Phys 344

Assignment # 1. Solutions

1.3 $T_K = 273.15 + T_c$

a) $T_c \approx 37°C$ $T_K = 310\ K$

b) $T_c = 100°C$ $T_K = 373.15\ K$

c) $T_c \approx -20°C$ $T_K = 250\ K$

d) $T_c = -196°C$ $T_K = 77\ K$

e) $T_c = 327°C$ $T_K = 600\ K$

1.12 $PV = NkT$ for N molecules

average volume per molecule:

$$\bar{V} = \frac{V}{N} = \frac{kT}{P} = \frac{1.381 \times 10^{-23} \times 300}{1.013 \times 10^{5}} = 4.09 \times 10^{-26}\ m^3$$

Average distance between molecules.

$$\bar{d} = \sqrt[3]{\bar{V}} = 3.45 \times 10^{-9}\ m \approx 3.5\ nm$$

size of H_2O $\sim 0.2\ nm$

size of N_2 $\sim 0.15\ nm$

$\therefore$ $\bar{d}$ is about ~~20~~ 10 times as large as a molecule.

1.14. N_2 : 0.78 mole,

$$m_N = 0.78 \times 28 = 21.84\ g$$

O_2 : 0.21 mole

$$m_O = 0.21 \times 32 = 6.72\ g$$

Ar : 0.01 mole.

$$m_A = 0.01 \times 40 = 0.4\ g$$

The mass of 1 mole of air.

$$M = m_N + m_o + m_A$$

$$= 21.84 + 6.72 + 0.4 = 28.96\ g$$

$$= 0.02896\ kg$$

$$\approx 0.029\ kg$$

1.18.

$$v_{rms} = \sqrt{\frac{3kT}{m}}$$

$$N_A m = 28 \times 10^{-3}\ kg.$$

$$m = \frac{28 \times 10^{-3}}{N_A} = \frac{28 \times 10^{-3}}{6.02 \times 10^{23}}$$

$$= \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 300 \times 6.02 \times 10^{23}}{28 \times 10^{-3}}}$$

$$= 517\ m/s$$

1.23

$$U = N \cdot f \cdot \frac{1}{2} kT$$

$$PV = N \cdot kT \Rightarrow N = \frac{PV}{kT}$$

$$U = \frac{PV}{kT} \cdot f \cdot \frac{1}{2} kT = \frac{1}{2} f PV$$

He: monatomic : $f = 3$

$$U_{He} = \frac{3}{2} PV$$

$$= \frac{3}{2} \times 1.013 \times 10^{5} \times 1.0 \times 10^{-3}$$

$$= 152 \text{ J}$$

Air: N_2 and O_2 : 99%, $f = 5$

Ar : 1%, $f = 3$

$$\therefore f = 0.99 \times 5 + 0.01 \times 3 = 4.98$$

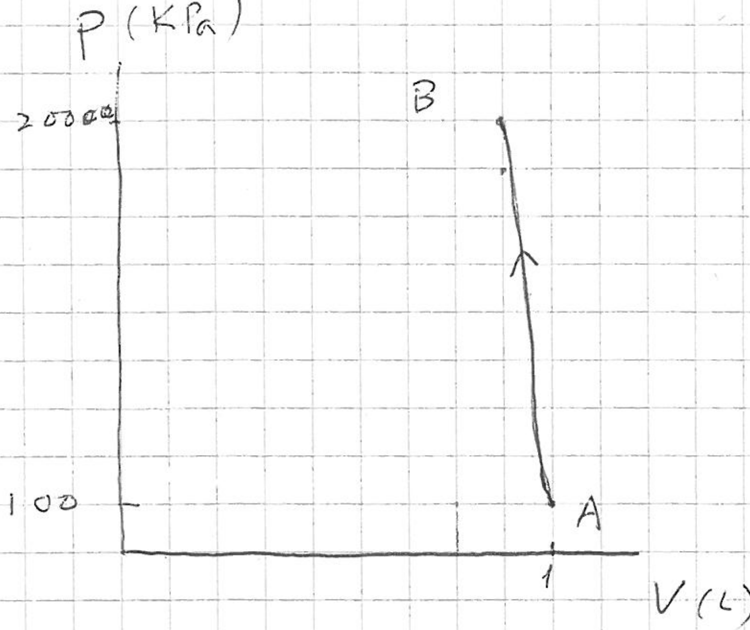
$$U_{air} = \frac{1}{2} \times 4.98 \times 1.013 \times 10^{5} \times 1.0 \times 10^{-3}$$

$$= 252 \text{ J}$$

OR, approximately, $f = 5$. (Mostly N_2 and O_2)

$$U_{air} = \frac{1}{2} \times 5 \times 1.013 \times 10^{5} \times 1.0 \times 10^{-3} = 250 \text{ J}$$

1.32.



