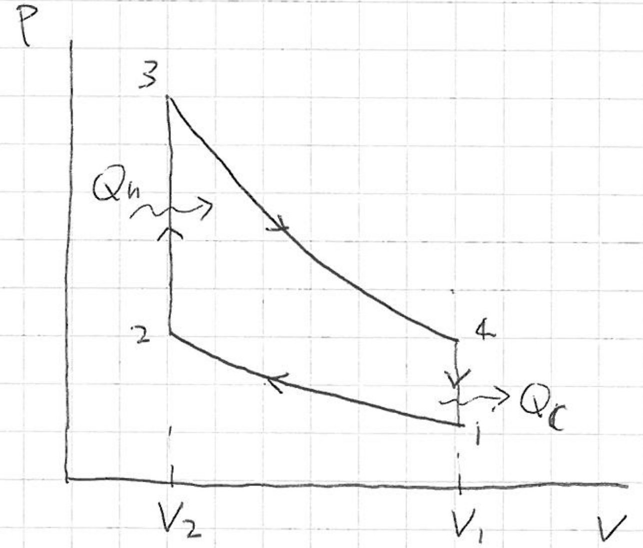
Phys 344

Assignment #6. Solutions.

4.18 Otto cycle — ideal gas quasistatic.



$$e = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h} \quad (1)$$

$$Q_h = \Delta U = \frac{f}{2} Nk (T_3 - T_2) \quad (2)$$

$$Q_c = \Delta U = \frac{f}{2} Nk (T_4 - T_1) \quad (3)$$

Adiabatic $TV^{\gamma-1} = \text{const}$:

$$T_3 V_2^{\gamma-1} = T_4 V_1^{\gamma-1} \quad (4)$$

$$T_2 V_2^{\gamma-1} = T_1 V_1^{\gamma-1} \quad (5)$$

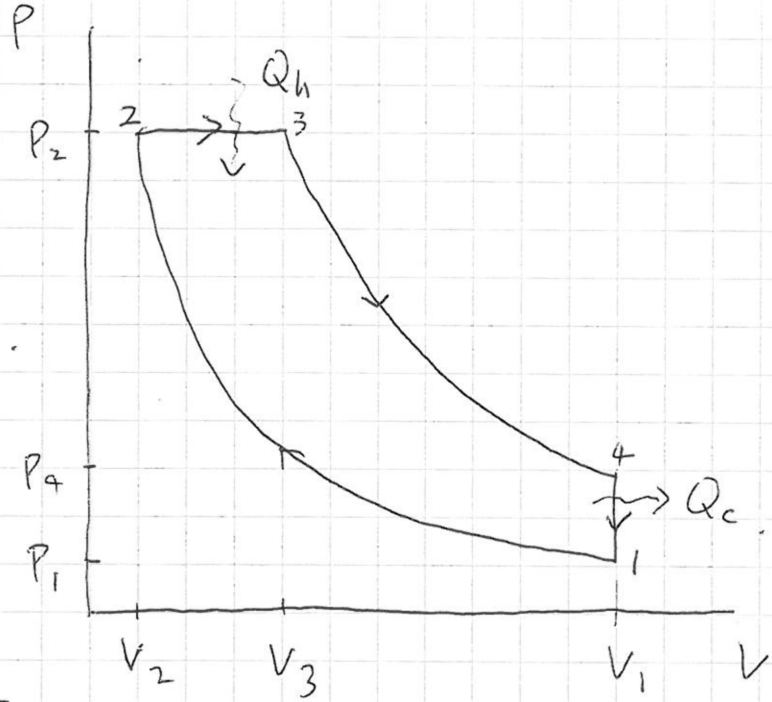
$\frac{(4)}{(5)}$: $\frac{T_3}{T_2} = \frac{T_4}{T_1}$.

$$\therefore \; e = 1 - \frac{Q_c}{Q_h} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{T_1\left(\frac{T_4}{T_1} - 1\right)}{T_2\left(\frac{T_3}{T_2} - 1\right)} = 1 - \frac{T_1}{T_2}.$$

from (5): $\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$.

