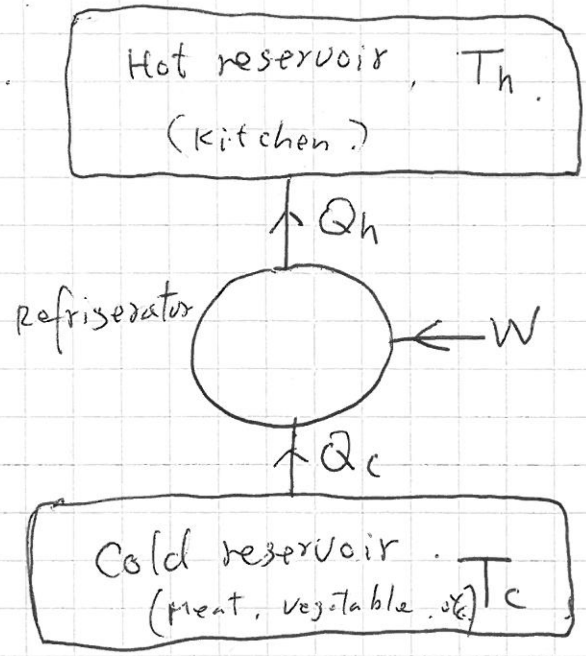
Phys 344.

Lecture 18 . Refrigerators.

- A refrigerator:
  Heat engine operated in reverse.



- Coefficient of performance.

$$COP = \frac{Q_c}{W} = \frac{\text{benefit}}{\text{cost}}.$$

$$\text{(1st law)}: \quad = \frac{Q_c}{Q_h - Q_c}.$$

$$= \frac{1}{Q_h/Q_c - 1}.$$

$$W + Q_c = Q_h. \quad (\Delta U = 0)$$

$\uparrow$ cyclic.

2nd Law: $\Delta S_{Total} \geq 0$.

$$\frac{Q_h}{T_h} - \frac{Q_c}{T_c} \geq 0 \qquad \left(\Delta S_{refrigerator} = 0 \quad \because \text{cyclic.} \right)$$

$$\frac{Q_h}{Q_c} \geq \frac{T_h}{T_c}.$$

$$\therefore \; COP \leq \frac{1}{T_h/T_c - 1} \quad \text{or.} \quad COP \leq \frac{T_c}{T_h - T_c}.$$

$$\begin{cases} < : \text{irreversible} \\ = : \text{reversible.} \end{cases}$$

e.g. $T_h = 298\,K$, $T_c = 255\,K$.

$$COP = \frac{255}{298-255} = 5.93$$

$\therefore$ COP can be greater than 1 !

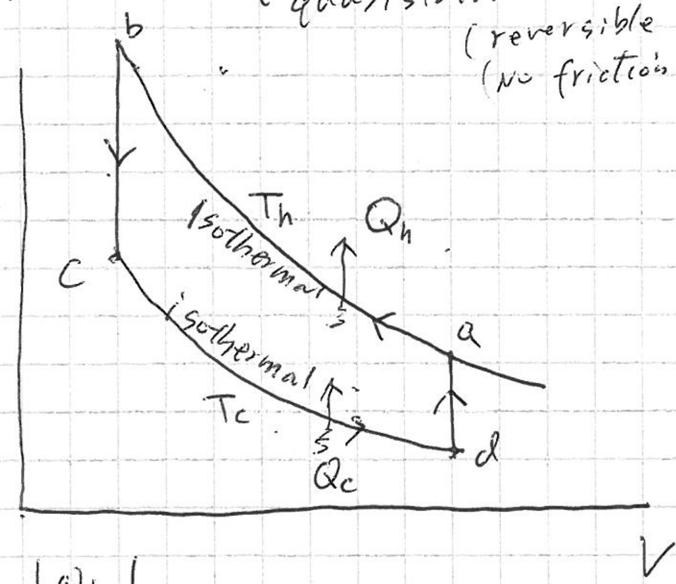
$\therefore$ Mechanical energy is more "valuable" than Thermal energy!

∘ e.g. Idealized Stirling refrigerator : $\begin{cases} \text{ideal gas} \\ \text{quasistatic} \end{cases}$ (reversible) (No friction)

$d \to a$: No work. ($V = \text{const}$).

$\Delta U = C_V (T_h - T_c) = Q_{ad}$

absorbed



$c \to b$: No work.

$\Delta U = C_V (T_c - T_h) = Q_{bc}$

release $|Q_{bc}|$.

~~Net heat~~ $\therefore$ Q_{ad} and Q_{bc} cancel

Net effect of heat: the refrigerator absorbs heat Q_c from the cold reservoir and release heat Q_h to the hot reservoir.

Price we have to pay: We need to do work W on the ~~refrigerator~~

$$W_{ab} = -\int_a^b P\,dV \qquad PV = nRT_h$$

$$P = \frac{nRT_h}{V}$$

$$= -nRT_h \ln \frac{V_b}{V_a}.$$

$$W_{cd} = -\int_c^d P\,dV = -nRT_c \ln \frac{V_d}{V_c} = -nRT_c \ln \frac{V_a}{V_b}.$$

$$\left(V_a = V_d \qquad V_c = V_b \right)$$

$$W = W_{ab} + W_{cd} = -nRT_h \ln \frac{V_b}{V_a} + nRT_c \ln \frac{V_b}{V_a}$$

$$= \left(nR \ln \frac{V_b}{V_a}\right)\left(T_c - T_h\right)$$

$$= \left(T_h - T_c\right) nR \ln \frac{V_a}{V_b}.$$

$$Q_c = -W_{cd}. \qquad \left(\because \Delta U_{cd} = 0 \quad \text{isothermal}\right)$$

