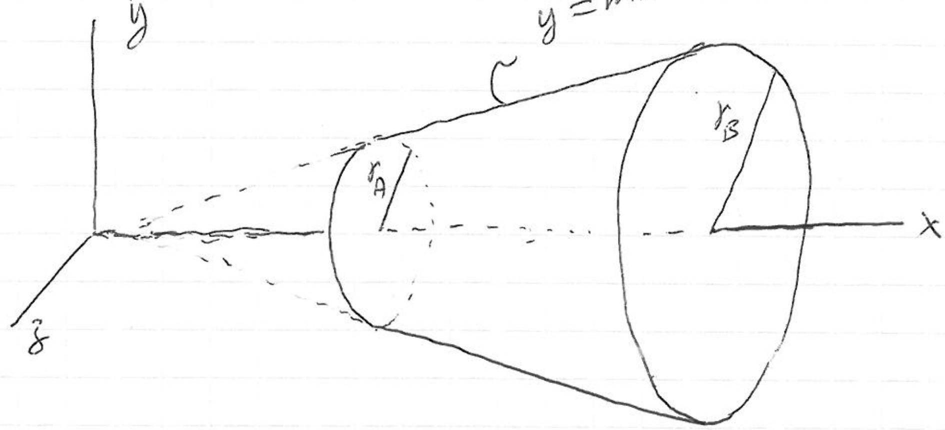
# Phys 344

## Assignment 9. Solutions.

1. Frustum




$$Q = -k A \frac{dT}{dx} = \text{constant}.$$

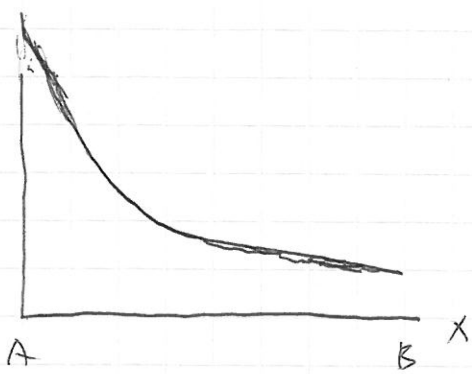
$$A = A(x) = \pi r^2 = \pi (mx)^2$$

$$\frac{dT}{dx} = \frac{-\text{constant}}{\pi k (mx)^2} = -\frac{C}{x^2}$$

$C$ — possitive constant.

As $x$ increases, the slope $\frac{dT}{dx}$ becomes shallower.

Therefor, (iii) is the answer.




* more quantitatively:

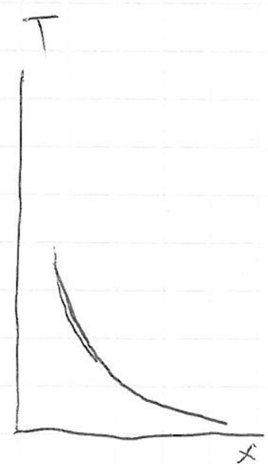
$$dT = -\frac{C\,dx}{x^2}.$$

$$T - T_0 = C\left(\frac{1}{x} - \frac{1}{x_0}\right)$$

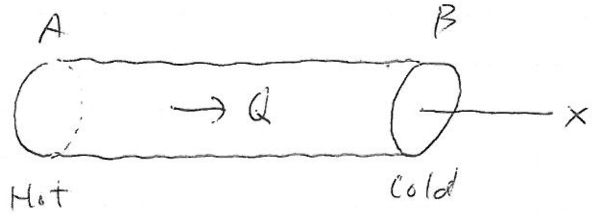
$$T = T_0 - \frac{C}{x_0} + \frac{C}{x}.$$

$$\therefore T = \text{Const}_1 + \frac{\text{const}_2}{x}.$$


OR



2. Cylindrical rod.



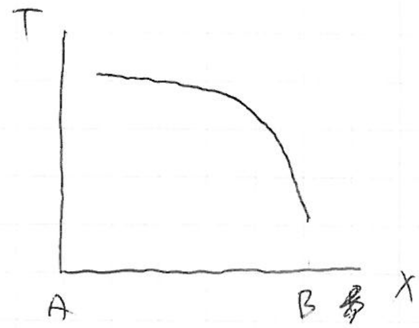

$$Q = -kA \frac{dT}{dx} = \text{const}.$$

$$\frac{dT}{dx} = -\frac{\text{const}}{Ak}.$$

as $x$ increases, $T$ decreases, $k$ decreases, $\frac{1}{k}$ increases.

slope $\frac{dT}{dx}$ becomes steeper. (negative, too).

$\therefore$ (iii) is the answer.