The details of the process are unknow, so the work can't be calculated accurately.

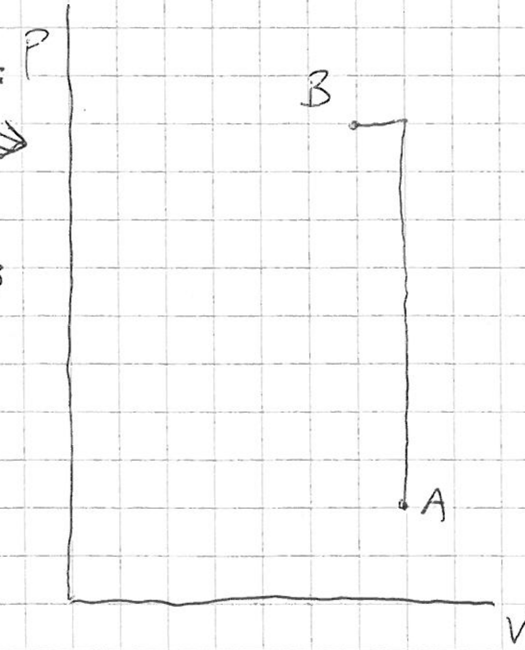
However, one can still estimate the work by taking the average pressure:

$$W = -\int_A^B P \cdot dV \approx -\bar{P}(V_B - V_A)$$

$$\approx -\frac{2\times10^7 + 1\times10^5}{2}\left(0.99\times10^{-3} - 1.00\times10^{-3}\right)$$

$\approx 100\ J$ NOT a huge amount, because the change in volume is small.

OR, one can design a process, for example:

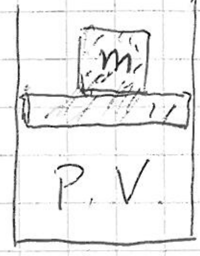


Then:
$$W = -\int_A^B P\,dV =$$

$$= -2\times10^7 \times (0.99 - 1.00)\times10^{-3}$$

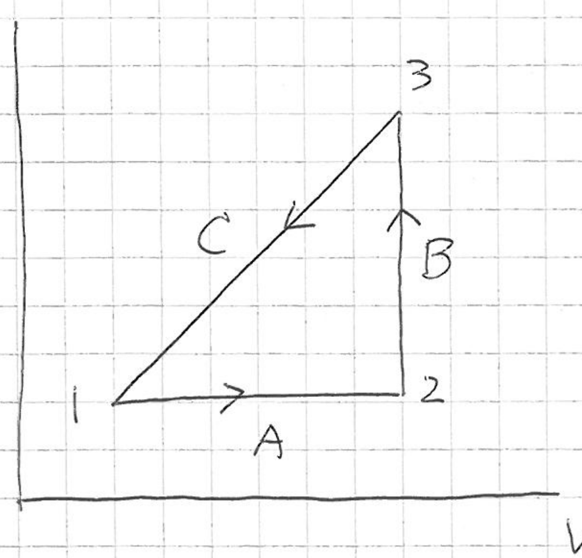
$$\approx 200\ J.$$

physically:



1.33 Ideal gas.

a) Work on gas W



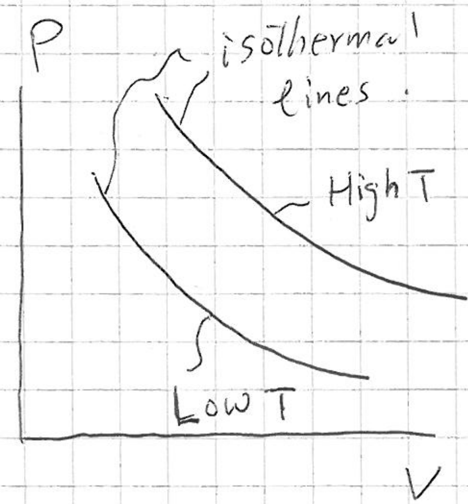
A) : $W<0$ (Expansion)

B) $W=0$ (isochoric)

C) $W>0$ (compression)

b) change in internal energy ΔU.

$U=\frac{1}{2}fNkT$. $\Delta U \propto \Delta T$.



A: $\Delta U > 0$ $(T_2 > T_1)$

B: $\Delta U > 0$ $(T_3 > T_2)$

C $\Delta U < 0$ $(T_1 < T_3)$

c) Heat to gas $Q = \Delta U - W$.