$$\therefore \; e = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma-1}.$$

4.20 . Diesel cycle.



$$Q_h = U_3 - U_2 + P_2(V_3 - V_2)$$

$$= \frac{f}{2} Nk (T_3 - T_2) + P_2(V_3 - V_2)$$

$$= \frac{f+2}{2} (T_3 - T_2) \qquad (1)$$

$$Q_c = U_4 - U_1$$

$$= \frac{f}{2} Nk (T_4 - T_1) \qquad (2)$$

$$e = 1 - \frac{Q_c}{Q_h} = 1 - \frac{f(T_4 - T_1)}{(f+2)(T_3 - T_2)}$$

$$= 1 - \frac{1}{\gamma} \frac{T_4 - T_1}{T_3 - T_2}$$

$$= 1 - \frac{1}{\gamma} \frac{T_1}{T_2} \frac{\frac{T_4}{T_1} - 1}{\frac{T_3}{T_2} - 1}$$

$$\left(= 1 - \frac{1}{\gamma} \left(\frac{V_2}{V_1}\right)^{\gamma-1} \frac{\frac{P_3 V_3^\gamma}{P_2 V_2^\gamma} - 1}{\frac{P_3 V_3}{P_2 V_2} - 1} \right)$$

$$= 1 - \frac{1}{\gamma} \left(\frac{V_2}{V_1}\right)^{\gamma-1} \frac{\frac{V_3^\gamma}{V_2^\gamma} - 1}{\frac{V_3}{V_2} - 1}$$

$$T_3 V_3^{\gamma-1} = T_4 V_4^{\gamma-1}$$

$$T_2 V_2^{\gamma-1} = T_1 (V_1)^{\gamma-1}$$

$$\Downarrow$$

$$\frac{T_3}{T_2} \left(\frac{V_3}{V_2}\right)^{\gamma-1} = \frac{T_4}{T_1}$$

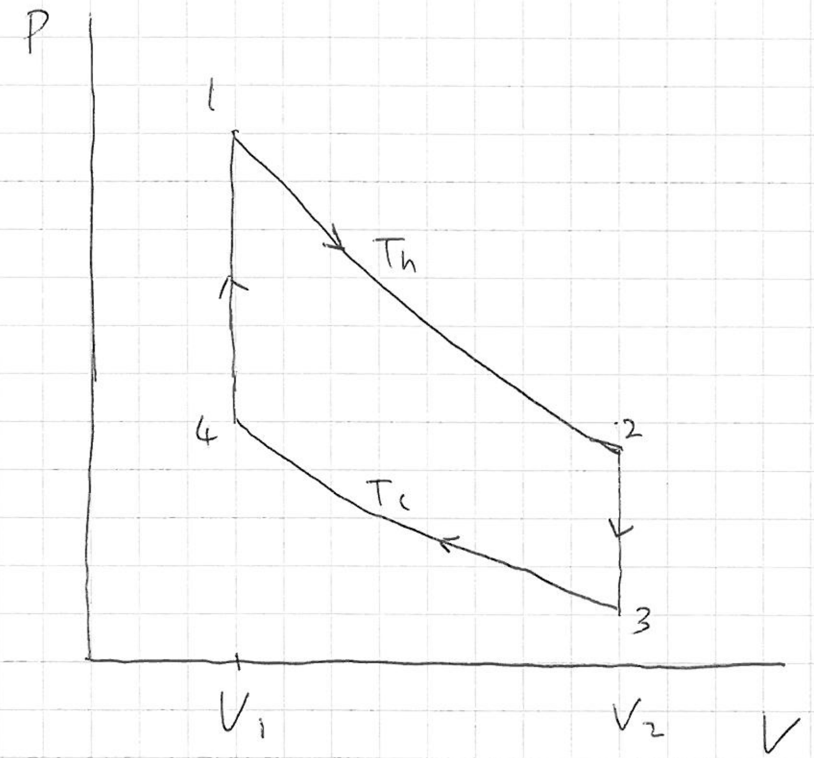
$$\frac{P_3 V_3^\gamma}{P_2 V_2^\gamma} = \frac{T_4}{T_1} = \frac{V_3^\gamma}{V_2^\gamma}$$

$$\frac{T_3}{T_2} = \frac{P_3 V_3}{P_2 V_2} = \frac{V_3}{V_2}$$

$$\therefore \boxed{e = 1 - \frac{1}{\gamma} \left(\frac{V_2}{V_1}\right)^{\gamma-1} \frac{(V_3/V_2)^\gamma - 1}{(V_3/V_2) - 1}}$$

