Phys 221.

Lecture 34.

Antenna Radiation Characteristics

- Antenna pattern.

$$F(\theta,\phi) = \frac{S(R,\theta,\phi)}{S_{max}}.$$

e.g. Hertzian dipole: $F(\theta,\phi) = \sin^2\theta$.

- Beamwidth (at half-power, or 3dB)

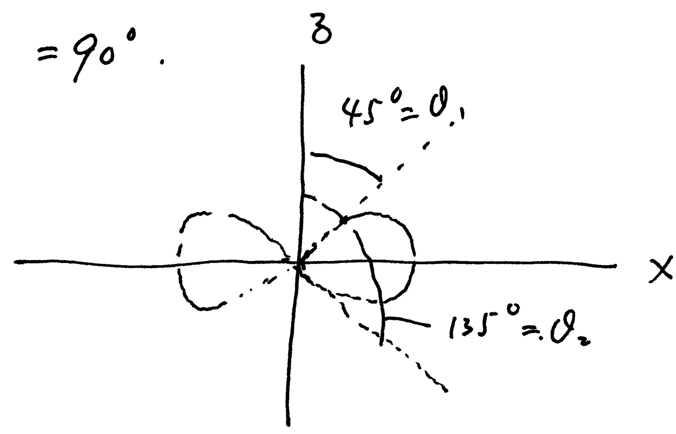
$\beta = \Delta\theta = \theta_2 - \theta_1$. ($\theta_1, \theta_2$ — 3dB points)

e.g. Hertzian dipole $F(\theta,\phi) = \sin^2\theta = F(\theta)$.

3dB. (half-power): $\sin^2\theta = \frac{1}{2}$.

$\theta_1 = 45°$, $\theta_2 = 135°$

$\therefore \beta = 135° - 45° = 90°$.



• Directivity.

$$D = \frac{F_{max}}{F_{av}} = \frac{1}{\frac{1}{4\pi}\iint_{4\pi} F(\theta,\phi)\, d\Omega} = \frac{4\pi}{\Omega_p}.$$

$$\Omega_p = \iint_{4\pi} F(\theta,\phi)\, d\Omega \quad \text{— pattern solid angle.}$$

e.g: For an isotropic antenna,

$$F(\theta,\phi) = 1, \quad \Omega_p = 4\pi, \quad D = 1.$$

e.g. Hertzian dipole: $F(\theta,\phi) = \sin^2\theta$.

Pattern solid angle: $\Omega_p = \iint \sin^2\theta \cdot \sin\theta \cdot d\theta \cdot d\phi$

$$= \int_{\phi=0}^{\phi=2\pi} \cdot \int_{\theta=0}^{\theta=\pi} \sin^3\theta\, d\theta \cdot d\phi$$

$$= \frac{8\pi}{3}.$$

$$D = \frac{4\pi}{\frac{8\pi}{3}} = \frac{3}{2} = 1.5.$$

~~e.g. $\frac{\lambda}{2}$ antenna : $F(\theta,\phi) = F(\theta) = \frac{\cos^2(\frac{\pi}{2}\cos\theta)}{\sin^2\theta}$~~

- Radiation efficiency.

$$\xi = \frac{P_{rad}}{P_t} = \frac{\text{Power radiated into space}}{\text{Transmitter power}}.$$

$$P_{rad} = R^2 \iint S(R,\theta,\phi) \sin\theta \, d\theta \, d\phi \quad \left(= \oiint S(R,\theta,\phi) \, dA \right)$$

$$= R^2 S_{max} \iint_{4\pi} F(\theta,\phi) \, d\Omega = R^2 S_{max} \Omega_p$$

$$= \frac{4\pi R^2 \cdot S_{max}}{D}$$

- Antenna gain.

$$G = \frac{4\pi R^2 S_{max}}{P_t} \quad \left(= \frac{4\pi R^2 S_{max}}{P_{rad}/\xi} = \frac{4\pi R^2 S_{max} \xi}{R^2 S_{max} \Omega_p} \right)$$

$$G = \xi D \quad \left(= \frac{4\pi \xi}{\Omega_p} \quad \cancel{\frac{4\pi\xi}{\Omega}} \quad = \xi D. \quad \left(D = \frac{4\pi}{\Omega_p} \right) \right)$$

- Radiation Resistance

R_{rad} : $\frac{1}{2} R_{rad} \cdot I_0^2 = P_{rad}$. $\quad I_0$ — amplitude. (NOT RMS)

$$P_t = P_{rad} + P_{Loss}$$

P_t: Transmitter power supplied by generator

P_{rad}: power radiated into space.

