Phys 344

Assignment #5 Solutions.

3.35. $\mu = \left(\frac{\partial U}{\partial N}\right)_{S,V}$ $\quad U = q\varepsilon. \quad N=3, \quad q=4$

$$\Omega = \binom{N+q-1}{q} = \binom{6}{4} = \frac{6!}{4!\,2!} = \frac{5\times 6}{2} = 15.$$

For $N=4, \quad q=3,$

$$\Omega = \binom{6}{3} = \frac{6!}{3!\,3!} = \frac{4\times5\times6}{2\times3} = 20.$$

For $N=4, \quad q=2.$

$$\Omega = \binom{5}{2} = \frac{5!}{2!\,3!} = \frac{4\times5}{2} = 10.$$

To keep S the same, ($\Omega = 15$), q should be between 2 and 3.

$$\mu = \frac{(q-4)\varepsilon}{1} = -(4-q)\varepsilon. \qquad (2 < q < 3)$$

$$-2\varepsilon < \mu < -\varepsilon.$$

3.37. $U = U(0) + Nmgz$.

a) $$\mu(z) = \left(\frac{\partial U}{\partial N}\right)_{S,V}.$$

$$= \left(\frac{\partial U(0)}{\partial S}\right)_{S,V} + mgz.$$

$$= \mu(0) + mgz.$$

$$\mu(0) = -kT \ln\left[\frac{V}{N}\left(\frac{2\pi m kT}{h^2}\right)^{3/2}\right] \quad \text{— (3.63)}$$

$$\left(\text{from } \mu = -T\left(\frac{\partial S}{\partial N}\right)_{U,V} \text{ for } z=0\right).$$

$$\therefore \quad \mu(z) = -kT \ln\left[\frac{V}{N}\left(\frac{2\pi m kT}{h^2}\right)^{3/2}\right] + mgz.$$

b). $$\mu(z) = \mu(0)$$

$$-kT \ln\left[\frac{V}{N(z)}\left(\frac{2\pi m kT}{h^2}\right)^{3/2}\right] + mgz = -kT \ln\left[\frac{V}{N(0)}\left(\frac{2\pi m kT}{h^2}\right)^{3/2}\right]$$

$$kT \ln N(z) + mgz = kT \ln N(0).$$

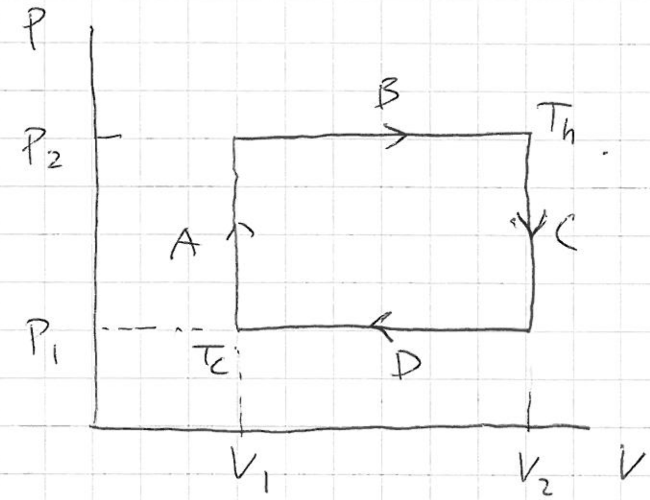
$$\ln\frac{N(z)}{N(0)} = -mgz/kT.$$

$$\frac{N(z)}{N(0)} = e^{-mgz/kT}.$$

$$N(z) = N(0)\, e^{-mgz/kT}.$$

AS-3.

4.1 . $f=5$. $\gamma = \frac{7}{5}$



a). $V_2 = 3V_1$, $P_2 = 2P_1$

Heat absorbed:

$$Q_A = C_V \Delta T$$

$$= \frac{5}{2} nR \cdot \frac{(P_2V_1 - P_1V_1)}{nR}$$

$$= \frac{5}{2} V_1 (P_2 - P_1)$$

$PV = nRT$.

