

Phys 221

Lecture 27.

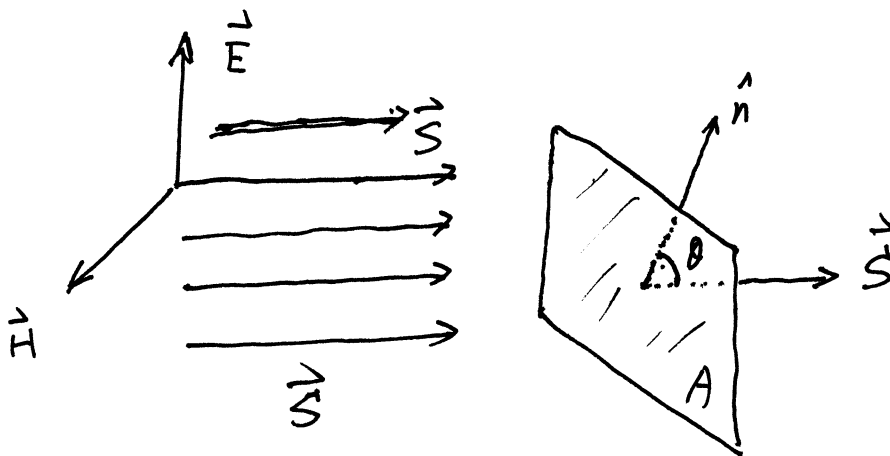
Ref. 7-7. Electromagnetic Power density.

• Poynting vector

$$\vec{S} = \vec{E} \times \vec{H}$$

Meaning: Direction: propagation direction ($\hat{S} = \hat{k}$)

Magnitude: Power through per unit area.



Power through area A:

$$P = \int_A \vec{S} \cdot \hat{n} \, dA = \int_A \vec{S} \cdot d\vec{A}$$

• In a lossless medium, e.g. plane wave travelling in $\hat{z}$ direction 27-2

— Time domain: $\vec{S}(z, t) = \vec{E}(z, t) \times \vec{H}(z, t)$

(Instantaneous relation) $= \hat{z} \frac{|E_{x0}|^2}{\eta} \cos^2(\omega t - kz) \quad (k = \beta)$

time-average (over a period T):

$$\vec{S}_{av} = \frac{1}{T} \int_0^T \vec{S}(z, t) dt = \hat{z} \frac{|E_{x0}|^2}{2\eta} \quad (7.157)$$

$$\left(\because \frac{1}{2\pi} \int_0^{2\pi} \cos^2 x dx = \frac{1}{2} \right)$$

— Phasor domain: $\vec{S} \neq \vec{E} \times \vec{H} \quad \left(\begin{array}{l} \text{because it's not a} \\ \text{linear operation. X} \end{array} \right)$

It turns out:

(because of 7.157) $\boxed{\vec{S}_{av} = \frac{1}{2} \text{Re} [\vec{\tilde{E}} \times \vec{\tilde{H}}^*]} \quad (7.158)$

$\vec{\tilde{H}}^*$ — complex conjugate of $\vec{\tilde{H}}$

↻ replace j with $-j$.

Verify 7.158: $\frac{1}{2} \text{Re} [\vec{\tilde{E}} \times \vec{\tilde{H}}^*] = \frac{1}{2} \text{Re} \left[\hat{x} E_{x0} e^{-jkz} \times \hat{y} \frac{E_{x0}^*}{\eta} e^{jkz} \right]$
 $= \frac{1}{2} \text{Re} \left[\hat{z} \frac{|E_{x0}|^2}{\eta} \right] = \hat{z} \frac{|E_{x0}|^2}{2\eta}$

• In a lossy medium.

$$\eta_c = |\eta_c| e^{j\theta_\eta}$$

Then, can be shown:

$$\vec{S}_{av}(z) = \hat{z} \frac{|E_0|^2}{2 \cdot |\eta_c|} e^{-2\alpha z} \cos(\theta_\eta)$$

at $z = \delta_s$ (skin depth).

$$E = E_0 e^{-1} = 37\% E_0$$

$$S_{av} = S_{av}(0) \cdot e^{-2} = 14\% S(0)$$

• Decibel Scale.

— For power ratio $G = \frac{P_1}{P_2}$.

$$G[\text{dB}] = 10 \log \frac{P_1}{P_2}$$

— For voltage, current, field $\vec{E}$, $\vec{H}$, $g = \frac{V_1}{V_2}$

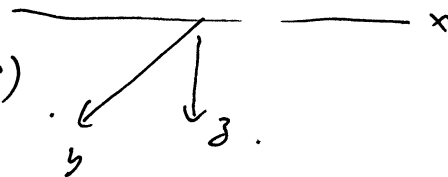
$$g[\text{dB}] = 20 \log \frac{V_1}{V_2}$$

(Why? $\because P = \frac{V^2}{R}$, so G and g in [dB]
 $P \propto I^2, E^2, H^2$, etc. have the same meaning)

e.g. 7-10. A plane wave travelling in ~~the~~ sea water.

$$\vec{E}(z, t) = \hat{x} 4.4 e^{-0.126z} \cos(2\pi \times 10^3 t - 0.126z + 60^\circ)$$

$$\vec{H}(z, t) = \hat{y} 100 e^{-0.126z} \cos(2\pi \times 10^3 t - 0.126z + 15^\circ)$$



A submarine at a depth of 200 m uses a wire antenna to receive signal transmission at 1 KHz. Determine The average power density incident upon the submarine antenna.

$$\vec{S}_{av}(z) = \frac{1}{2} \text{Re} [\tilde{\vec{E}} \times \tilde{\vec{H}}]$$

$$= \hat{z} \cdot \frac{|E_0|^2}{2|\eta_c|} e^{-2\alpha z} \cos \theta_\eta$$

$$= \hat{z} \frac{(4.44 \times 10^{-3})^2}{2 \times 0.044} e^{-0.252z} \cos 45^\circ$$

$$= \hat{z} 0.16 e^{-0.252z} \quad (\text{mW/m}^2)$$

$$|E_0| = 4.44 \text{ mV/m}$$

$$\alpha = 0.126$$

$$\eta_c = 0.044 \angle 45^\circ$$

at $z = 200 \text{ m}$.

$$\vec{S}_{av} = \hat{z} 0.16 \times 10^{-3} e^{-0.252 \times 200}$$

$$= 2.1 \times 10^{-26} \text{ W/m}^2$$