For $\frac{V_1}{V_2} = 18$, $\frac{V_3}{V_2} = 2$, and $\gamma = 1.4$: $e = 63\%$.

4.21. stirling engine.



Work done by gas:

$1 \to 2$:

$$W_{12} = \int_1^2 P\,dV$$

$$= NkT_h \ln \frac{V_2}{V_1}$$

$3 \to 4$.

$$W_{34} = \int_3^4 P\,dV$$

$$= NkT_c \ln \frac{V_4}{V_3}$$

$$= -NkT_c \ln \frac{V_2}{V_1}$$

Total work by gas:

$$W = W_{12} + W_{34} = (T_h - T_c) Nk \ln \frac{V_2}{V_1}$$

Heat input from hot reservoir: $Q_h = Q_{12} + Q_{41}$

$$Q_{12} = W_{12} = NkT_h \ln \frac{V_2}{V_1}.$$

$$Q_{41} = U_1 - U_4 = \frac{f}{2} Nk (T_h - T_c)$$

$$Q_h = Nk T_h \ln \frac{V_2}{V_1} + \frac{f}{2} Nk (T_h - T_c)$$

$$e = \frac{W}{Q_h} = \frac{(T_h - T_c)\ln\frac{V_2}{V_1}}{T_h \ln\frac{V_2}{V_1} + \frac{f}{2}(T_h - T_c)}$$

$$\frac{1}{e} = \frac{T_h}{T_h - T_c} + \frac{f}{2\ln\frac{V_2}{V_1}}$$

$$= \frac{1}{e_c} + \frac{f}{2\ln\frac{V_2}{V_1}} \qquad (V_2 > V_1)$$

i.e. $\frac{1}{e} > \frac{1}{e_c} \Rightarrow e_c > e$.

e.g. $T_h = 600\,K$, $T_c = 300\,K$ $\frac{V_2}{V_1} = 10$.

$$e_c = 1 - \frac{T_c}{T_h} = 1 - \frac{300}{600} = 0.5 = 50\%.$$

$$e = \frac{300 \cdot \ln 10}{600 \cdot \ln 10 + \frac{5}{2} \times 300} = 0.32 = 32\%.$$

c). Q_{41} is not supplied by the hot reservoir.

$$\therefore \; Q_h = Q_{12} = Nk T_h \ln\frac{V_2}{V_1}$$

$$e = \frac{W}{Q_h} = \frac{T_h - T_c}{T_h} = 1 - \frac{T_c}{T_h} = e_c.$$

d).* Not required.
It's easier to build as compared to a Carnot engine
(A sudden change from isothermal to adiabatic processes is difficult.)

4.24

Original data.