$T = \frac{PV}{nR}$.

$$Q_B = C_p \Delta T$$

$$= \frac{7}{2} nR \cdot \frac{P_2V_2 - P_2V_1}{nR}$$

$$= \frac{7}{2} P_2 (V_2 - V_1).$$

$$Q_h = Q_A + Q_B = \frac{5}{2} V_1 (P_2 - P_1) + \frac{7}{2} P_2 (V_2 - V_1)$$

$$= \frac{5}{2} V_1 P_1 + \frac{7}{2} 2P_1 \cdot 2V_1$$

$$= \frac{33}{2} P_1 V_1.$$

Total work output: $W = \text{area} = (V_2 - V_1)(P_2 - P_1)$

$$= 2V_1 \cdot P_1$$

$$e = \frac{W}{Q_h} = \frac{4}{33} \approx 12\%.$$

b). $e_{carnot} = 1 - \frac{P_1V_1/nR}{P_2V_2/nR} = 1 - \frac{P_1 \cdot V_1}{2P_1 3V_1} = 1 - \frac{1}{6} = 83\%$

4.2. a) $e_1 = 1 - \frac{T_c}{T_h}$ $\quad T_h = 773\text{ K}. \quad T_c = 293\text{ K}$

$= 1 - \frac{293}{773}$

$= 62\% \quad (62.1\%)$

b). $e_2 = 1 - \frac{293}{873} = 66.4\%$.

~~Energy per second:~~ $\cancel{= 10^9 \times \frac{66.4}{62.1} = 1.070 \times 10^9}$ ~~watts~~

~~Additional energy per second:~~ $\cancel{7.0 \times 10^7}$ ~~J~~

New power: $= 10^9 \times \frac{66.4}{62.1} = 1.070 \times 10^9$ watts.

Additional power: 7.0×10^7 watts $= 7.0 \times 10^4$ KW.

Additional Energy per year: $365 \times 24 \times 7.0 \times 10^4$

$= 6.13 \times 10^8$ KW-hr.

$\$ = 6.13 \times 10^8 \times 0.05 = 3.1 \times 10^7$ (Dollars).

4.4.

a). $e_{max} = 1 - \frac{T_c}{T_h}$ $T_c = 4°C = 277K$

$T_h = 22°C = 295K$.

$e_{max} = 1 - \frac{277}{295} = 6.1\%$

b). $e = \frac{W}{Q_h}$. $Q_h = \frac{W}{e} = \frac{1\times10^9}{0.061} = 1.64\times10^{10}$ J.

$Q = mC \cdot \Delta T$. $\Delta T = 22° - 4°C = 18K$

$m = \frac{Q_h}{C\Delta T}$ $c = 4186$ J/kg·K

$= \frac{1.64\times10^{10}}{4186\times18}$

$= 2.2\times10^5$ kg.

$V = \frac{m}{\rho}$ $\rho = 1000$ kg/m^3

$= \frac{2.2\times10^5}{1000}$

$= 220$ m^3.

4.5 . (2→3 and 4→1 are adiabatic.)

1→2.

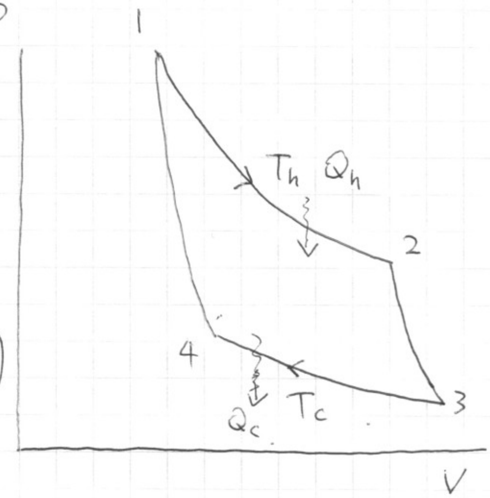
$$Q_h = Q_{12}$$

$$= -W_{12} \quad (\Delta U = 0).$$

$$= \int_1^2 P\,dV \quad \left(W_{12} \text{ — work on system}\right)$$

$$= nRT_h \int_1^2 \frac{dV}{V} \qquad P = \frac{nRT_h}{V}$$

$$= nRT_h \ln\frac{V_2}{V_1}.$$



3→4. $Q_c = -Q_{34}$ Q_{34} — heat absorbed by system.

