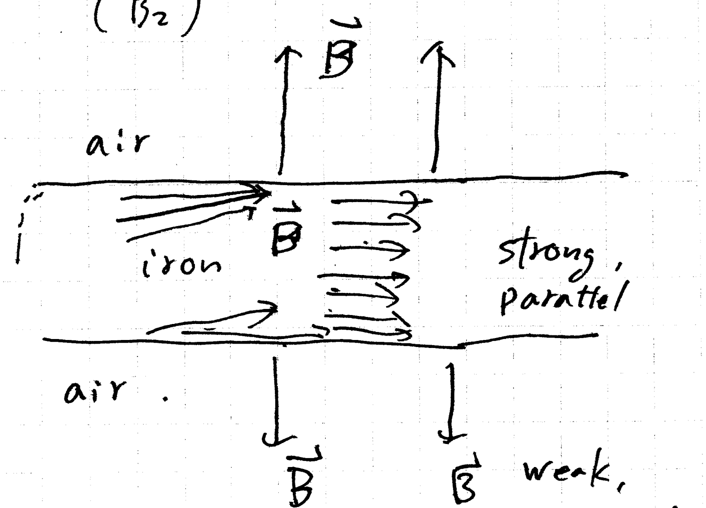
for $\mu_1 = 1$ (air)

$\mu_2 \gg 1$. (iron $\sim 10^5$)

$$\frac{\mu_1}{\mu_2} \sim 0.$$

$\theta_1 \sim 0$

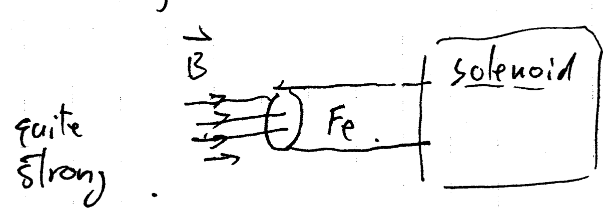
$\theta_2 \sim 90°$.



The $\vec{B}$-lines are mostly "flowing" through the iron core. (and very strong). — can even turn along the iron core.

Demo: Let the iron rod stick out of solenoid.

and test the intensity of $\vec{B}$

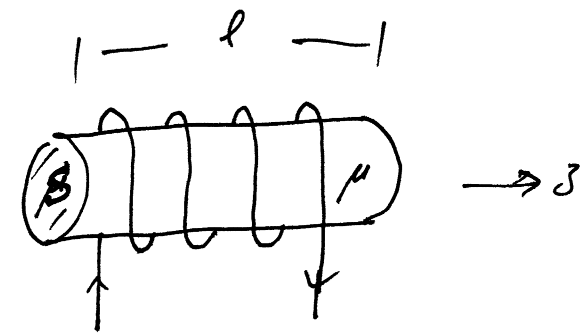


- Inductance

— An inductor stores magnetic energy.
(just like a capacitor stores electric energy. $W_e = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$)

- Self inductance:

e.g. A solenoid



$$\vec{B} = \hat{z}\,\frac{\mu N I}{l}.$$

flux through a loop:
$$\Phi = \frac{\mu N I}{l} S \qquad S \text{ — area}.$$

flux through N loops (Total flux):
$$\Lambda = N\Phi = \frac{\mu N^2 S}{l} I.$$

$$\Lambda \propto I. = L\,I.$$

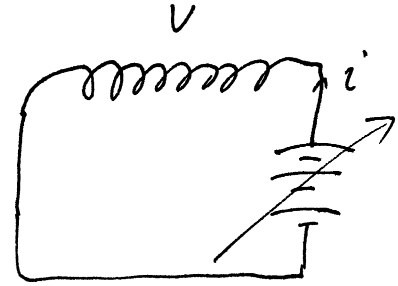
L — self inductance.

for a solenoid:
$$L = \mu \frac{N^2}{l} S.$$

- Note: Faraday's law:
$$\mathcal{E}_{emf} = -N\frac{d\Phi}{dt} = -\frac{d\Lambda}{dt} = -L\cdot\frac{dI}{dt}.$$

electromotive force.

- Magnetic energy.



$$W_m = \int p \cdot dt$$

$$= \int v\, i\, dt$$

$$= \int L \frac{di}{dt} \cdot i\, dt.$$

$$= \int_0^I L\, i\, di$$

$$= \frac{1}{2} L I^2.$$

$$V = -\mathcal{E}_{emf} = L \cdot \frac{di}{dt}.$$

Solenoid:

$$W_m = \frac{1}{2} \mu \frac{N^2}{\ell} S I^2. \qquad \left(H = \frac{NI}{\ell} \right)$$

$$H^2 = \frac{N^2 I^2}{\ell^2}$$

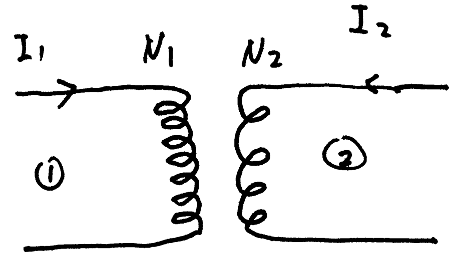
$$= \frac{1}{2} \mu \underbrace{\ell S}_{\text{volume}} \cdot H^2.$$

$$= \frac{1}{2} \mu \underbrace{V}_{\text{volume}} H^2.$$

general: magnetic energy density: $w_m = \frac{1}{2} \mu H^2$.

Total magnetic energy: $W_m = \int \frac{1}{2} \mu H^2 \underbrace{dV}_{\text{volume}}$.

Mutual Inductance



$\vec{B}$-field at ② due to ① : $\vec{B}_1$

Flux through a loop in ② : $\Phi_{12} = \int_{S_2} \vec{B}_1 \cdot \vec{ds}$

Total magnetic flux linkage through ②:

$$\Lambda_{12} = N_2 \Phi_{12} = N_2 \int_{S_2} \vec{B}_1 \cdot \vec{ds}$$

$$\Lambda_{12} \propto I_1 = L_{12} I_1$$

Mutual Inductance :

$$L_{12} = \frac{\Lambda_{12}}{I_1} = \frac{N_2}{I_1} \int_{S_2} \vec{B}_1 \cdot \vec{ds}$$

Can be shown :

$$L_{21} = L_{12}$$

$$\left(L_{21} = \frac{N_1}{I_2} \int_{S_1} \vec{B}_2 \cdot \vec{ds} \right)$$