Phys 344.

Assignment 2. Solutions.

(Assignment #3. 2.1, 2.5b, 2.9)

1.36. $V_i = 1L = 1.0\times10^{-3}\ m^3$ $\qquad f=5.$ $\qquad PV = NkT = nRT$

$P_f = 7\ atm = 7.0\times10^5\ Pa.$ $\qquad PV^\gamma = const.$

$P_i = 1\ atm = 1.0\times10^5\ Pa.$ $\qquad \gamma = \frac{f+2}{f} = \frac{7}{5} = 1.4.$

T_i = Room Temp = 300 K

a) $P_f V_f^\gamma = P_i V_i^\gamma.$

$$V_f = V_i\left(\frac{P_i}{P_f}\right)^{1/\gamma} = 1\times10^{-3}\times\left(\frac{1\times10^5}{7\times10^5}\right)^{5/7} = 2.5\times10^{-4}\ m^3$$

$= 0.25\ L.$

b). $W = -\int P\,dV \qquad P = \frac{P_i V_i^\gamma}{V^\gamma}$

$$= -P_i V_i^\gamma \int_{V_i}^{V_f} V^{-\gamma}\,dV$$

$$= -\frac{P_i V_i^\gamma}{1-\gamma}\left(V_f^{1-\gamma} - V_i^{1-\gamma}\right)$$

$$= \frac{10^5\times(10^{-3})^{1.4}}{1.4-1}\left[(2.5\times10^{-4})^{-0.4} - (10^{-3})^{-0.4}\right]$$

$$= 15.8\times(27.59 - 15.85)$$

$= 186\ J$

$\approx 190\ J$

c). quasistatic adiabatic process.

$$PV^{\gamma} = \text{const.} \qquad (\text{ideal gas}: PV = NkT)$$

$$NkT \cdot V^{\gamma-1} = \text{const}.$$

$$\therefore \quad TV^{\gamma-1} = \text{const}. \qquad \gamma - 1 = \frac{f+2}{f} - 1 = \frac{f+2-f}{f} = \frac{2}{f}.$$

$$\text{OR.} \quad TV^{\frac{2}{f}} = \text{const}.$$

$$T_f V_f^{\frac{2}{f}} = T_i V_i^{2/f}.$$

$$T_f = T_i \left(\frac{V_i}{V_f}\right)^{2/f}$$

$$= 300\text{ K} \left(\frac{10^{-3}}{0.25 \times 10^{-3}}\right)^{2/5}$$

$$\approx 520\text{ K}.$$

1-38. When a bubble expands, it does work to outside, $\Delta U < 0$.

— bubble A expands adiabatically, the gas inside has to lower its temperature to do the expansion work. $(T \propto U)$ $(Q = 0)$

— bubble B. has a constant temperature which is higher than the temperature of bubble A. Therefore, Bubble B is larger.

1.41. Metal: $T_i = 100°C = 373\,K$. mass: m_M

$T_f = 24°C = 297\,K$.

Water: $T_i = 20°C = 293\,K$ mass: $m_W = 0.25\,kg$

$T_f = 24°C = 297\,K$. specific heat:

$c_W = 4.186 \times 10^3\ J/kg$.

a) $Q_W = m \cdot c_W \cdot (\Delta T)_W$

$= 0.25 \times 4.186 \times 10^3 \times (4) = 4186\,J$.

b) $4186\,J$. (Heat gained by water = heat loss in metal)

c) $C_M = \dfrac{Q}{\Delta T} = \dfrac{-4186}{-76} = 55\ J/K$.

d) $c_M = \dfrac{C_M}{m_M} = \dfrac{55}{0.1} = 550\ J/kg$

1.45. $w = xy$, $x = yz$. $\left(y = \frac{x}{z}\right)$

a). $w = x \cdot \frac{x}{z} = \frac{x^2}{z}$.

b). $\left(\frac{\partial w}{\partial x}\right)_y = y$.

$\left(\frac{\partial w}{\partial x}\right)_z = \frac{2x}{z}$.

c) $\left(\frac{\partial w}{\partial y}\right)_x = x$.

$w = y^2 z$.

$\left(\frac{\partial w}{\partial y}\right)_z = 2yz$.