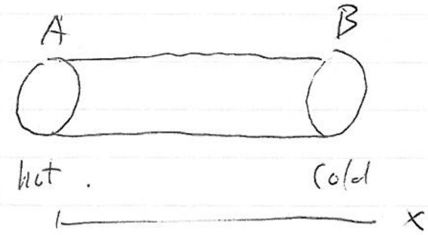



* more quantitatively,

$$\frac{d^2T}{dx^2} = \frac{C}{Ak^2} \cdot \frac{dk}{dx} \quad \text{where } \frac{dk}{dx} < 0. \ (T_B < T_A.$$

$\therefore \frac{d^2T}{dx^2} < 0.$ (concave down. )

3. For a non-radioactive rod,




T decreases with x monotonically.

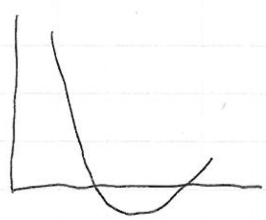
for $x_A < x < x_B$, $T \geq T_B$.

For a radioactive rod, there are heat sources throughout the rod.

~~The tempe~~ $\therefore$ $T > T_B$ for $A < x < B$.

i.e, it is impossible that the temperature anywhere in the rod is lower than $T_B$.

$\therefore$ (ii) is the impossible case.



* More quantitatively.

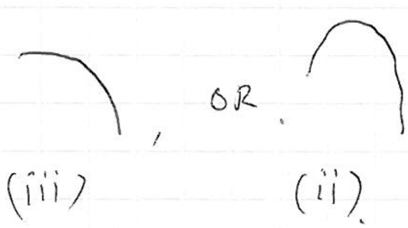
$$\nabla^2 T + \frac{q}{k} = 0.$$

$$\nabla^2 T = -\frac{q}{k}.$$

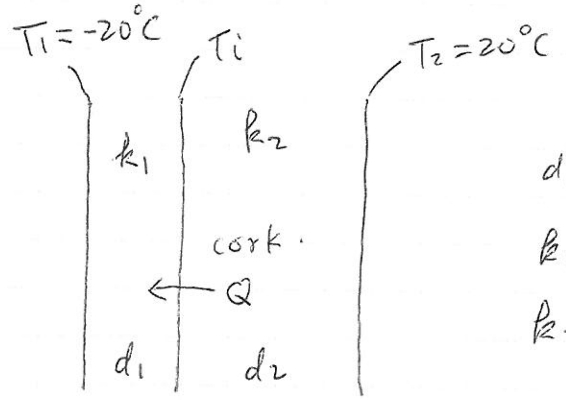
for 1-D long rod. (as a model). $\frac{\partial^2 T}{\partial y^2} = 0$, $\frac{\partial^2 T}{\partial z^2} = 0$

$$\frac{d^2 T}{dx^2} = -\frac{q}{k} < 0.$$

$T \sim x$ should be concave down.



4.




$d_1 = 0.1$ m

$k_1 = 5$ W/mK

$k_2 = 0.045$ W/mK.

$$\frac{Q}{A} = k_1 \frac{\Delta T_1}{\Delta X_1}$$

$$= k_1 \frac{T_i - T_1}{d_1}.$$

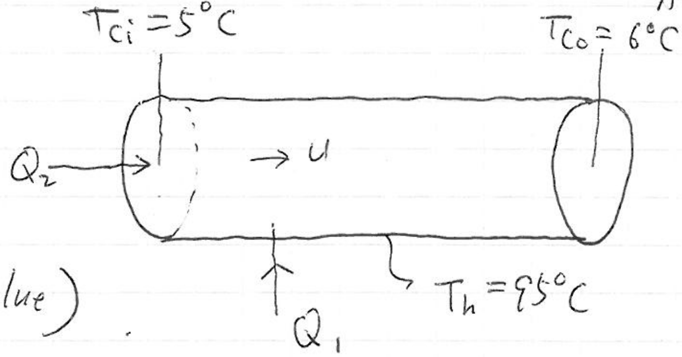
$$10 = 5 \frac{T_i + 20}{0.1}$$

$$T_i = -20 + 0.2 = -19.8\,°C.$$

$$\frac{Q}{A} = k_2 \frac{T_2 - T_i}{d_2}.$$

$$d_2 = \frac{k_2 (T_2 - T_i)}{Q/A} = \frac{0.045\,[20 - (-19.8)]}{10} = 0.18 \text{ m}.$$

6. Water,
Turbulent flow.




$u = 2u'$.