$P_1 = P_4 = 0.023$ bar

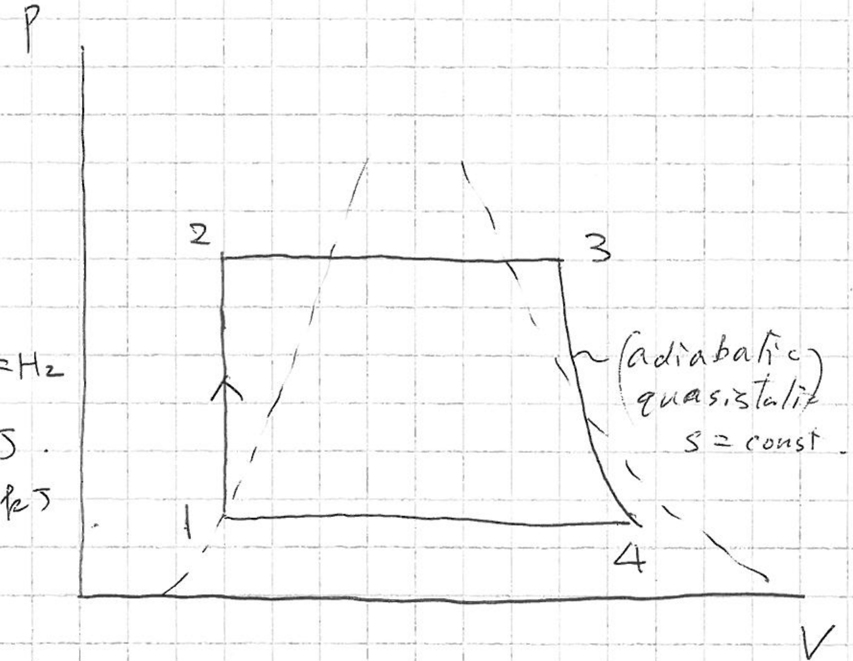
$P_2 = P_3 = 300$ bar.

$T_3 = 600°C$. $H_1 = 84$ kJ $= H_2$

$T_1 = 20°C$. $H_3 = 3444$ kJ.

$H_4 = 1824$ kJ

$$e = 1 - \frac{H_4 - H_1}{H_3 - H_1} = 0.48$$



a). $T_3 = 500°C$

$$H_1 = 84 \text{ kJ}, \quad H_3 = 3081 \text{ kJ}$$

$$S_3 = 5.791 \text{ kJ/K} = S_4.$$

let x = fraction of liquid water in state 4.

$$0.297x + (1-x)8.667 = 5.791.$$

Solve for x: $x = \dfrac{8.667 - 5.791}{8.667 - 0.297} = 0.3436$.

$$H_4 = 84x + (1-x)2538 = 1695 \text{ kJ}.$$

$$e = 1 - \frac{1695 - 84}{3081 - 84} = 0.46,$$

slightly less than the original efficiency 0.48.

because the maximum temperature is lower.

b) $P_2 = P_3 = 100$ bars. $T = 600°C$.

$H_3 = 3625$ kJ. $S_3 = S_4 = 6.903$ kJ/K.

$$0.297x + (1-x)\,8.667 = 6.903.$$

$$x = \frac{8.667 - 6.903}{8.667 - 0.297} = 0.211$$

$$H_4 = 84x + (1-x)\,2538 = 2020 \text{ kJ}.$$

$$e = 1 - \frac{2020 - 84}{3625 - 84} \approx 0.45.$$

c) $T_1 = 10°C$, $H_1 = 42$ kJ $= H_2$.

$H_3 = 3444$ kJ ($T_3 = 600°C$. $P_3 = 300$ bar)

$S_3 = S_4 = 6.233$ kJ/K.

$$0.151x + (1-x)\,8.901 = 6.233.$$

Solve for x: $x = \dfrac{8.901 - 6.233}{8.901 - 0.151} = 0.305$.

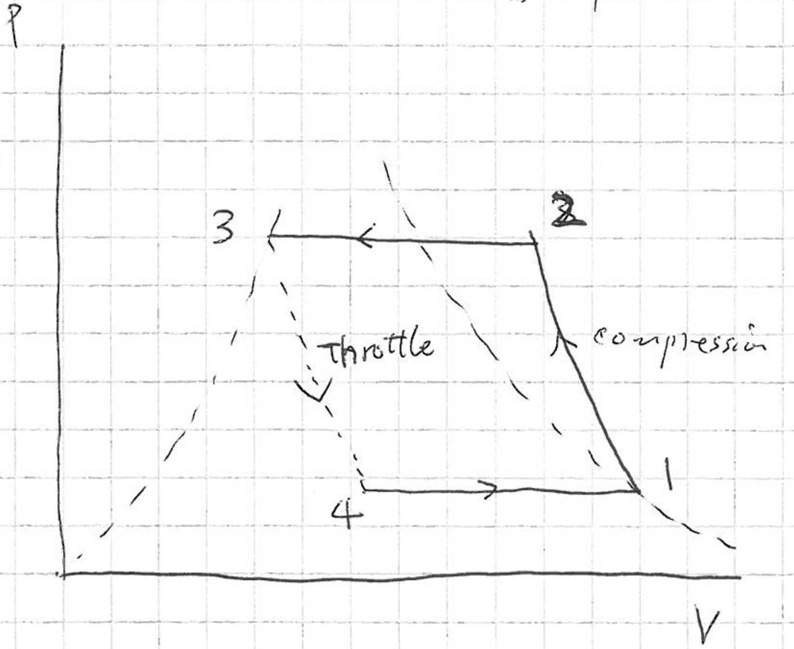
$$H_4 = 42x + (1-x)\,2520 = 1764 \text{ kJ}.$$

