Phys 221.

Lecture 28.

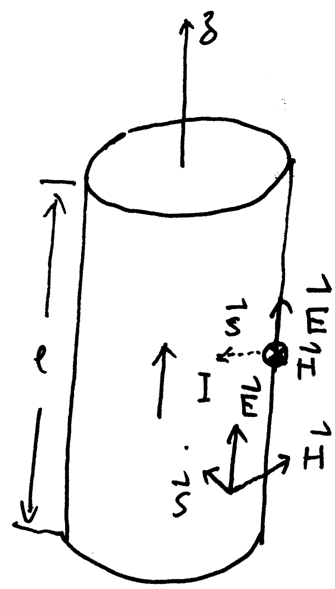
Ref. 7-7, 8-1, 8-2, 8-3, 8-4.

- Poynting vector: $\vec{S} = \vec{E} \times \vec{H}$ (Power density of E & M Fields)

For harmonic fields,

$$\vec{S}_{av} = \frac{1}{2} \mathrm{Re}\left[\tilde{\vec{E}} \times \tilde{\vec{H}} \right]$$

- e.g. A long conducting wire of radius a and conductivity σ carries a DC current I.



at surface:

$$\vec{J} = \sigma \vec{E} = \frac{I}{\pi a^2} \hat{z}$$

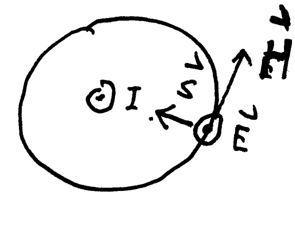
$$\vec{E} = \hat{z} \frac{I}{\pi a^2 \sigma}$$

$$\vec{H} = \hat{\phi} \frac{I}{2\pi a} \quad \text{(Ampere's law)}$$

$$\vec{S} = \vec{E} \times \vec{H}$$

$$= \underbrace{-\hat{r}}_{\uparrow} \frac{I^2}{2\pi^2 a^3 \sigma}$$

electromagnetic field energy flows across the surface into the wire.



Field energy flowing in to wire of length l, per unit time:

$$P_{field} = -\int \vec{S} \cdot d\vec{A}$$

$$= \frac{I^2}{2\pi^2 a^3 \sigma} \cdot 2\pi a \cdot l .$$

$$= \frac{I^2 l}{\pi a^2 \sigma}$$

but: $\frac{1}{\sigma} \frac{l}{\pi a^2} = R$.

$$\therefore \quad P_{field} = \underbrace{I^2 R = P_{heat}}_{\text{Joule's law}} .$$

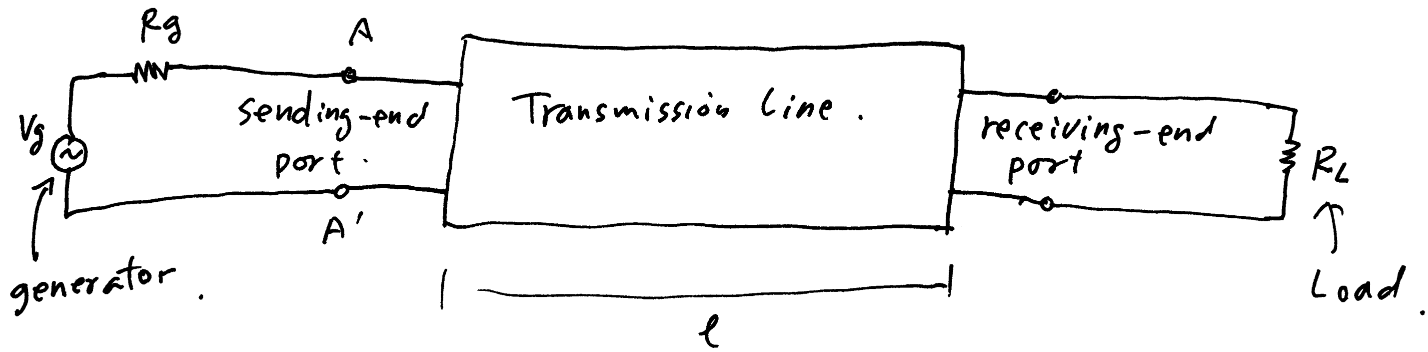
i.e. ~~The ohmic~~ The ohmic power loss I^2R in the wire equals the power of EDM Field entering the wire.

(energy conservation

energy in = energy used)

• Transmission Lines.

A transmission line — Two-port network.



e.gs. figure 8-4 on page 248.

Two-wire line

Coaxial line

parallel-plate line.

etc.

typically 2-conductor lines.

In this course, we deal with TEM-mode lines only.

• For low-frequencies, the transmission line doesn't play an important role, and thus can be treated as a short.

However, for high frequencies, $\frac{\ell}{\lambda} > 1\%$ of λ, we need to consider the effects of the line.

why? Because it takes time for the E & M wave to travel through.
└ leads to a phase shift. (for high-freq. signals)

e.g. at AA' $V_{AA'}(t) = V_0 \cos(\omega t)$

at BB' $V_{BB'}(t) = V_0 \cos[\omega(t - \frac{l}{c})]$

phase difference : $\Delta\phi = \frac{\omega l}{c} = \frac{2\pi}{T}\frac{l}{c} = 2\pi\frac{l}{\lambda}$

$\Delta\phi$ is negnegible only if $\Delta\phi \ll 2\pi$.

i.e. $\frac{l}{\lambda} \ll 1$, or $l \ll \lambda$.

e.g. If $f = 1\,kHz$, $\lambda = \frac{c}{f} = \frac{3\times10^8}{10^3} = 3\times10^5\,m = 300\,km$.

for a short line of $l = 5\,cm = 0.05\,m$, $\ll \lambda$.

~~$V_{BB'}(t) =$~~ phase shift $\frac{l}{\lambda}\cdot 2\pi \sim 1\times10^{-6}$ rad.

$V_{AA'}(0) = V_0$, $V_{BB'}(0) = 0.999999999998\,V_0$

for a long line of $l = 20\,km = 2\times10^4\,m$.

$\frac{l}{\lambda} = 0.067$.

$V_{BB'}(0) = 0.91\,V_0$ — the effects of transimission line needs to be analyzed.

- Besides phase shift, there are other effects of Transmission lines.
  - — reflection of E & M signals.
  - — Power loss
  - — dispersion — wave velocity depends on freq.

    $\left(u \sim \frac{1}{\sqrt{\varepsilon\mu}} \text{, e.g. } \varepsilon = \varepsilon(\omega) \right)$

    — leads to distortion of signals.

Note: A "long line" doesn't have to be long!

L relative to λ.

e.g.: at $f = 10$ GHz, $\lambda = 3$ cm $= 0.03$ m.

a line of 1 mm, $\frac{l}{\lambda} \approx 0.033$ (considered as "long") even though its length is only 1 mm.

This can happen in an IC chip.