Phys 344

Assignment #4. Solutions.

3.1. $\quad T = \left(\dfrac{\partial U}{\partial S}\right)_{N,V}$. $\qquad U = q\varepsilon$.

when $q_A = 1$. $\quad T_A = \dfrac{\Delta U_A}{\Delta S_A} = \dfrac{(2\varepsilon - 0\varepsilon)}{(10.7-0)k} = 0.19\ \varepsilon/k$.

$$= \frac{0.19 \times 0.1 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23}}$$

$$= 220\ \text{K}.$$

~~when $q_A < 60$~~. $\quad T_B = \dfrac{\Delta U_B}{\Delta S_B} = \dfrac{(98-100)\varepsilon}{(185.3-187.5)k} = 0.91\ \varepsilon/k$

$$= 1060\ \text{K}$$

when $q_A = 60$. $\quad T_A = \dfrac{\Delta U_A}{\Delta S_A} = \dfrac{(61-59)\varepsilon}{(160.9-157.4)k} = 0.57\ \varepsilon/k$

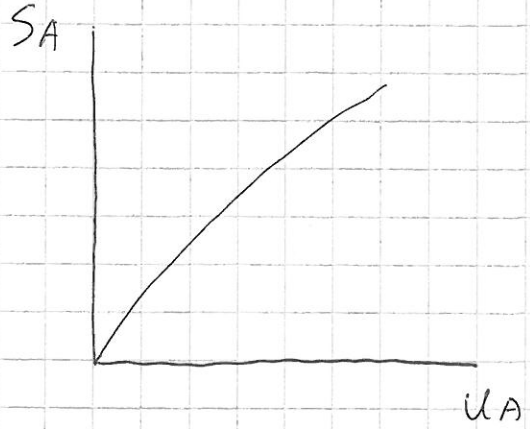
$$= 660\ \text{K}$$

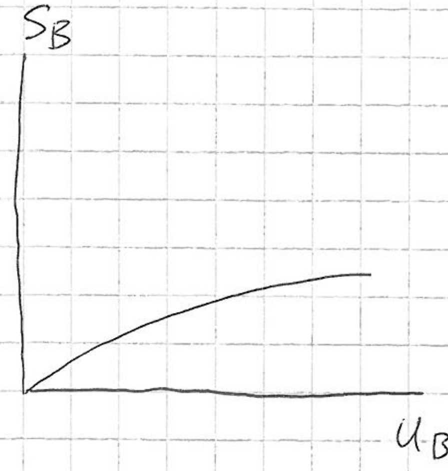
$$T_B = \frac{\Delta U_B}{\Delta S_B} = \frac{(39-41)\varepsilon}{(103.5-107.0)k}$$

$$= 660\ \text{K}.$$

3.3

$$S = S_A + S_B.$$

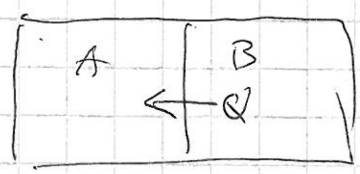




$$\frac{\partial S_A}{\partial U_A} > \frac{\partial S_B}{\partial U_B} > 0.$$

If energy is transfered from B to A, the gain in S_A is greater than the loss in S_B, which leads to the increase in S_{Total}.

∴ That's the spontaneous tendency.



3.10. $m = 30g = 0.03\ kg$.

a). $\Delta S = \dfrac{Q}{T}$

Latent heat for melting ice

$L = 333\ J/g = 80\ cal/g$.

$= \dfrac{30 \times 333}{273}$

$= 36.6\ J/K$

b) $\Delta S = \int \dfrac{dQ}{T}$

specific heat of water

$C = 4.186\ J/gK$

$= \int_{273}^{298} \dfrac{C\ dT}{T}$

$= C \ln \dfrac{T_f}{T_i}$

$= 4.186 \times 30 \times \ln \dfrac{298}{273}$

$= 11.0\ J/K$.

c) $\Delta S_1 = \dfrac{Q}{T} = \dfrac{-30 \times 333}{298} = -33.52\ J/K$.

$\Delta S_2 = \dfrac{-30 \times 4.186 \times (298 - 273)}{298} = -44.1\ J/K$

$\Delta S = \Delta S_1 + \Delta S_2 = -$.

d) $\Delta S_{Total} = 36.6 + 11.0 - 44.1$

$= 23.5\ J/K$.

