Phys 221

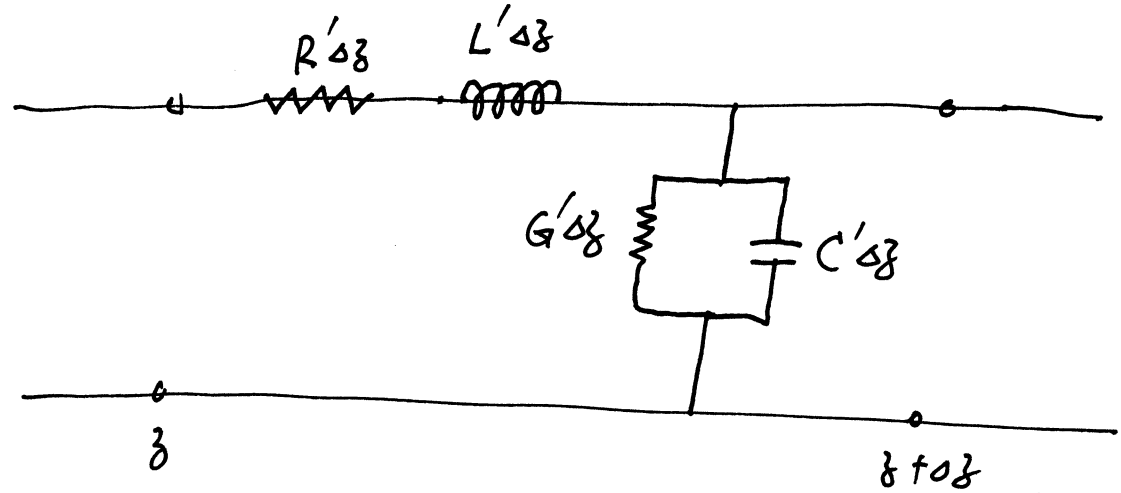
Lecture 30

Ref. 8-3, 8-4, 8-5*

- Last lecture

Transmission line equations

(using the lumped-element model)



phasors:

$$-\frac{d\tilde{V}(z)}{dz} = (R'+j\omega L')\,\tilde{I}(z) \quad (8.18a)$$

$$-\frac{d\tilde{I}(z)}{dz} = (G'+j\omega C')\,\tilde{V}(z) \quad (8.18b)$$

$\{R', L', G', C'\}$ determines the behavior of the transmission line.

differentiate 8.18 a:

$$-\frac{d^2\tilde{V}(z)}{dz^2} = (R'+j\omega L)\frac{d\tilde{I}(z)}{dz}.$$

$$-\frac{d^2\tilde{V}(z)}{dz^2} = -(R'+j\omega L)(G'+j\omega C)\tilde{V}(z).$$

$$= -\gamma^2 \tilde{V}(z)$$

i.e.

$$\boxed{\frac{d^2\tilde{V}(z)}{dz^2} - \gamma^2 \tilde{V}(z) = 0} \quad (8.21)$$

$$\boxed{\gamma = \sqrt{(R'+j\omega L')(G'+j\omega C')}} \quad (8.22)$$

wave eq!

Similarly.

$$\boxed{\frac{d^2 I(z)}{dz^2} - \gamma^2 \tilde{I}(z) = 0} \quad (8.23).$$

$$\gamma = \alpha + j\beta$$

α – attenuation constant

β – phase constant

Solution of 8-21 and 8-23:

$$\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} .$$

$$\tilde{I}(z) = I_0^+ e^{-\gamma z} + I^- e^{\gamma z} .$$

$V_0^+ e^{-\gamma z}$ — propagating along $+z$ direction

$V_0^- e^{\gamma z}$ — " " $-z$ "

(reflection).

• characteristic impedance of a transmission line.

$$Z_0 \triangleq \frac{V_0^+}{I_0^+} = \frac{\text{V of right-going wave } \cancel{\text{of}}\ V_0^+ e^{-\gamma z}}{\text{I of right-going wave } \cancel{\text{of}}\ I_0^+ e^{-\gamma z}} .$$

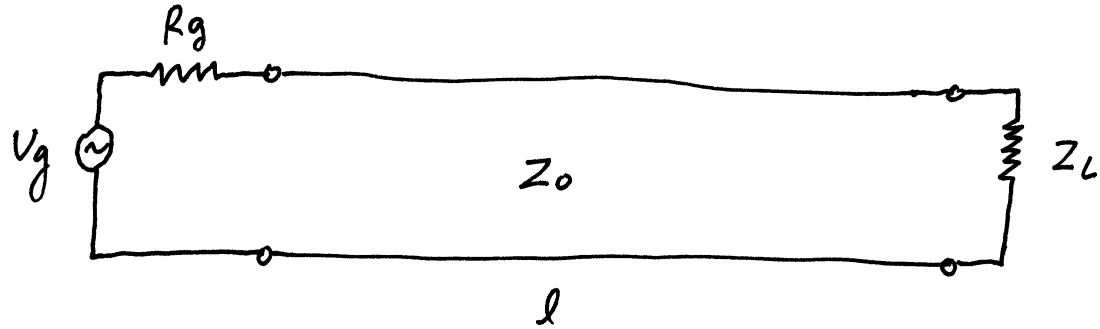
$$(8.18a) \Rightarrow I_0^+ e^{-\gamma z} = \frac{-1}{R'+j\omega L'} \frac{d}{dz}\left(V_0^+ e^{-\gamma z}\right)$$

