Review of Electromagnetics Theory.

- Time - independent electromagnetics

Maxwell's equations

1. Gauss' law : $\nabla \cdot \vec{D} = \rho_v$, $\oint_S \vec{D} \cdot d\vec{s} = Q$

free charge

2. $\vec{E}$-field is conservative : $\nabla \times \vec{E} = 0$. $\oint_c \vec{E} \cdot d\vec{l} = 0$

any closed c.

Then: $\vec{E} = -\nabla V$

Poisson's eq: $\nabla^2 V = -\frac{\rho_v}{\varepsilon}$.

3. No monopole : $\nabla \cdot \vec{B} = 0$ $\oint_S \vec{B} \cdot d\vec{s} = 0$

any closed S

Then: $\vec{B} = \nabla \times \vec{A}$

4. Ampere's Law : $\nabla \times \vec{H} = \vec{J}$, $\oint_c \vec{H} \cdot d\vec{l} = I$.

free current.

*5 Coulomb's Gauge : $\nabla \cdot \vec{A} = 0$. ($\because$ $\vec{B} = \nabla \times \vec{A}$ does not uniquely define $\vec{A}$)

Then. $\nabla^2 \vec{A} = -\mu \vec{J}$

(Poisson's equation for each component of $\vec{A}$)

- *Time-dependent Theory.*

1. Gauss' law: Same as t-ind. cases.

2. Faraday's law: $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$. $\oint_C \vec{E} \cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$.

   — A changing $\vec{B}$-field can contribute to $\vec{E}$-field.

   — Then, $\vec{E}$-field is no longer conservative.

   — Then, $\vec{E} \neq -\nabla V$. (Need ~~to~~ to be modified).

   — Then, V does not obey Poisson's equation.

3. No monopole: Same as t-ind. cases.

4. Ampere's law: Needs to be modified — Maxwell's contribution

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}, \qquad \oint_C \vec{H} \cdot d\vec{l} = \int_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{s}$$

A time-varying $\vec{E}$-field should contribute to $\vec{B}$-field too!

[redacted].

5. Potentials and Lorentz Gauge.

$$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}.$$

$$\vec{B} = \nabla \times \vec{A}$$ ← same as t-ind. cases

coulomb gauge should be replaced by Lorentz gause:

$$\nabla\cdot\vec{A} + \varepsilon_0\mu_0 \frac{\partial V}{\partial t} = 0 .$$

Then the potentials $V(\vec{r}, t)$ and $\vec{A}(\vec{r}, t)$ will obey the wave equation:

$$\nabla^2 V - \varepsilon\mu \frac{\partial^2 V}{\partial t^2} = -\frac{\rho_v}{\varepsilon}$$

$$\nabla^2 \vec{A} - \varepsilon\mu \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu_0 \vec{J} .$$

wave source.

6. In source-free space, $\rho_v = 0$, $\vec{J} = 0$,

$\vec{E}$, $\vec{B}$, V, $\vec{A}$ all obey the "No source" ~~equa~~ wave equation.

$$\nabla^2 V - \varepsilon\mu \frac{\partial^2 V}{\partial t^2} = 0$$

$$\nabla^2 \vec{A} - \varepsilon\mu \frac{\partial^2 \vec{A}}{\partial t^2} = 0 .$$

$$\nabla^2 \vec{E} - \varepsilon\mu \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$$\nabla^2 \vec{B} - \varepsilon\mu \frac{\partial^2 \vec{B}}{\partial t^2} = 0 .$$

speed of wave : $\frac{1}{\sqrt{\varepsilon\mu}}$.