3.13.

a). In a second. $Q = 1000\,J$

In a day: $Q = 1000\,J \times 8\,hr \times 3600 = 2.88 \times 10^7\,J$

In a year: $Q = 2.88 \times 10^7 \times 365 = 1.05 \times 10^{10}\,J$.

$$\Delta S = \frac{Q}{T} = \frac{1.05 \times 10^{10}\,J}{300} = 3.5 \times 10^7\,J/K.$$

b) This square meter of earth is not an isolated system. Therefore, its entropy does not have to increase. In fact, although the growth of grass decreases the entropy, the total entropy of the combined system that includes the sun and the surroundings increases by a much larger amount than the decrease of entropy due to the growth of grass.

3.14. $C_V = aT + bT^3$, $a = 0.00135\ J/K^2$

$b = 2.48 \times 10^{-5}\ J/K^2$

$$S(T) = S(T) - S(0) = \int_0^T \frac{dQ}{T}$$

$$= \int_0^T \frac{C_V(T)\,dT}{T}$$

$$= \int_0^T (a + bT^2)\,dT$$

$$= aT + \frac{1}{3}bT^3$$

$$S(1K) = 0.00135 \times 1 + \frac{1}{3} \times 2.48 \times 10^{-5} \times 1^3 = 0.00136\ J/K.$$

$$S(10K) = 0.00135 \times 10 + \frac{1}{3} \times 2.48 \times 10^{-5} \times 10^3 = 0.0218\ J/K.$$

$$\frac{S(1K)}{k} = \frac{0.00136}{1.38 \times 10^{-23}} = 9.8 \times 10^{19}.$$

$$\frac{S(10K)}{k} = \frac{0.0218}{1.38 \times 10^{-23}} = 1.58 \times 10^{21}$$

3.31. 1 mol of graphite.

$$C_p = a + bT - \frac{c}{T^2}. \qquad \begin{cases} a = 16.86 \text{ J/K} \\ b = 4.77 \times 10^{-3} \text{ J/K}^2 \\ c = 8.54 \times 10^{5} \text{ J}\cdot\text{K}^2 \end{cases}$$

$$\Delta S = \int_i^f \frac{dQ}{T} = \int_i^f \frac{C_p}{T} dT = \int_i^f \left(\frac{a}{T} + b - \frac{c}{T^3}\right) dT$$

quasistatic.

$$= \left(a \ln T + bT + \frac{c}{2T^2}\right)\Big|_{298}^{500}$$

$$= 16.86 \cdot \ln\frac{500}{298} + b(500-298) + \frac{8.54\times10^5}{2}\left(\frac{1}{500^2} - \frac{1}{298^2}\right)$$

$$=$$

$$= 6.59 \text{ J/K}.$$

at 298 K. $S = 5.74$ J/K.

at 500 K. $S(500) = S(298) + \Delta S$

$$= 5.74 + 6.59$$

$$= 12.33 \text{ J/K}.$$

3.32

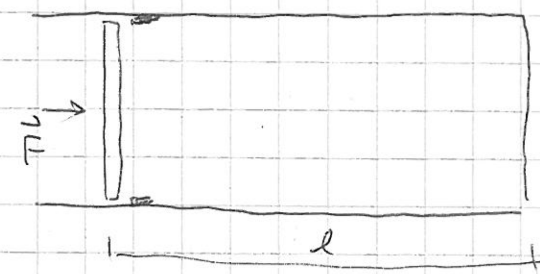
$V = 1L = 10^{-3}\,m^3$.

$T = 300\,K$

$P = 10^5\,Pa$.

$A = 0.01\,m^2$.

$F = 2000\,N$. (very suddenly).



$dl = -10^{-3}\,m$. — small.

a) $W = \vec{F} \cdot \vec{dl}$

$= 2000 \times 10^{-3} = 2\,J$.

b) $Q = 0$. — very suddenly. — adiabatic (No time for heat transfer)

c) $\Delta U = W + Q$

$= W = 2J$.

$$dS = \frac{1}{T} \cdot dU + \frac{P}{T} \cdot dV.$$

$$\Delta S \approx \frac{1}{T} \Delta U + \frac{P}{T} \Delta V.$$

$$= \frac{2J}{300\,K} + \frac{10^5\,Pa}{300K}(-10^{-5})$$

$$= \frac{1}{300}\,J/K$$

$$= 3.33 \times 10^{-3}\,J/K.$$

$\Delta V = A \cdot dl$

$= 0.01 \times (-10^{-3})$

$= -10^{-5}\,m^3$.