$$= \frac{\gamma}{R'+j\omega L'} V_0^+ e^{-\gamma z} .$$

$$Z_0 = \frac{R'+j\omega L'}{\gamma}$$

$$= \sqrt{\frac{R'+j\omega L'}{G'+j\omega C'}} .$$

- Meaning of Z_0



① . $Z_0 = \dfrac{V_+}{I_+} = \dfrac{V_-}{I_-}$ (It's the $\frac{V}{I}$ ratio of individual wave, → or ←, NOT the total wave!)

but in general $Z_0 \neq \dfrac{V}{I}$.

② . $Z_0 \neq$ input impedance, (in general) i.e., $Z_0 \neq \dfrac{V(z=0)}{I(z=0)}$

but $Z_0 =$ input impedance if the line was extended to infinity (i.e, no reflection)

③ . Impedance matching

can be shown: (8-5) If $Z_L = Z_0$, No reflection.

Then, the transmission line can be equivalent to $l \to \infty$.

That's what you want when you try to send a signal through.

e.g. A parallel-plate transmission line made of two brass strips : $\sigma_c = 1.6 \times 10^7$ S/m.

$W = 20$ mm $= 0.02$ m $= 2 \times 10^{-2}$ m

separated by a lossy dielectric slab:

$$\begin{cases} d = 0.0025 \text{ m} \quad (2.5 \text{ mm}) = 2.5 \times 10^{-3} \\ \mu = \mu_0 \\ \varepsilon_r = 3 \\ \sigma = 10^{-3} \ (\text{S/m}) \end{cases}$$

The operating frequency is $f = 500$ MHz $= 5 \times 10^8$ Hz.

a). Calculate R', L', G' and C'

b). Find γ and Z_0.

[solution]. use the last column of Table 8-1. on page 251.

a). $R_s = \sqrt{\pi f \mu_c / \sigma_c}$ (μ_c, σ_c — properties of conductor)

$$= \left(\frac{\pi \times 5 \times 10^8 \times 4\pi \times 10^{-7}}{1.6 \times 10^7} \right)^{1/2} = 1.11 \times 10^{-2} \ \Omega$$

$$R' = \frac{2R_s}{W} = \frac{2 \times 1.11 \times 10^{-2}}{0.02} = 1.11 \ \Omega/\text{m}.$$

$$L' = \frac{\mu d}{W} = \frac{4\pi \times 10^{-7} \times 2.5 \times 10^{-3}}{2 \times 10^{-2}} = 1.57 \times 10^{-7} \text{ H/m} = 0.157 \ \mu\text{H/m}$$

$$G' = \frac{\sigma W}{d} = \frac{10^{-3} \times 2 \times 10^{-2}}{2.5 \times 10^{-3}} = 0.008 \text{ S/m}$$

$$C' = \frac{\varepsilon W}{d} = \frac{3 \times 8.854 \times 10^{-12} \times 2 \times 10^{-2}}{2.5 \times 10^{-3}} = 2.12 \times 10^{-10} \text{ F/m} = 0.212 \text{ nF/m}$$

b). γ and Z_0.

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$$

$$\left(\omega = 2\pi f = 2\pi \times (5 \times 10^8) Hz = \pi \times 10^9 \text{ rad/s.} \right.$$

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

$$\omega L' = \pi \times 10^9 \times 1.57 \times 10^{-7} = 493$$

$$\omega C' = \pi \times 10^9 \times 2.12 \times 10^{-10} = 0.666.$$

$$R' + j\omega L' = 1.11 + j\,493 = 493\,e^{j89.87^\circ}$$

$$G' + j\omega C' = 0.008 + j0.666 = 0.666\,e^{j89.31^\circ}$$

$$\gamma = \left[493\,e^{j89.87^\circ} \cdot 0.666\,e^{j89.31^\circ} \right]^{1/2}$$

$$= 18.12\,e^{j89.59}$$

$$= 0.130 + j18.12$$

$$Z_0 = \left[\frac{493\,e^{j89.87^\circ}}{0.666\,e^{j89.31^\circ}} \right]^{1/2}$$

$$= 27.207\,e^{j0.28^\circ}$$

$$= 27.21 + j0.13$$