Phys 344

Assignment #7. Solutions.

5.1. Ar, $f=3$, $n=1$, $T=300\,K$, $P=1.01\times10^5\,Pa$, ideal gas. $N=6.02\times10^{23}$

$$U=\frac{3}{2}NkT=\frac{3}{2}nRT=\frac{3}{2}\times1\times8.31\times300=3.74\,kJ$$

$$S=Nk\left\{\ln\left[\frac{V}{N}\left(\frac{4\pi mU}{3Nh^2}\right)^{3/2}\right]+\frac{5}{2}\right\}$$

$PV=NkT$.

$\frac{V}{N}=\frac{kT}{P}$.

$\frac{U}{3N}=\frac{1}{2}kT$

$$=nR\left\{\ln\left[\frac{kT}{P}\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\right]+\frac{5}{2}\right\}$$

$$=8.31\left\{\ln\left[\frac{(1.38\times10^{-23}\times300)^{5/2}}{1.01\times10^5}\left(\frac{2\pi\times40\times1.67\times10^{-27}}{(6.63\times10^{-34})^2}\right)^{3/2}\right]+\frac{5}{2}\right\}$$

$$=8.31\left\{\ln\left[(1.092\times10^{-56})(9.33\times10^{62})\right]+\frac{5}{2}\right\}$$

$$=155\ J/K.$$

$$H=U+PV$$

$$=U+nRT$$

$$=3.74\times10^3+8.31\times300$$

$$=6.23\ kJ.$$

$$F=U-TS=3740-300\times155=-42.7\ kJ$$

$$G=H-TS=6230-300\times155=-40.3\ kJ.$$

5-5.

$$CH_4 + 2O_2 \longrightarrow 2H_2O + CO_2$$

	CH_4	O_2	H_2O	CO_2
$\Delta_f H$	-74.8	0	-285.8	-393.5
$\Delta_f G$	-50.7	0	-237.1	-394.4

a) $\Delta H = (\Delta_f H)_{products} - (\Delta_f H)_{reactants}$

$= (-285.8 \times 2 - 393.5) - (-74.8)$

$= -890.3$ kJ.

$\Delta G = (-237.1 \times 2 - 394.4) - (-50.7) = -817.9$ kJ.

b) $(W_{electron})_{max} = |\Delta G| = 817.9$ kJ.

c) $|\Delta H| - |\Delta G| = 890.3 - 817.9 = 72.4$ kJ.

d) $$\frac{817.9 \times 10^3}{8 \times 6.02 \times 10^{23}} = 1.70 \times 10^{-19} \text{ J} = 1.06 \text{ eV}.$$

$\therefore$ Voltage $= 1.06$ V.

5.8. $G = U - TS + PV$.

$dG = dU - TdS - SdT + PdV + VdP$.

$dU = TdS - PdV + \mu dN$

$\therefore\ dG = TdS - PdV + \mu dN - TdS - SdT + PdV + VdP$

$= -SdT + VdP + \mu dN$

Then: $-S = \left(\frac{\partial G}{\partial T}\right)_{P,N} \Rightarrow S = -\left(\frac{\partial G}{\partial T}\right)_{P,N}$.

$V = \left(\frac{\partial G}{\partial P}\right)_{T,N}$. $\mu = \left(\frac{\partial G}{\partial N}\right)_{T,P}$.

5.11. $H_2 + \frac{1}{2}O_2 \longrightarrow H_2O$ at $T = 75^\circ C$.

$\Delta T = 75 - 25 = 50\,K$.

	H_2	$\frac{1}{2}O_2$	H_2O
$G(25^\circ C)$	0	0	-237 kJ
$S(25^\circ C)$	131 J/K	$205 \times \frac{1}{2}$ J/K	70 J/K
$G(75^\circ C)$	-6.55 kJ	$-10.25 \times \frac{1}{2}$ kJ	-240.5 kJ

at const P, N: $dG = -SdT = -S(25^\circ C) \cdot 50\,K$.

$G(75^\circ C) = G(25^\circ C) + dG = G(25^\circ C) - S(25^\circ C) \cdot 50^\circ C$.

$\Delta G(75^\circ C) = (-240.5) - (-6.55 - 10.25 \times \frac{1}{2}) = -229$ kJ.

max electric work $W_{other} = -\Delta G = 229$ kJ.

Less useful work than 25°C. because need larger ΔS to dump more heat to the environment. ($T\Delta S$) ↑ higher T.