$$= W_{34} \qquad W_{34} \text{ — work on system}$$

$$= -\int_3^4 P\,dV \qquad \Delta U = 0 \ \text{(isothermal)}$$

$$= -nRT_c \ln\frac{V_4}{V_3}$$

$$= nRT_c \ln\frac{V_3}{V_4}.$$

2→3, 4→1 : $T_2 V_2^{\gamma-1} = T_3 V_3^{\gamma-1}$, $T_4 V_4^{\gamma-1} = T_1 V_1^{\gamma-1}$.

$$\frac{T_2}{T_3} = \left(\frac{V_3}{V_2}\right)^{\gamma-1} = \frac{T_h}{T_c} = \frac{T_1}{T_4} = \left(\frac{V_4}{V_1}\right)^{\gamma-1}.$$

$$\therefore \frac{V_3}{V_2} = \frac{V_4}{V_1} \Rightarrow \frac{V_2}{V_1} = \frac{V_3}{V_4}$$

$$e = \frac{Q_h - Q_c}{Q_h} = \frac{T_h - T_c}{T_h} = 1 - \frac{T_c}{T_h}.$$

$\left(nR \ln\frac{V_2}{V_1} = nR \ln\frac{V_3}{V_4} \text{ cancels}\right)$

4.10. $COP = \frac{T_c}{T_h - T_c}$. $\left.\begin{array}{l} T_c = 255\text{ K} \\ T_h = 300\text{ K} \end{array}\right\}$ Typically.

$$= \frac{255}{300-255}$$

$$= 5.7$$

$(\text{Power needed}) \times COP = \text{Power wasted}$.

$$\text{Power needed} = \frac{\text{Power wasted}}{COP}$$

$$= \frac{300\text{ Watts}}{5.7}$$

$$= 53\text{ watts}$$

$$\approx 50\text{ watts}.$$

4.14. a). because Q_h is the benefit — heat pumped into the building.

W is the cost.

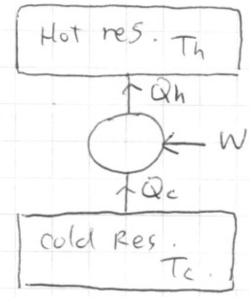
$\therefore$ $COP = Q_h / W$.

b). $W + Q_c = Q_h$. $\Rightarrow COP = \dfrac{Q_h}{Q_h - Q_c}$.

Yes, $COP > 1$, since $Q_h > W$.

c). $\Delta S_{Total} = \Delta S_{HR} + \Delta S_{CR} + \underset{\substack{\downarrow \\ 0 \\ \text{cyclic}}}{\Delta S_{HP}}$

$\geqslant 0$.



$\therefore$

$$\Delta S_{HR} + \Delta S_{CR} = \frac{Q_h}{T_h} - \frac{Q_c}{T_c} \geqslant 0.$$

$$\frac{Q_h}{Q_c} \geqslant \frac{T_h}{T_c}.$$

$$COP = \frac{Q_h}{Q_h - Q_c} = \frac{1}{1 - Q_c/Q_h}.$$

$$\frac{Q_c}{Q_h} \leqslant \frac{T_c}{T_h}.$$

$$-\frac{Q_c}{Q_h} \geqslant -\frac{T_c}{T_h}$$

$$1 - \frac{Q_c}{Q_h} \geqslant 1 - \frac{T_c}{T_h}.$$

$$\frac{1}{1 - \frac{Q_c}{Q_h}} \leqslant \frac{1}{1 - T_c/T_h}$$

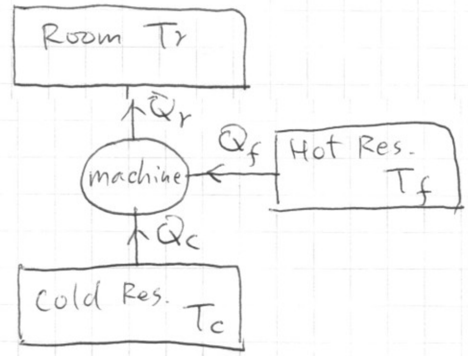
$$\therefore COP = \frac{1}{1 - Q_c/Q_h} \leqslant \frac{1}{1 - T_c/T_h}.$$

$$COP_{max} = \frac{1}{1 - T_c/T_h} = \frac{T_h}{T_h - T_c}.$$

d). $COP > 1$. whereas an electric furnace can only reach 100%.

