Phys 344

Final Exam Solutions. (Spring 2009)

1. In a cycle: $\Delta U = 0$, $\Delta T = 0$

If C is a constant, $Q = \oint C\,dT = C \cdot \Delta T = 0$

But: $W = -\text{enclosed area} \neq 0$.

Then $\Delta U \neq W + Q$.

This violates the first law.

$\therefore$ C can not be a constant.

2. $a \to b$ is isothermal, $U_b - U_a = 0$ (ideal gas)

$$\therefore \quad Q_1 = -W_{ab} = \int_a^b P\,dV = \int_a^b \frac{NRT_1\,dV}{V} = NRT_1 \ln \frac{V_b}{V_a}$$

$$W_{cd} = -\int_c^d P\,dV = -NRT_2 \ln \frac{V_d}{V_c} = -NRT_2 \ln \frac{V_a}{V_b}$$

$$W' = -W_{ab} - W_{cd} = NRT_1 \ln \frac{V_b}{V_a} + NRT_2 \ln \frac{V_a}{V_b}$$

$$= NR(T_1 - T_2) \ln \frac{V_b}{V_a}$$

$$\eta = \frac{W'}{Q_1} = \frac{NR(T_1 - T_2) \ln \frac{V_b}{V_a}}{NRT_1 \ln \frac{V_b}{V_a}} = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}.$$

3. Heat transfer from you to the environment in 3 hours:

$$Q = 100\,J \times 3 \times 3600 = 1.08\ MJ.$$

$$\Delta S_{you} = \frac{-Q}{T_{you}} = \frac{-1.08\times10^6}{310} = -3.484\ kJ/K$$

$$\Delta S_{env.} = \frac{Q}{T_{env.}} = \frac{1.08\times10^6}{295} = 3.661\ kJ/K$$

Total entropy created:

$$\Delta S = \Delta S_{you} + \Delta S_{env} = 3.661 - 3.484 = 0.177\ kJ/K.$$

$$= 177\ J/K$$

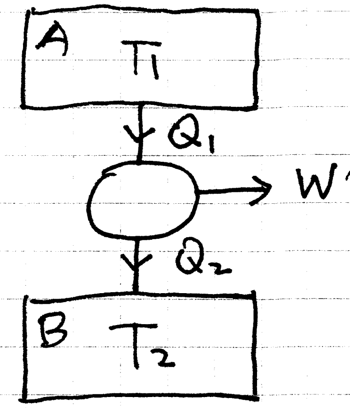
4. $$\frac{dP}{dT} = \frac{L}{T(v_2 - v_1)}, \quad \rho = \frac{m}{V}, \quad v = \frac{V}{m} = \frac{1}{\rho}.$$

$$v_1 = \frac{1}{920} = 1.087\times10^{-3}\ m^3/kg, \quad v_1 = \frac{1}{1000} = 1.000\times10^{-3}\ m^3/kg$$

$$\Delta P = \frac{L}{T(v_2 - v_1)}\Delta T = \frac{3.3\times10^5}{273.15\,(1.087 - 1.000)\times10^{-3}} \cdot (1)$$

$$= 1.39\times10^7\ Pa.$$

$$P = P_0 + \Delta P = 1.01\times10^5 + 1.39\times10^7 = 1.40\times10^7\ Pa.$$



5. Maximum work is performed in a reversible process via a Carnot engine.

$$\Delta S_1 = \int_{T_{1i}}^{T_{1f}} \frac{-dQ_1}{T_1} = \int_{T_{1i}}^{T_{1f}} \frac{C\,dT_1}{T_1} = C \cdot \ln \frac{T_{1f}}{T_{1i}}$$

$$\Delta S_2 = \int_{T_{2i}}^{T_{2f}} \frac{dQ_2}{T_2} = \int_{T_{2i}}^{T_{2f}} \frac{C\,dT_2}{T_2} = C \cdot \ln \frac{T_{2f}}{T_{2i}}$$

