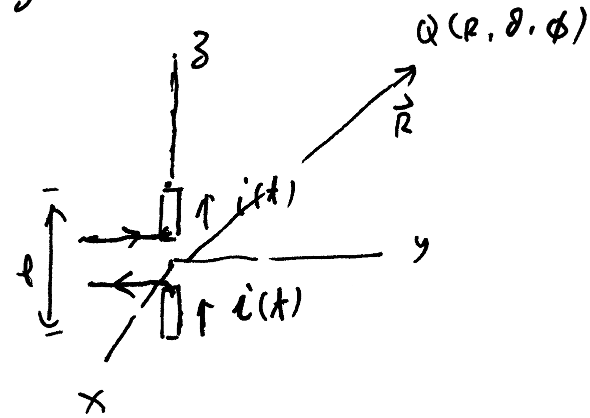
Phys 221

Lecture 32.

Ref: 10-2, 3.

Last lecture: The short dipole (Hertzian dipole)

$$\frac{l}{\lambda} < \frac{1}{50}$$



Then, $i(t)$ ind. of z.

$i(t) = I_0 \cos(\omega t)$.

Method: retarded vector potential.

$$\tilde{\vec{A}}(\vec{R}) = \frac{\mu_0}{4\pi} \int_{V'} \frac{\tilde{\vec{J}} e^{-jkR'}}{R'} dV' \quad \left(\text{Then } \tilde{\vec{H}}, \text{ Then } \tilde{\vec{E}}\right)$$

Result:
$$\tilde{\vec{A}}(\vec{R}) = \hat{z} \frac{\mu_0}{4\pi} I_0 l \frac{e^{-jkR}}{R}$$

$$= (\hat{R}\cos\theta - \hat{\theta}\sin\theta) \frac{\mu_0 I_0 l}{4\pi} \frac{e^{-jkR}}{R} \quad (A_\phi = 0)$$

spherical system.

θ – zenith angle

ϕ – azimuth angle

Then,
$$\tilde{\vec{H}} = \frac{1}{\mu_0} \nabla \times \tilde{\vec{A}}$$

$$\tilde{\vec{E}} = \frac{1}{j\omega\varepsilon} \nabla \times \tilde{\vec{H}}.$$

$$\tilde{H}_\phi = \frac{I_0 l k^2}{4\pi} e^{-jkR} \left[\frac{j}{kR} + \frac{1}{(kR)^2} \right] \sin\theta .$$

$$\tilde{E}_R = \frac{2 I_0 l k^2}{4\pi} \eta_0 e^{-jkR} \left[\frac{1}{(kR)^2} - \frac{j}{(kR)^3} \right] \cos\theta$$

$$\tilde{E}_\theta = \frac{I_0 l k^2}{4\pi} \eta_0 e^{-jkR} \left[\frac{j}{kR} + \frac{1}{(kR)^2} - \frac{j}{(kR)^3} \right] \sin\theta .$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 120\pi .$$

$$(H_R = 0 , \quad H_\theta = 0 , \quad E_\phi = 0)$$

- Far field. $kR \gg 1$. $\left(2\pi \frac{R}{\lambda} \gg 1 \right)$: $\left(\frac{1}{R^2}\right) \left(\frac{1}{R^3}\right)$ die out

$$\tilde{E}_\theta = j \frac{I_0 l k \eta_0}{4\pi} \frac{e^{-jkR}}{R} \sin\theta \quad \sim \frac{1}{R} .$$

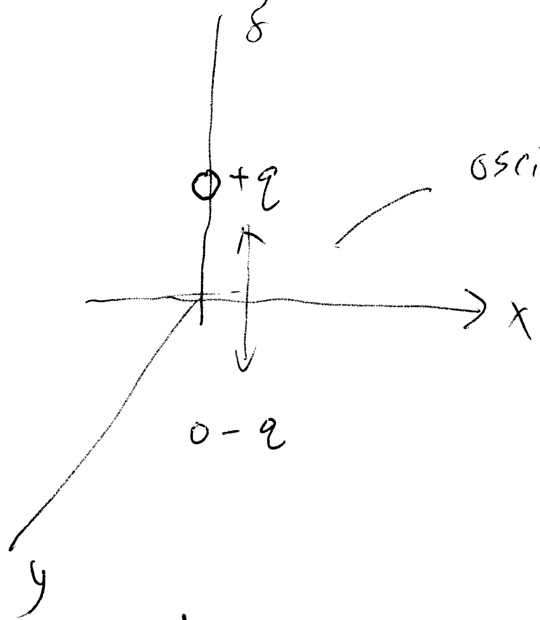
$$\tilde{H}_\phi = \frac{\tilde{E}_\theta}{\eta_0} \quad \text{also} \sim \frac{1}{R} .$$

$$\left(\tilde{E}_R \sim \frac{1}{R^2} \sim 0 \right)$$

fig. 10-6. — draw directions of $\vec{E}$ and $\vec{H}$, $\vec{k}$ $(\vec{S})$.