$$= nRT_c \ln \frac{V_a}{V_b}.$$

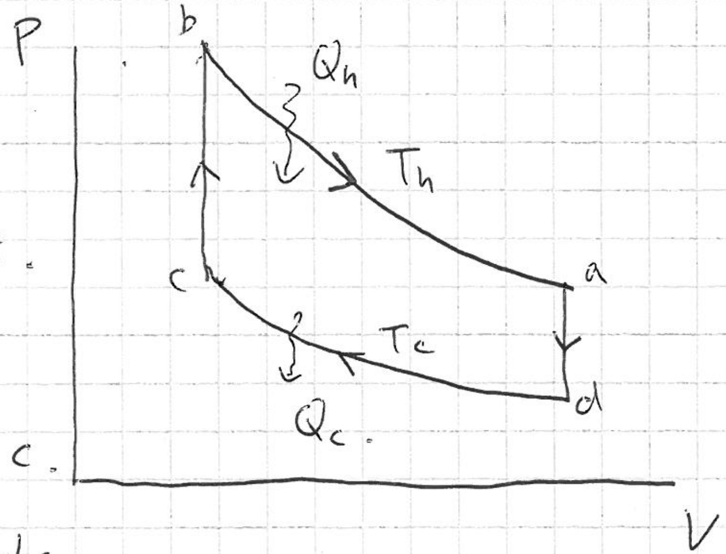
$$COP = \frac{Q_c}{W} = \frac{T_c}{T_h - T_c}.$$

∘ Stirling Engine {reverse of stirling refrigerator}.

· $a \to d$ and $c \to b$.

No work,

heat cancels. $\Rightarrow$ No heat.



∴ we only need to deal with $b \to a$ and $d \to c$.

i.e., the ~~e~~ engine absorbs heat Q_h and release heat Q_c, and do work $\underline{\underline{W}}$ to outside
at T_c at T_c. by system.

$$W = \int_b^a P dV + \int_d^c P dV$$

$$= nRT_h \ln \frac{V_a}{V_b} + nRT_c \ln \frac{V_c}{V_d} \qquad \begin{cases} V_a = V_d \\ V_b = V_c \end{cases}.$$

$$= nRT_h \ln \frac{V_a}{V_b} + nRT_c \ln \frac{V_b}{V_a}.$$

$$= \left(nR \ln \frac{V_a}{V_b}\right)(T_h - T_c).$$

$$Q_h = \int_b^a P dV = nRT_h \ln \frac{V_a}{V_b}.$$

$$e = \frac{W}{Q_h} = \frac{T_h - T_c}{T_h} = 1 - \frac{T_c}{T_h}.$$

more than 2 reservoirs.

("same" as 2nd ~~law~~ reversible. $e = e_{max}$).

- Real Heat engines.

idealized machines — give the limit in efficiency and COP.

(but maybe slow. or inconvenient.)

- Internal Combustion Engines.

Gasoline engine in your car.

Otto cycle:

1 — injection of air + fuel vapour.

1→2. compression (adiabatic)

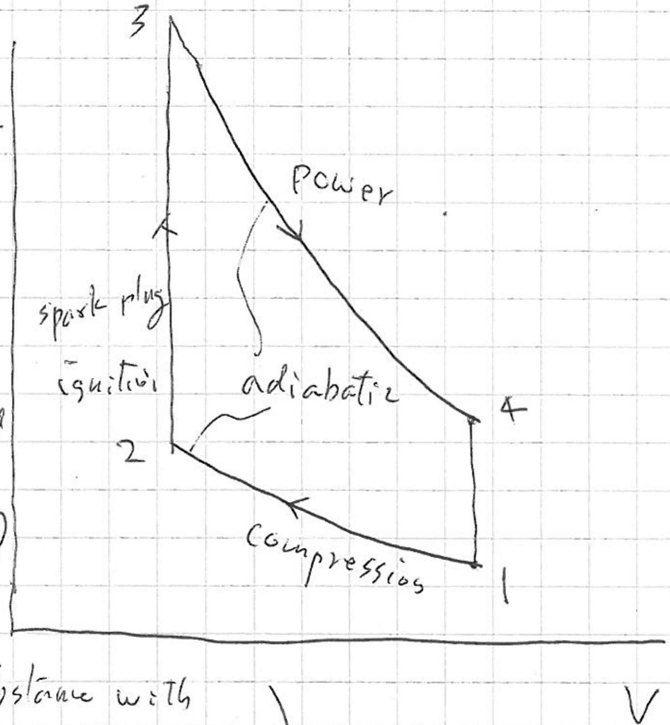
2 — ignition.

2→3 pressure increase which V = fixed (burning of fuel)

3→4. power. (adiabatic expansion)

4→1 exhaust.

(replace old working substance with the new one, without doing work)



Efficiency. (assignment 6. # 4.18).

$$e = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma - 1}$$

$\frac{V_1}{V_2}$ = compression ratio

Note: $\frac{V_1}{V_2}$ ~ depends on T_1 and T_2 (T_3, T_4)

e.g. $\gamma = 7/5$. $\frac{V_1}{V_2} = 8$. (Typical)

$$e = 1 - \left(\frac{1}{8}\right)^{2/5} = 0.56$$

The higher the $\frac{V_1}{V_2}$, the higher the e.

but there is a limit: if $\frac{V_1}{V_2}$ is too high,

the feul would burn before

you turn on the spark plug.

Solution: Diesel engine.

Typically $\frac{V_1}{V_2} \simeq 20$. (limited by material of the piston, cylinder

$T_2 = 820K$