P_{Loss}: power dissipated as heat in antenna.

• Loss resistance R_{loss} :

$$\frac{1}{2} I_0^2 R_{loss} = P_{loss}$$

• $$\xi = \frac{P_{rad}}{P_t} = \frac{P_{rad}}{P_{rad} + P_{loss}} = \frac{\frac{1}{2} I_0^2 R_{rad}}{\frac{1}{2} I_0^2 R_{rad} + \frac{1}{2} I_0^2 R_{loss}}$$

$$= \frac{R_{rad}}{R_{rad} + R_{loss}} .$$

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e.g. A Hertzian dipole: $l = 0.04\ m$, $f = 75\ MHz = 7.5\times10^7\ Hz$ $\Big\}\ \frac{l}{\lambda} < \frac{1}{50}$

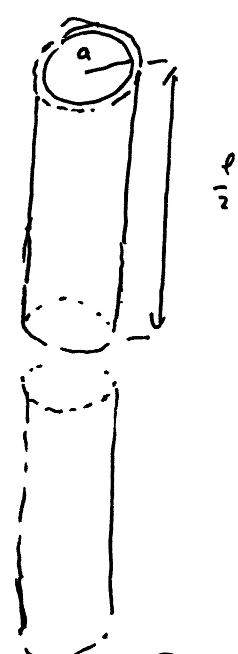
$\lambda = \frac{c}{f} = \frac{3\times10^8}{7.5\times10^7} = 4m$.

Copper : $\sigma_c = 5.8\times10^7\ S/m$, radius: $a = 4\times10^{-4}\ m = 0.4\ mm$.

find ξ :

$$R_{loss} = \frac{l}{2\pi a \delta_s \sigma_c} , \quad \delta_s = \sqrt{\pi f \mu_c \sigma_c}$$

$$= 0.036\ \Omega .$$



$$P_{rad} = \frac{4\pi R^2}{D} S_{max}$$

$$= \frac{4\pi R^2}{1.5} \cdot \frac{15\pi I_0^2}{R^2} \cdot \left(\frac{l}{\lambda}\right)^2$$

$$= 40\pi^2 I_0^2 \left(\frac{l}{\lambda}\right)^2$$

compare to: $P_{rad} = \frac{1}{2} I_0^2 R_{rad}$, $R_{rad} = \frac{2P_{rad}}{I_0}$

∴ Radiation resistance : $R_{rad} = 80\pi^2 \left(\frac{l^2}{\lambda^2}\right) = 0.08\ \Omega$.

Radiation efficiency: $\xi = \dfrac{P_{rad}}{P_t} = \dfrac{P_{rad}}{P_{rad} + P_{loss}}$

$$= \frac{R_{rad}}{R_{rad} + R_{loss}}$$

$$= 0.69 .$$

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e.g. If we change the length to $l = 2m$.

it becomes a $\frac{\lambda}{2}$ dipole. $(l = \frac{\lambda}{2})$.

$R_{loss} = 1.8\,\Omega$. (50 times longer than it was before)

Directivity: $D = \dfrac{4\pi R^2 S_{max}}{P_{rad}}$. $\left(S_{max} = \dfrac{15 I_0^2}{\pi R^2} \right)$

$$P_{rad} = R^2 \iint S(R, \theta)\, d\Omega = \frac{15 I_0^2}{\pi} \int_0^{2\pi} d\phi \underbrace{\int_0^{\pi} \frac{\cos^2(\frac{\pi}{2}\cos\theta)}{\sin^2\theta}\, d\theta}$$

$$= 36.6\, I_0^2 .$$

can't be integrated.
Has to be done numerically.

$$D = \frac{4\pi R^2 \cdot 15 I_0^2}{36.6 I_0^2 \cdot \pi R^2} = 1.64 .$$

$(10 \log 1.64 = 2.15\ dB)$

Radiation resistance: $R_{rad} = \dfrac{P_{rad}}{\frac{1}{2} I_0^2} = \dfrac{36.6 I_0^2}{0.5 I_0^2} = 73\,\Omega .$

Radiation efficiency:

$$\xi = \frac{R_{rad}}{R_{rad} + R_{loss}} = \frac{73}{73 + 1.8} \approx 98\%.$$

Better than the Hertzian dipole (69%).

More important, it's much easier to match the impedance:

$\frac{\lambda}{2}$ dipole: $R_{rad} + R_{loss} \approx 75\,\Omega$.

Hertzian: $R_{rad} + R_{loss} \approx 0.1\,\Omega$. (Too small, Hard to match).