Finally A and B reach thermal equilibrium: $T_{1f} = T_{2f} = T_f$

change in entropy for the composite system:

$$\Delta S = \Delta S_1 + \Delta S_2 = C \cdot \ln \frac{T_f}{T_{1i}} + C \ln \frac{T_f}{T_{2i}} = C \cdot \ln \frac{T_f^2}{T_{1i} T_{2i}}.$$

Since it's reversible and adiabatic, $\Delta S = 0$.

$$\therefore \quad \frac{T_f^2}{T_{1i} = T_{2i}} = 1. \quad \Rightarrow \quad T_f = \sqrt{T_{1i} T_{2i}} = 346.4\,K.$$

$$Q_1 = -C \cdot \Delta T_1 = 1.00\,(400 - 346.41) = 53.59\ kJ.$$

$$Q_2 = C \Delta T_2 = 1\,(346.4 - 300) = 46.41\ kJ.$$

$$W'_{max} = Q_1 - Q_2 = 7.81\ kJ.$$

6. mass flow rate : 5 kg/sec.

a). Initial : 1 kPa, 7°C. Same enthalpy as 6 kPa, 30°C.
(because it's throttling).
(assuming adiabatic)

$$h_i = 0.13 \text{ MJ/kg}.$$

final: 1 kPa, 10°C.

$$h_f = 2.51 \text{ MJ/kg}.$$

$$q = h_f - h_i = 2.38 \text{ MJ/kg} \quad \text{(isobaric)}$$

$$Q = M \cdot q = 5 \times 2.38 = 11.9 \text{ MW}.$$

b). Now $h_i = 2.51 \text{ MJ/kg}$.

$h_f = 2.98 \text{ MJ/kg}$ (at 6 kPa, 250°C)

$$w = \Delta h = 0.47 \text{ MJ/kg}.$$

$$W = M \cdot w = 2.35 \text{ MW}.$$

7. a: $T = 550°C$, $P = 1\times10^7$ Pa

$h_a = 3.5$ MJ/kg, $S_a = 6.75$ kJ/K·kg

b: $T = 350°C$, $P = 1\times10^7$ Pa

$h_b = 3.16$ MJ/kg, $s_b = 7.3$ kJ/K·kg

$T_{res} = 20°C = 293.15$ K, flow rate: 200 kg/sec.

a). $W_r = \Delta h - T_{res}\,\Delta S = (h_b - h_a) - T_{res}(s_b - s_a)$

$= -0.34\times10^6 - 293.15\,(0.55\times10^3) = -0.501\times10^6$ J.

$W'_{max} = -W_r = 0.501$ MJ

Max Power $= 200 \times 0.501 = 100$ MW.

b). Adiabatic (irreversible)

$W' = h_a - h_b = 0.34\times10^6$ J $= 0.34$ MJ

Actual power $= 200 \times 0.34 = 68$ MW.

Power lost $= 100 - 68 = 32$ MW.

8. (a). More ordered means lower entropy.

$$\therefore \left(\frac{\partial S}{\partial L}\right)_T < 0$$

(b). $dF = -S\,dT + \tau\,dL$

$$-\left(\frac{\partial S}{\partial L}\right)_T = \left(\frac{\partial \tau}{\partial T}\right)_L = -\frac{\left(\frac{\partial L}{\partial T}\right)_\tau}{\left(\frac{\partial L}{\partial \tau}\right)_T}$$

$$\therefore \left(\frac{\partial L}{\partial T}\right)_\tau = \left(\frac{\partial S}{\partial L}\right)_T \cdot \left(\frac{\partial L}{\partial \tau}\right)_T$$

Since $\left(\frac{\partial S}{\partial L}\right)_T < 0$ (from X-ray data)

and $\left(\frac{\partial L}{\partial \tau}\right)_T > 0$. (when you pull, the rubber band gets longer)

$$\therefore \left(\frac{\partial L}{\partial T}\right)_\tau < 0$$

physical meaning: When the tension is kept constant, heating the rubber band would shorten it.