4.15 . a) Benefit $= Q_c$, Cost $= Q_f$.

$\therefore \ COP = \dfrac{Q_c}{Q_f}$.



b) . $Q_c + Q_f = Q_r$.

(input = output) .

Can't tell if $\dfrac{Q_c}{Q_f}$ can be > 1 .

c) . $\Delta S_{Total} \geq 0$. (also, $\Delta S_{machine} = 0$ cyclic)

$\therefore \ \Delta S_{Room} + \Delta S_{HR} + \Delta S_{CR} \geq 0$.

$$\frac{Q_r}{T_r} - \frac{Q_f}{T_f} - \frac{Q_c}{T_c} \geq 0 . \quad (Q_r = Q_f + Q_c)$$

$$\frac{Q_f}{T_r} + \frac{Q_c}{T_r} - \frac{Q_f}{T_f} - \frac{Q_c}{T_c} \geq 0$$

$$\frac{Q_f}{T_r} - \frac{Q_f}{T_f} \geq \frac{Q_c}{T_c} - \frac{Q_c}{T_r} .$$

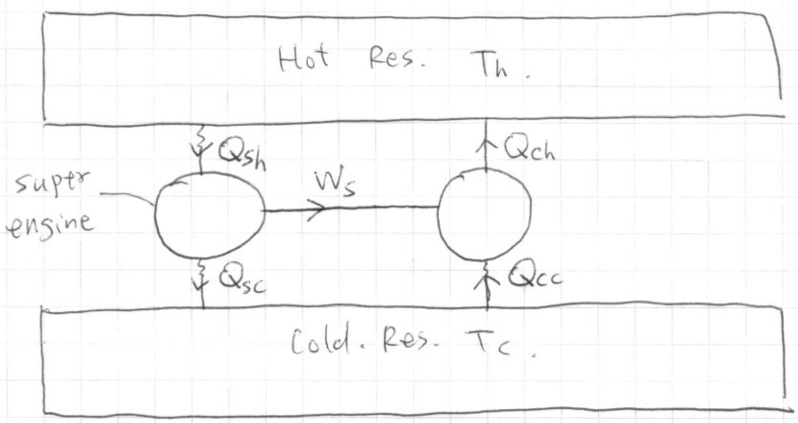
$$\frac{\frac{1}{T_c} - \frac{1}{T_r}}{\frac{1}{T_r} - \frac{1}{T_f}} \geq \frac{Q_c}{Q_f} .$$

$$COP = \frac{Q_c}{Q_f} \leq \frac{\left(\frac{1}{T_c} - \frac{1}{T_r}\right) T_c T_r T_f}{\left(\frac{1}{T_r} - \frac{1}{T_f}\right) T_c T_r T_f} = \frac{(T_r - T_c) T_f}{(T_f - T_r) T_c} .$$

4.16 . For the "super engine": $e_s > 1 - \frac{T_c}{T_h} = \frac{T_h - T_c}{T_h}$

For the Carnot refrigerator: $COP_c = \frac{T_c}{T_h - T_c}$.

We can hook up the 2 machines between 2 reservoirs.



$$\frac{W}{Q_{sh}} > \frac{T_h - T_c}{T_h}.$$

$$Q_{sh} < \frac{T_h}{T_h - T_c} W_s.$$

$$Q_{sc} = Q_{sh} - W.$$

$$Q_{sc} < \frac{T_c}{T_h - T_c} W_s$$

$$\frac{Q_{cc}}{W_s} = \frac{T_c}{T_h - T_c}.$$

$$Q_{cc} = \frac{T_c}{T_h - T_c} \cdot W_s.$$

$$Q_{ch} = W_s + Q_{cc} = \frac{T_h}{T_h - T_c} W_s.$$

Net effect after a cycle: No work input.

Heat out of Cold Res: $Q_c = Q_{cc} - Q_{sc} > 0$.

Heat into Hot Res: $Q_h = Q_{ch} - Q_{sh} > 0$.

That's a refrigerator that requires no work input.