$\left(\frac{\partial w}{\partial z}\right)_x = -\frac{x^2}{z^2}$.

$\left(\frac{\partial w}{\partial z}\right)_y = y^2$.

1.47.

Heat to remove from Tea: use. °C . g , cal

$$(200 \times 1\,cal/1^\circ g)(35^\circ C) = 7000\ cal.$$

Ice : mass $= m$. $c = 0.5\ c/g \cdot {}^\circ C$

— warm up to $0^\circ C$ from $-15^\circ C$.

— melt at $0^\circ C$ $80 = L$ (cal/g)

— warm up to $65^\circ C$ (water)

$$m(0.5)(15) + m(80) + m(1\,cal/g \cdot {}^\circ C)(65) = 7000.$$

Solve for $m = 46\,g$.

1.50. 1 mole of CH_4:

$$CH_4 + 2O_2 \longrightarrow CO_2 + 2H_2O.$$

$T = 298\ K$. $P = 10^5\ Pa$ (const).

a). $CH_4 \longrightarrow C + 2H_2$

ΔH for forming 1 mole of CH_4 $= -74.8$ (KJ)

∴ ΔH for converting a mole of methane to C and H_2 gas:

$$\Delta H = 74.8\ (KJ).$$

b). (1 mole)

$C + O_2 \longrightarrow CO_2$: $\Delta H = -393.5$ (kJ)

$H_2 + \frac{1}{2}O_2 \longrightarrow H_2O$: $\Delta H = -241.8$ (KJ).

∴ ΔH for forming ~~mole of CO_2 and~~ 2 moles of water vapor:

$$\Delta H = 2 \times (-241.8) = -483.6\ (kJ).$$

c) $CH_4 + 2O_2 \longrightarrow C + 2H_2$ ($\Delta H = 74.8$ KJ)

↳ $2H_2 + O_2 \rightarrow 2H_2O$

↳ $C + O_2 \rightarrow CO_2$

$$\Delta H = 74.8 - 393.5 - 483.6 = -802.3\ KJ.$$

d). 802.3 kJ. (Releasing heat)

e)

$$\Delta U = \Delta H - P\Delta V$$

$\Delta V = 0$.

$$V_i = \frac{n_i RT}{P_i}$$

$$V_f = \frac{n_f RT}{P_f}$$

$n_i = 3$.
1 mole of CH_4
2 moles of O_2.

$n_f = 3$.
1 mole of CO_2
2 moles of H_2O gas.

$\therefore \Delta U = \Delta H$
$= -802.3$ kJ.

However, if ~~H_2O~~ the 2 moles of H_2O is liquid.

Then, for $2\times(H_2 + \frac{1}{2}O_2 \rightarrow H_2O_{(liquid)})$

$$\Delta H = 2\times(-285.83) = -571.7 \text{ kJ}.$$

Then Total $\Delta H = 74.8 - 393.5 - 571.7$
$= -890.4$ kJ

$$V_f = \frac{1RT}{P_f}$$

($n_f = 1$ now)
CO_2 gas.

$\Delta U = \Delta H - P\Delta V$
$= \Delta H - (P_f V_f - P_i V_i$
$= \Delta H - (RT - 3RT)$
$= \Delta H + 2RT = -890.4 + 2\times 8.31\times 298 \times 10^{-3} = -885.4$ KJ.

f). 1 mole of CH_4 reacts with 2 moles of O_2.

$\therefore$ Total mass of rectance to produce 8.0×10^5 J.

$$m = (12 + 4 + 2 \times 32) = 16 + 64 = 80 \text{ g} = 0.08 \text{ kg}.$$

Total mass of the sun$/m$:

$$\frac{2 \times 10^{30}}{0.08} = 2.5 \times 10^{31}$$ moles of methane (with ideal proports with O_2).

Total energy it can give off.

$$2.5 \times 10^{31} \times 8 \times 10^5 = 2 \times 10^{37} \text{ J}.$$

time it can last:

? $$\frac{2 \times 10^{37}}{3.9 \times 10^{26}} = 5 \times 10^{10} \text{ sec} \approx 1600 \text{ years}.$$

That must NOT be the mechanism.