$$e = 1 - \frac{1764 - 42}{3444 - 42} = 0.49.$$

lower T_c should increase e.

4.30 . $H_3 = H_4$. $P_2 = P_3$

$S_1 = S_2$. $P_1 = P_4$.



$P_1 = P_4 = 1.0$ bar.

$P_2 = P_3 = 10$ bar.

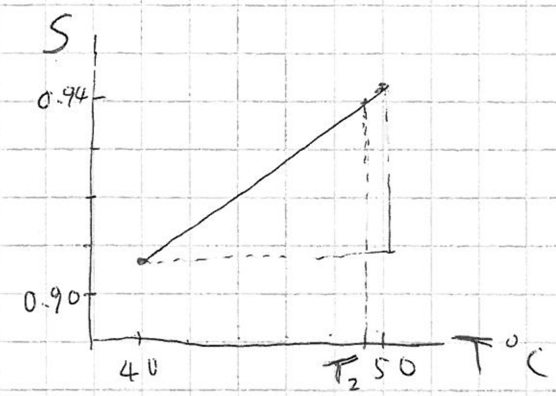
$T_1 = -26.4\,^\circ C$
$= 246.8\ K$

$T_3 = 39.4\,^\circ C = 312.6\ K$

a). $S_1 = 0.94\ kJ/K = S_2$.

$$\frac{\Delta S}{\Delta T} = \frac{S_2 - 0.943}{T_2 - 50} = 0.036.$$

$$T_2 = \frac{0.94 - 0.943}{0.0036} + 50 = 49.2\,^\circ C$$
$$= 322\ K$$



$$\frac{\Delta S}{\Delta T} = \frac{0.943 - 0.907}{50 - 40} = 0.0036$$

b). $H_1 = 231\ kJ$

$H_2 = 279\ kJ$.

$H_3 = 105\ kJ$.

$H_4 = H_3 = 105\ kJ$.

$$COP = \frac{H_1 - H_3}{H_2 - H_1} = \frac{231 - 105}{279 - 231} = 2.6$$

$$COP_C = \frac{T_c}{T_h - T_c} = \frac{246.8}{312.6 - 246.8} \approx \frac{247}{313 - 247} = 3.7$$

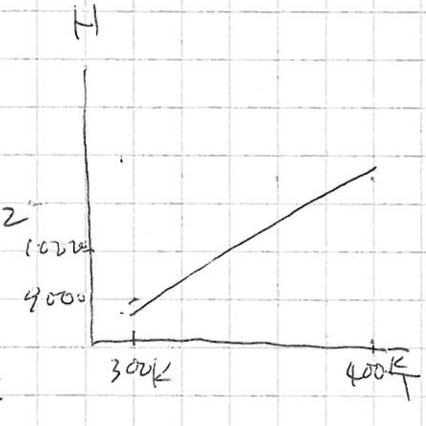
c). $x \cdot 231 + (1-x)16 = 105$.

$$x = \left(\frac{231 - 16}{105 - 16}\right)^{-1} = 0.41 \qquad 41\%\ \text{vapourizes.}$$

4.33. $P_i = 100$ bar, $P_f = 1$ bar.