A: $Q>0$ (since $\Delta U > 0$, $W<0$)

B: $Q = \Delta U > 0$

C: $Q<0$ (since $\Delta U<0$, $W>0$)

For the whole cycle: $\begin{cases} W>0 & \text{(Counter clockwise)} \\ \Delta U = 0 & \text{(cyclic)} \\ Q = -W < 0 \end{cases}$

$\begin{cases} \text{Input work} \\ \text{output heat} \end{cases}$

* like a refrigerator. — NOT a practical one.

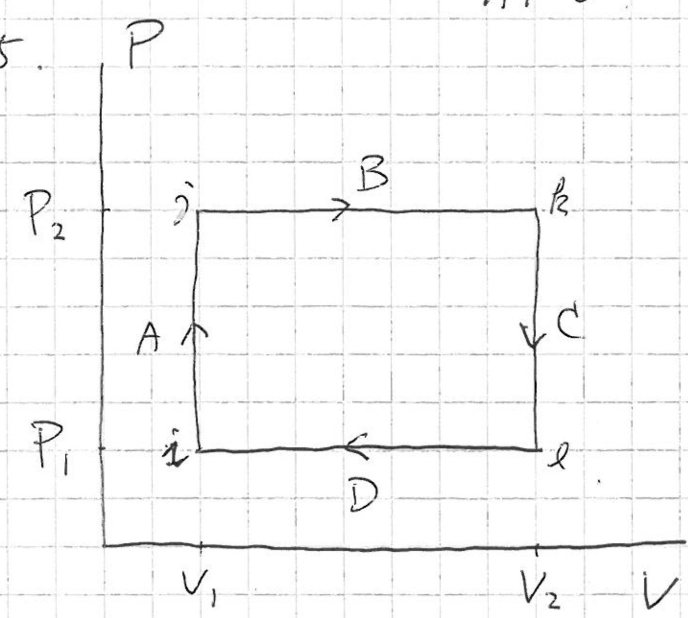
1.34 ideal diatomic gas: $f=5$.

$U = \frac{5}{2}NkT = \frac{5}{2}nRT$.

$W = -\int p\,dV$

$pV = NkT = nRT$.

$\Delta U = W + Q$



a).

A: $W = 0$.

$$\Delta U = \frac{5}{2}Nk(T_j - T_i)$$

$$= \frac{5}{2}(P_2V_1 - P_1V_1) = \frac{5}{2}V_1(P_2 - P_1).$$

$$Q = \Delta U - W = \frac{5}{2}V_1(P_2 - P_1)$$

B: $W = -P_2(V_2 - V_1) = P_2(V_1 - V_2)$

$$\Delta U = \frac{5}{2}Nk(T_k - T_j) = \frac{5}{2}(P_2V_2 - P_2V_1) = \frac{5}{2}P_2(V_2 - V_1)$$

$$Q = \Delta U - W = \frac{5}{2}P_2(V_2 - V_1) + P_2(V_2 - V_1) = \frac{7}{2}P_2(V_2 - V_1).$$

C: $W = 0$.

$$\Delta U = Q = \frac{5}{2}Nk(T_l - T_k) = \frac{5}{2}(P_1V_2 - P_2V_2) = \frac{5}{2}V_2(P_1 - P_2)$$

D: $W = -P_1(V_1 - V_2) = P_1(V_2 - V_1)$

$$\Delta U = \frac{5}{2}Nk(T_i - T_l) = \frac{5}{2}(P_1V_1 - P_1V_2) = \frac{5}{2}P_1(V_1 - V_2)$$

$$Q = \frac{5}{2}P_1(V_1 - V_2) + P_1(V_1 - V_2) = \frac{7}{2}P_1(V_1 - V_2).$$

b).

A : Heat is added to the gas, but no work (piston is fixed)

The gas gains thermal energy.

B: Heat is added to the gas, some of it becomes output work, some of it raises the energy of the gas.

C. Heat is released to the surroundings, No work.

As a result, the energy of gas decreases.

D. The energy of gas decreases, partly becomes output work, partly becomes heat loss.

c) Net work on gas for the cycle W: $W<0$ (Clockwise)

$$|W| = \text{area} = (P_2-P_1)(V_2-V_1).$$

$$\therefore \quad W = (P_2-P_1)(V_1-V_2) \quad < 0$$

$$\Delta U = 0.$$

$$Q = \Delta U - W = (P_2-P_1)(V_2-V_1) \quad > 0$$

It's a heat engine. The system converts the input heat to output work.