(prime ' means old value).

$$Re = \frac{u \rho d}{\mu} = 2 Re'$$

$$Nu = 0.023 Re^{0.8} Pr^{0.33} = 2^{0.8} Nu' \quad \left(= \frac{hd}{k}\right)$$

$$h = 2^{0.8} h' = 1.74 h'.$$

$$Q_1 = hA(T_h - T_a) \qquad \left( \begin{array}{l} \text{assume } T_h - T_a = (T_h - T_a)' \\ \text{since } T_h \gg (T_{co} - T_{ci}') \end{array} \right)$$

$$= 1.74 Q_1'$$

$$Q_2 = \dot{m} C (T_{co} - T_{ci}) = 2\dot{m}' C (T_{co} - T_{ci})' \frac{T_{co} - T_{ci}}{(T_{co} - T_{ci})'}$$

$$= 2 Q_2' \frac{T_{co} - T_{ci}}{(T_{co} - T_{ci})'}$$

$Q_1 = Q_2$, $\quad Q_1' = Q_2'$ (energy conservation).

$$1.74 Q_1' = 2 Q_2' \frac{T_{co} - T_{ci}}{T_{co}' - T_{ci}'} \qquad T_{co}' - T_{ci}' = 1°C.$$

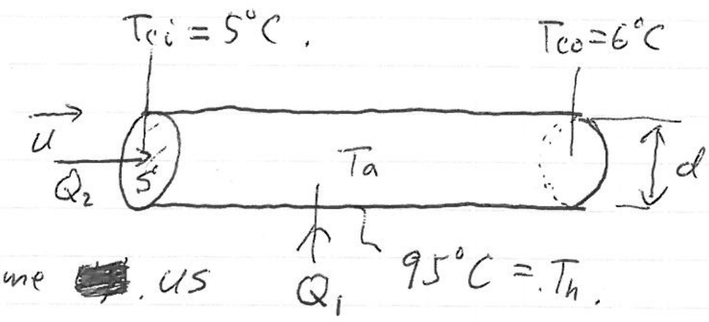
$$1.74 = 2(T_{co} - T_{ci})$$

$$T_{co} = T_{ci} + \frac{1.74}{2} = 5 + 0.87 = 5.87°C.$$

$\therefore$ The exit temperature decreases to 5.87°.
(i.e, increases less than it did originally)

7. Water,

Turbulent flow.




Same $\dot{m} = u\rho S \Rightarrow$ Same . $uS$

$d = \frac{1}{2} d'$ ( ' means old value ).

$S = \frac{1}{4} S'$, $u = 4u'$

$$Re = \frac{\rho d u}{\mu} = 2 Re'$$

$$Nu \propto Re^{0.8} \Rightarrow Nu = Nu' \cdot 2^{0.8} = 1.74\, Nu'$$

$$h = \frac{Nu\, k}{d} = h' \frac{1.74}{0.5} = 3.48\, h'$$

$(T_h - T_a) \approx (T_h - T_a)'$

$$Q_1 = hA(T_h - T_a) = h \cdot \pi d L (T_h - T_a)$$

$$\approx (3.48)\, h' \cdot (0.5) A (T_h - T_a) = 1.74\, Q_1'$$

$$Q_2 = \dot{m} c (T_{co} - T_{ci}) = \dot{m} c \cdot (T_{co} - T_{ci})' \cdot \frac{T_{co} - T_{ci}}{(T_{co} - T_{ci})'}$$

$$= Q_2' \cdot \frac{T_{co} - T_{ci}}{(T_{co} - T_{ci})'}$$

Conservation of energy: $Q_1 = Q_2$, $Q_1' = Q_2'$

$$1.74 = \frac{T_{co} - T_{ci}}{(T_{co} - T_{ci})'}$$

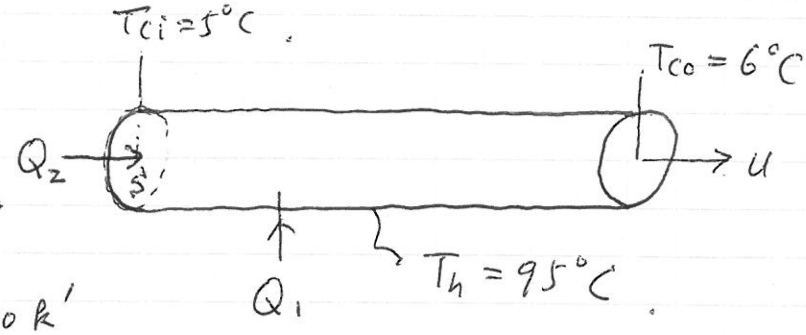
$(T_{co} - T_{ci})' = 6°C - 5°C = 1°C$.

$$T_{co} = T_{ci} + 1.74 \times (1) = 6.74°C$$

$\therefore$ The exit temperature will increase to 6.74°C.

8. Oil (replacing water).

Turbulent flow.