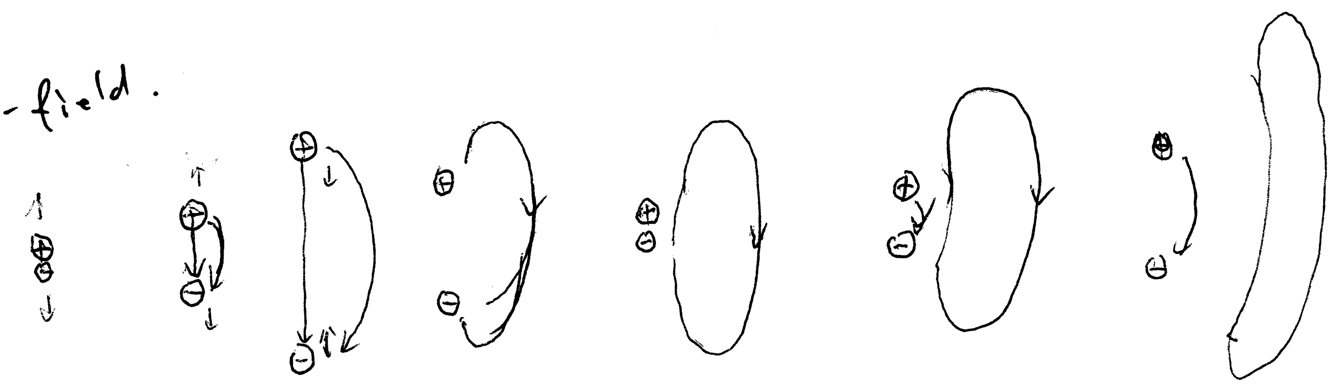
fig. 10-7 — How $\vec{E}$-field is generated and propagates

- Another way to view an oscillating dipole.

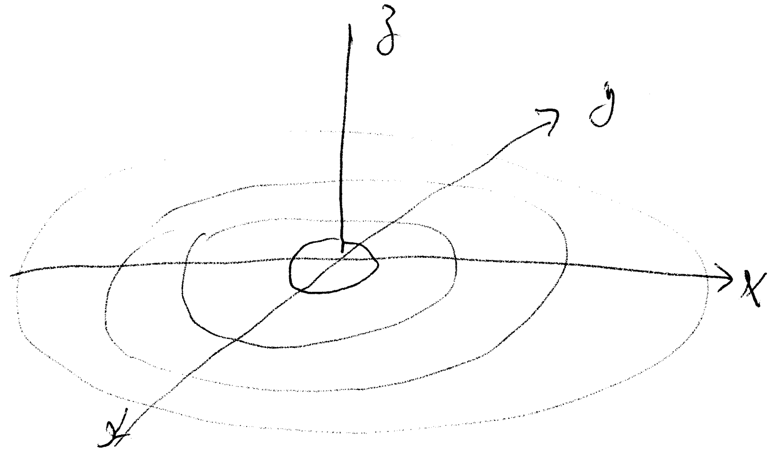


oscillating. $l(t) = l_0 \cos(\omega t)$

E-field.



Magnetic field: $\vec{H}$ - lines: in x-y plane



- Power density

time-average Poynting vector

$$\vec{S}_{av} = \frac{1}{2} Re\left(\tilde{\vec{E}} \times \tilde{\vec{H}}^* \right) = \hat{R}\, S(\theta, \phi)$$

$$S(\theta,\phi) = \frac{1}{2} Re\left(\tilde{E}_\theta \cdot \tilde{H}_\phi^* \right)$$

$$= \frac{1}{2} Re\left[\frac{j I_0 \ell k \eta_0}{4\pi} \cdot \frac{e^{-jkR}}{R} \sin\theta \cdot \frac{(-j)}{\eta_0} \frac{I_0 \ell k \eta_0}{4\pi} \frac{e^{jkR}}{R} \sin\theta \right]$$

$$= \frac{1}{2} Re\left[\frac{I_0^2 \ell^2 k^2 \eta_0}{16\pi^2 R^2} \sin^2\theta \right]$$

$$= \frac{I_0^2 \ell^2 k^2 \eta_0}{32\pi^2 R^2} \sin^2\theta$$

$$= S_0 \cdot \sin^2\theta$$

- Directional Pattern : $F(\theta,\phi)$

Normalized radiation intensity

$$F(\theta,\phi) = \frac{S(R,\theta,\phi)}{S_{max}}.$$

S_{max} — Max value of $S(R,\theta,\phi)$ at a given R.