9. $\left(P+\frac{a}{v^2}\right)(v-b) = RT$

$$P = \frac{RT}{v-b} - \frac{a}{v^2} \qquad ①$$

Critical Point: $\left(\frac{\partial P}{\partial v}\right)_T = 0$, $\left(\frac{\partial^2 P}{\partial v^2}\right)_T = 0$

i.e. $\frac{-RT}{(v-b)^2} + \frac{2a}{v^3} = 0 \qquad \frac{RT}{(v-b)^2} = \frac{2a}{v^3} \qquad ②$

$\frac{2RT}{(v-b)^3} - \frac{6a}{v^4} = 0 \qquad \frac{2RT}{(v-b)^3} = \frac{6a}{v^4} \qquad ③$

$\frac{②}{③}$: $\frac{v-b}{2} = \frac{v}{3} \Rightarrow 2v = 3v - 3b \Rightarrow v = 3b$.

subs into ②: $\frac{RT}{(2b)^2} = \frac{2a}{(3b)^3} \Rightarrow T = \frac{8a}{27Rb}$

subs into ①:

$$P = \frac{R}{3b-b}\left(\frac{8a}{27Rb}\right) - \frac{a}{(3b)^2}$$

$$= \frac{4a}{27b^2} - \frac{a}{9b^2}$$

$$= \frac{a}{27b^2}$$

$\therefore \quad v_c = 3b, \quad T_c = \frac{8a}{27Rb}, \quad P_c = \frac{a}{27b^2}$

10. $C_p = T\left(\frac{\partial S}{\partial T}\right)_P$, $C_v = T\left(\frac{\partial S}{\partial T}\right)_V$ ①

$$S(T,P) = S[T, V(T,P)]$$

$$\left(\frac{\partial S}{\partial T}\right)_P = \left(\frac{\partial S}{\partial T}\right)_V + \left(\frac{\partial S}{\partial V}\right)_T \cdot \left(\frac{\partial V}{\partial T}\right)_P . \quad ②$$

$$\left(\frac{\partial T}{\partial V}\right)_S = - \frac{\left(\frac{\partial S}{\partial V}\right)_T}{\left(\frac{\partial S}{\partial T}\right)_V} . \Rightarrow \left(\frac{\partial S}{\partial V}\right)_T = -\left(\frac{\partial T}{\partial V}\right)_S \left(\frac{\partial S}{\partial T}\right)_V$$

subs into ② :

$$\left(\frac{\partial S}{\partial T}\right)_P = \left(\frac{\partial S}{\partial T}\right)_V - \left(\frac{\partial T}{\partial V}\right)_S \left(\frac{\partial S}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_P .$$

$$\frac{C_p}{T} = \frac{C_v}{T} - \left(\frac{\partial T}{\partial V}\right)_S \cdot \frac{C_v}{T} V\alpha . \qquad \left(\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P\right)$$

$$C_p = C_v - C_v V \cdot \alpha \cdot \left(\frac{\partial T}{\partial V}\right)_S .$$

$$\left(\frac{\partial T}{\partial V}\right)_S = \frac{C_v - C_p}{C_v \cdot V\alpha} = \left(1 - \frac{C_p}{C_v}\right) \cdot \frac{1}{V\alpha} = \frac{1-\gamma}{V \cdot \alpha}$$

7. a: $T = 550°C. = 823.15 K$ $h = 3.5$ MJ/kg , s=

$P = 1 \times 10^7$ Pa. $s = 6.6$ kJ/kgK.

b: $T = 350°C = 623.15 K$ $h = 3.16$ MJ/kg

$P = 1 \times 10^6$ Pa. $s = 7.3$ kJ/K·kg:

$T_{res} = 20°C = 293.15$

c). $W_{\cancel{max}\, r.} = \Delta h - T_{res} \Delta S$

$= (3.16 - 3.5) - 293.15 \cdot (\cancel{6.6 - 7.3}\ 7.3 - 6.6)$

$= -0.34 - 0.176$

$= -0.516$.

$W_{max} = 0.516$ MJ.

Power $= 0.516$ MW $\cdot \times 200 = 103$ MW.

b). $W = \cancel{\Delta h}\ (h_a - h_b)\, m$

$= \cancel{0.3} = 0.34 \times 200$

$= 68$ MW.

lost: $103 - 68 = 35$ MW.