$\rho = 800\ kg/m^3 = 0.80\rho'$

$k = 0.132\ W/m \cdot K = 0.210\,k'$

$\mu = 4.5 \times 10^{-3}\ N s\, m^{-2} = 4.5\,\mu'$

$c = 300\ J/kg \cdot K = 0.0714\, c'$

Same $\dot{m} = \rho u \cdot S$, same $d$, same $S$.

$\therefore$ same $\rho u$.

[again, $'$ (prime) means old value, (water)].

$$Re = \frac{\rho d u}{\mu} = \frac{Re'}{4.5} = 0.222 Re'$$

$$Pr = \frac{c\mu}{k} = Pr' \frac{0.0714 \times 4.5}{0.21} = 1.53\, Pr'$$

$$Nu = 0.023\, Re^{0.8}\, Pr^{0.33} = Nu' \cdot (0.222)^{0.8}\, (1.53)^{0.33} = 0.343\, Nu'$$

$$h = \frac{k\, Nu}{d} = h' \cdot (0.343)(0.21) = 0.0720\, h'$$

$$Q_1 = hA(T_h - T_a) \approx 0.0720\, Q_1' \quad [\ T_h - T_a \approx (T_h - T_a)'\ ]$$

$$Q_2 = \dot{m} c\,(T_{co} - T_{ci}) = \dot{m}' c' (0.0714) \cdot (T_{co} - T_{ci})' \cdot \frac{T_{co} - T_{ci}}{(T_{co} - T_{ci})'}$$

$$= 0.0714\, Q_2' \cdot \frac{T_{co} - T_{ci}}{(T_{co} - T_{ci})'} = 0.0714\, Q_2' (T_{co} - T_{ci})$$

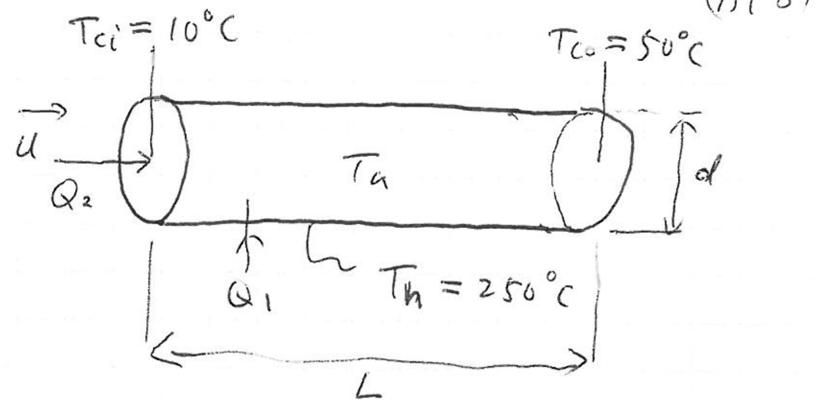
$Q_1 = Q_2$, $Q_1' = Q_2'$

$\therefore\ 0.0720 = 0.0714\,(T_{co} - T_{ci})$

$T_{co} = T_{ci} + 1.01 = 6.01°C$.

$\therefore$ The exit temperature will marginally increase.
(or approximately unchanged)

9. Air.




$d = 20\ mm = 0.02\ m$.

$r = 0.01\ m = 1 \times 10^{-2}\ m$.

$u = 1\ m/s$.

$\rho = 1\ kg/m^3$

$\mu = 2 \times 10^{-5}\ N \cdot s\, m^{-2}$.

$k = 0.03\ W/m \cdot K$.

$c = 1000\ J/kg \cdot K$.

$\Rightarrow$. $Re = \dfrac{\rho \cdot d \cdot u}{\mu} = \dfrac{1 \times 0.02 \times 1}{2 \times 10^{-5}} = 1000$.

Laminar flow !

find $L = ?$

Laminar flow at constant wall temp : $Nu = 3.658$

$$h = \frac{Nu \cdot k}{d} = \frac{3.658 \times 0.03}{0.02} = 5.487.$$

$$Q_1 = h \cdot \pi d L (250° - 30°) = \dot{m} c (50° - 10°)$$

($T_a \approx 30°$. Since $250° >> 30°$)

$$L = \frac{\rho u \cdot \pi r^2 \cdot c (50° - 10°)}{\pi h d (250° - 30°)} = \frac{\rho u d c\ (40°C)}{4 h\ \ (220°C)}$$

$$= \frac{1 \times 1 \times 0.02 \times 1000 \times 40}{4 \times 5.487 \times 220}$$

$$= 0.166\ m.$$

10. The counter-flow heat exchanger gives a higher exit temperature for the initially cool fluid. The exit temperature is ~~a~~ close to the initial temperature of the hot fluid. The parallel one would yield an exit temp $\approx$ average temp.