$\cancel{H_i}$

a). $T_i = 300K$, $H_i = 8174 = H_f$



$$\frac{\Delta H}{\Delta T} = \frac{H_f - 8717}{T_f - 300} = \frac{11635 - 8717}{400 - 300} = 29.2$$

$$T_f = 300 + \frac{8174 - 8717}{29.2} = 281K$$

b) $T_i = 200K$. $H_i = 4442 = H_f$.

$$\frac{\Delta H}{\Delta T} = \frac{H_f - 2856}{T_f - 100} = \frac{5800 - 2856}{200 - 100} = \frac{2944}{100} = 29.44$$

$$T_f = 100 + \frac{4442 - 2856}{29.44} = 154\ K$$

c). $T_i = 100K$. $H_i = -1946 = H_f$.

$T_f = 77K$. fraction of liquid nitrogen $= x$.

$$-3407x + 2161(1-x) = -1946.$$

$$x = \frac{1946 + 2161}{3407 + 2161} = 0.74.$$

d). $H_i = 2161$. $= H_f$.

$$\frac{\Delta H}{\Delta T} = \frac{4442 - (-1946)}{200 - 100} = \frac{H_i - (-1946)}{T_i - 100} = 63.88.$$

$$T_i = 100 + \frac{2161 + 1946}{63.88} = 164\ K.$$

e). $H_f > H_i$, it won't cool down. $T_f > T_i$

Temperature will increase.

4.35. Magnetic field due to a dipole that's 1 nm away:

a). $$B = \frac{\mu_0}{4\pi}\frac{\mu}{r^3} = \frac{4\pi \times 10^{-7} \times 9 \times 10^{-24}}{4\pi \times (10^{-9})^3} = 9 \times 10^{-4}\ T$$

The average effect of all the neighbouring dipoles would be.

$B \sim$ a few times greater.

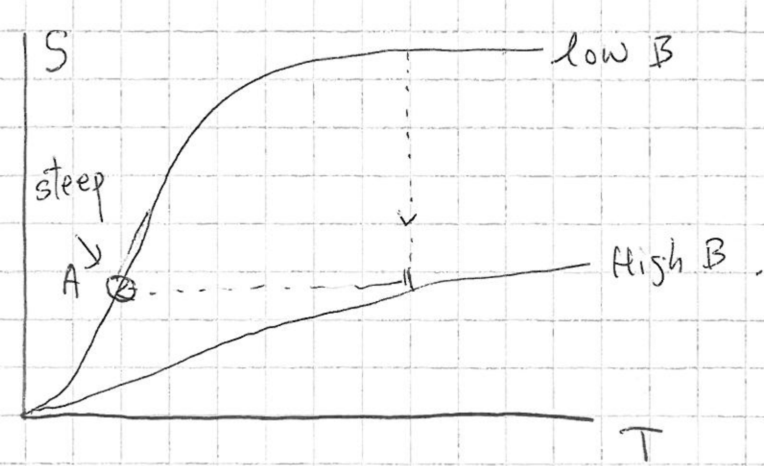
$\therefore\ B \approx 2 \times 10^{-3}\ T.$

b). $B_i = 1T$, $B_f = 2 \times 10^{-3}\ T$. (decreased by a factor of 500)

Since $\frac{B}{T}$ remains more less the same.

T will decrease by a factor of 500 too.

c).



At point A,

low field
(Bexternal = off).

$kT = \mu B$.

$S \sim T$ – steep slope.

$$T \approx \frac{\mu B}{k} = \frac{9 \times 10^{-24} \times 2 \times 10^{-3}}{1.38 \times 10^{-23}} \approx 10^{-3}\ K \approx 1\ mK.$$

d). $\frac{\partial S}{\partial T}$ = small at low T. $\Rightarrow T \cdot \frac{\partial S}{\partial T}$ = small.

Heat capacity $= \frac{Q}{\delta T} = T\frac{\partial S}{\partial T}$ = small.

$\therefore$ small leakage of heat $\Rightarrow$ increase T significantly. can't cool down much more.