- For the Hertzian dipole

$$S_{max} = S_0 = \frac{I_0^2 \ell^2 k^2 \eta_0}{32\pi^2 R^2}$$

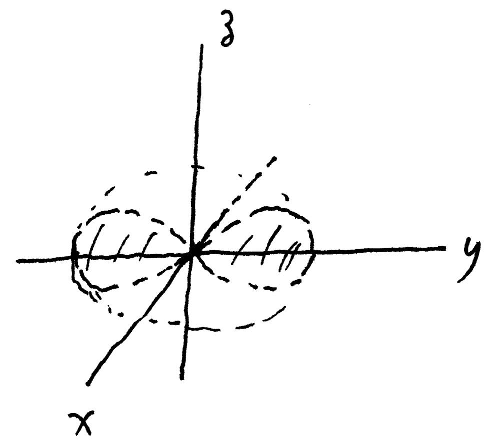
$$= \frac{15\pi I_0^2}{R^2} \left(\frac{\ell}{\lambda}\right)^2$$

$$F(\theta, \phi) = \sin^2\theta .$$

Figure 10-8 on page 355:

Doughnot shape.

No energy is radiated in the z-direction.



- Elevation pattern

On the $\underline{\theta\text{-plane}}$ (elevation plane) (zenith plane)

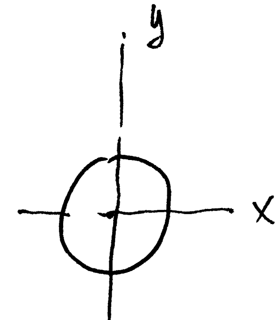
$\llcorner$ constant ϕ.

- Azimuth pattern

on the ϕ-plane (azimuth plane)

$\theta = 90°$.

(or, just the x-y plane)

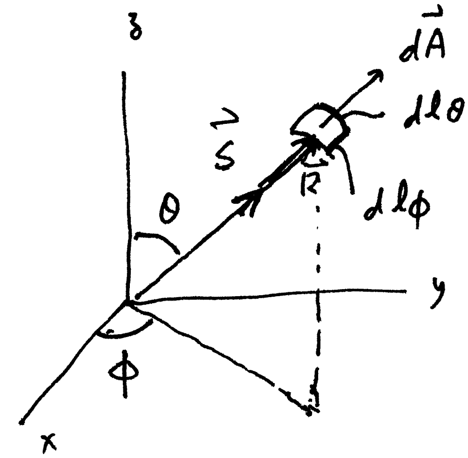


• Differential power radiated by antenna through an elemental area dA.

$$dl_\theta = R\,d\theta$$

$$dl_\phi = R\sin\theta \cdot d\phi$$

$$dA = R^2 \sin\theta \cdot d\theta \cdot d\phi$$



$$dP_{rad.} = \vec{S}_{av} \cdot d\vec{A}$$

$$= (\hat{R} S) \cdot (\hat{R}\, dA) = S\, dA$$

• Solid angle associated with dA.

$$d\Omega = \frac{dA}{R^2} = \sin\theta \cdot d\theta \cdot d\phi. \quad (dA = R^2 d\Omega)$$

(in sr. — steradians)

$$dP_{rad} = \underbrace{R^2 S(R, \theta, \phi)}_{\text{power per unit solid angle}} d\Omega.$$

• Total radiated power.

$$P_{rad} = R^2 \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} S(R, \theta, \phi)\, d\Omega = R^2 S_{max} \iint F(\theta, \phi)\, d\Omega$$

$$= R^2 S_{max} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} F(\theta, \phi) \sin\theta\, d\theta\, d\phi.$$

Note: Total solid angle of a ~~sp~~ sphere : $= 4\pi$.

$$\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} d\Omega = 4\pi .$$

OR: $$\frac{\text{surface area of sphere}}{R^2} = \frac{4\pi R^2}{R^2} = 4\pi .$$

- Antenna pattern
  - described by the normalised radiation intensity.

$$F(\theta,\phi) = \frac{S(R,\theta,\phi)}{S_{max}} .$$

How to plot $F(\theta,\phi)$ — Next.