

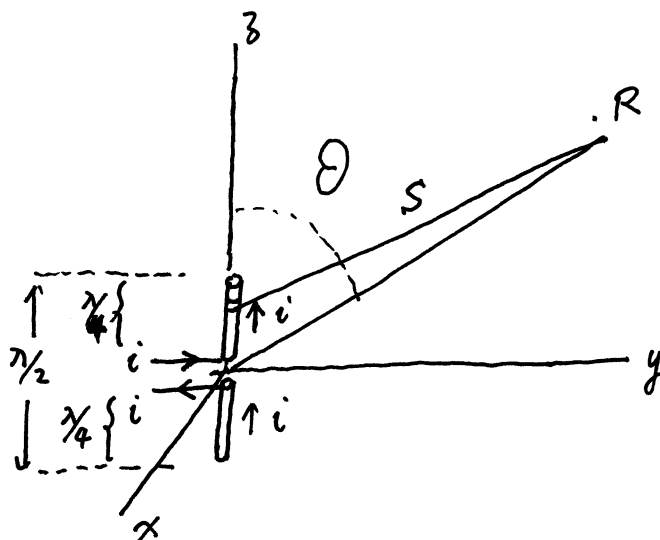
Lecture 33 .

• Half-wave dipole antenna

Since the current at the ends is zero .

$$i = I_0 \cos(\omega t) \cos(kz)$$

$$= \text{Re}[I_0 \cos(kz) e^{j\omega t}]$$



i.e: $\tilde{I}(z) = I_0 \cos(kz)$.

(The current in the antenna is like a standing wave)

$$\tilde{I}(\frac{\lambda}{4}) = \tilde{I}(-\frac{\lambda}{4}) = 0$$

• Analysis: Use the Hertzian dipole as an elemental current source .

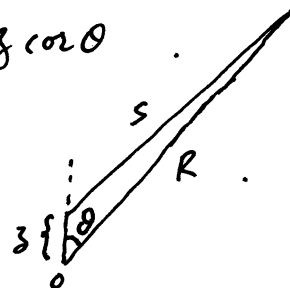
Hertzian dipole : dl

Now : $\tilde{I}(z) dz$. — Then integrate over dz .

• Far field results : $R \gg \lambda$, $S \approx R - z \cos \theta$.

$$d\tilde{E}_\theta = \frac{j k \eta_0}{4\pi} \tilde{I}(z) dz \frac{e^{-jks}}{S} \sin \theta$$

$$d\tilde{H}_\phi = \frac{d\tilde{E}_\theta}{\eta_0}$$



• Note : $R \gg \lambda$, $S \gg \lambda$.

but $S - R \approx z \cos \theta$ can be comparable to λ
(or greater than λ)

$$\therefore \frac{e^{-jkS}}{S} \approx \frac{e^{-j k R}}{R} \cdot e^{j k z \cos \theta}$$

$$\begin{aligned} \tilde{E}_\theta &= \int_{-\lambda/4}^{\lambda/4} \frac{j k \eta_0}{4\pi} I_0 \cdot \cos(kz) \frac{e^{-jkR}}{R} \sin \theta \cdot e^{j k z \cos \theta} dz \\ &= j 60 I_0 \frac{e^{-jkR}}{R} \sin \theta \cdot \int_{-\lambda/4}^{\lambda/4} \cos(kz) e^{j k z \cos \theta} dz \end{aligned}$$

$$\begin{aligned} &= j 60 I_0 \frac{e^{-jkR}}{R} \left[\frac{\cos\left(\frac{\pi}{2} \cos \theta\right)}{\sin \theta} \right] \left[\int e^{ax} \cos(bx) dx \right] \\ \tilde{H}_\phi &= \frac{\tilde{E}_\theta}{\eta_0} \left[= \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) \right] \end{aligned}$$

$\eta_0 = 120 \pi$

Both $\tilde{E}_\theta$ and $\tilde{H}_\phi$ are ind. of ϕ .

- Time-average power density.

$$\begin{aligned}
 S(R, \theta) &= \frac{1}{2} \operatorname{Re} [\tilde{E}_\theta \tilde{H}_\phi^*] = \frac{1}{2\eta_0} \tilde{E}_\theta \tilde{E}_\theta^* = \frac{1}{2\eta_0} |\tilde{E}_\theta|^2 \\
 &= \frac{(60 I_0)^2}{2\eta_0} \frac{1}{R^2} \frac{\cos^2\left(\frac{\pi}{2} \cos \theta\right)}{\sin^2 \theta} \\
 &= \frac{15 I_0^2}{\pi R^2} \frac{\cos^2\left(\frac{\pi}{2} \cos \theta\right)}{\sin^2 \theta}
 \end{aligned}$$

$$S_{\max} = S_0 = \frac{15 I_0^2}{\pi R^2}$$

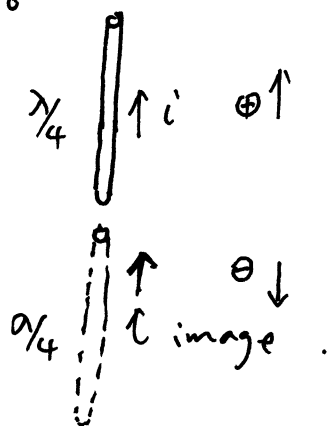
$$F(\theta, \phi) = F(\theta) = \frac{S(R, \theta)}{S_0} = \left[\frac{\cos\left(\frac{\pi}{2} \cos \theta\right)}{\sin \theta} \right]^2 \quad (10.55)$$

(Normalized radiation intensity)

- Quarter-wave Monopole antenna.

Idea: use the image method.

It's equivalent to

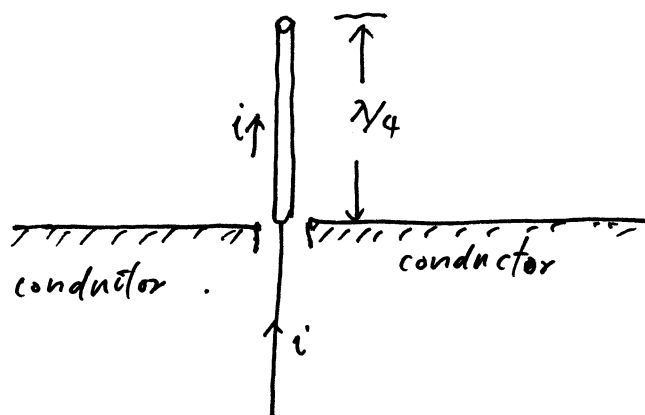


That's a $\frac{\lambda}{2}$

dipole antenna!

but only in the

upper-half plane: $0 \leq \theta \leq \frac{\pi}{2}$.



(Exercise problems:
10.3, 5, 11, 14.)