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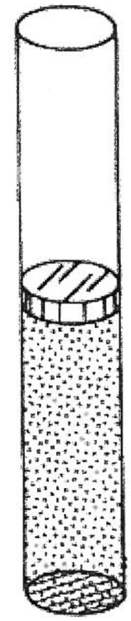
Physics 344 – Mid-term 1

February 8, 2012

Constants: $k=1.38\times10^{-23}$ J/K; $R=8.31$ J/mol·K. $g = 10\ m/s^2$

1$_{(10marks)}$. A cylinder containing an ideal monatomic gas is fitted with a frictionless piston. The mass of the piston is 15kg and its cross-sectional area is 1.00×10^{-3} m^2. The system is initially in equilibrium at a temperature of 300K. Initially the height of the gas column is 1.0m. The external pressure on the piston is the atmospheric pressure (10^5Pa). The cylinder and the piston are insulated to prevent heat transfer into or out of the gas.
A student gently lowers a mass of 10kg on top of the piston so that the gas can be considered undergoing a quasi-static process.



$T_i = 300\ K$

a) Determine the final height of the piston; $A = 1.0\times10^{-3}\ m^2$

b) Find the work done on the gas. $h_i = 1.0\ m$

(1) $f=3,\quad \gamma = 5/3$ $\qquad V_i = Ah_i = 1.0\times10^{-3}\ m^3$

(1) $\begin{cases} P_i = P_0 + \dfrac{15\times g}{A} = 2.5\times10^5\ Pa \\ P_f = P_0 + \dfrac{25g}{A} = 3.5\times10^5\ Pa \end{cases}$

a) quasistatic adiabatic (1) $PV^\gamma = const$

$P_iV_i^\gamma = P_fV_f^\gamma$ (1) $\quad V_f = \left(\dfrac{P_i}{P_f}\right)^{1/\gamma}\cdot V_i = \left(\dfrac{2.5}{3.5}\right)^{3/5}\times10^{-3} = 8.17\times10^{-4}\ m^3$

(1) $h_f = \dfrac{V_f}{A} = \dfrac{8.17\times10^{-4}}{1\times10^{-3}} = 0.82\ m$

b) $P_fV_f = nRT_f$ (1) $\quad \begin{cases} P_iV_i = nRT_i \quad n = \dfrac{P_iV_i}{RT_i} \\ n = \dfrac{2.5\times10^5\times10^{-3}}{8.31\times300} = 0.1\ mol \end{cases}$ (1)

$T_f = \dfrac{P_fV_f}{nR}$

(1) $= \dfrac{3.5\times10^5\times8.17\times10^{-4}}{0.1\times8.31} = 344\ K$

$\underline{W} = \Delta U = \dfrac{f}{2}nR\Delta T = \dfrac{3}{2}\times0.1\times8.31\times44 = 55\ J$

(1)

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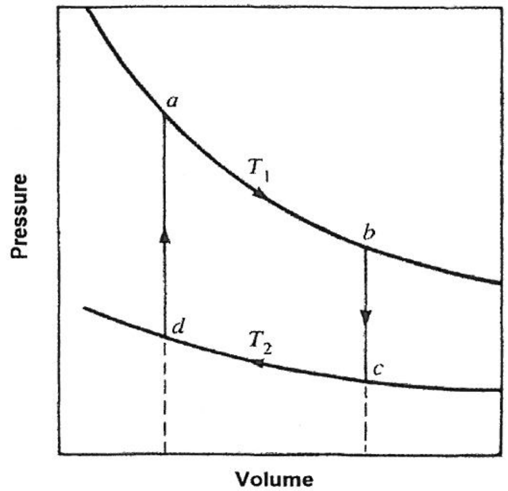
2(10marks). The Stirling cycle of a monatomic ideal gas, shown in the figure below, consists of four quasistatic processes. The steps $a \rightarrow b$ and $c \rightarrow d$ are isothermal processes at temperatures T_1 and T_2 respectively; the steps $b \rightarrow c$ and $d \rightarrow a$ are isochoric processes. $P_a = 1.0 MPa$, $V_a = 1.0 \times 10^{-3} m^3$, $V_b = 2.0 \times 10^{-3} m^3$, $T_1 = 500K$, $T_2 = 300K$.

(a) Calculate the work done on the gas in process $c \rightarrow d$;

(c) Calculate the heat input to the gas in process $a \rightarrow b$.



(a). $PV = nRT$ $\quad P = \frac{nRT}{V}$

$$W = \int_c^d -PdV = -nRT_2 \int_c^d \frac{dV}{V}$$

$$= -nRT_2 \ln \frac{V_d}{V_c}$$

0.5 $\quad = -0.24 \times 8.31 \times 300 \ln \frac{10^{-3}}{2 \times 10^{-3}}$

0.5 $\quad = 415 J$.

$P_a V_a = nRT_a$.

$$n = \frac{P_a V_a}{RT_a} = \frac{10^6 \times 10^{-3}}{8.31 \times 500}$$

$= 0.24$ mol.

(b). $Q = -W$

$$= \int_a^b PdV$$

$$= nRT_1 \int_a^b \frac{dV}{V} = nRT_1 \ln \frac{V_b}{V_a} =$$

$$= 0.24 \times 8.31 \times 500 \times \ln \frac{2 \times 10^{-3}}{1 \times 10^{-3}}$$

0.5 $\quad = 691 J$

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3(15marks). An ice cube (mass 20g) at 0°C is left sitting on the kitchen table, where it gradually melts. The temperature in the kitchen is 27°C. The latent heat for melting ice is 333 J/g. The specific heat of water is 4.186 J/g.

(a) Calculate the change in entropy of the ice cube as it melts into water;

(b) Calculate the change in the entropy of the water as it warms up from 0°C to 27°C.

(c) Calculate the net change in the entropy of the universe during this process.

a). $\Delta S_1 = \frac{Q}{T} = \frac{mL}{T} = \frac{20 \times 333}{273} = 24.4 \ J/K$.

b) $\Delta S_2 = \int \frac{dQ}{T} = \int \frac{C\,dT}{T} = mc \cdot \int_{273}^{300} \frac{dT}{T} = 20 \times 4.186 \ln \frac{300}{273}$

$= 7.9 \ J/K$.

c). Change in entropy of the environment.

$$\Delta S_3 = \frac{-mL - mc(\Delta T)}{300\ K}$$

$$= \frac{(-20 \times 333 - 20 \times 4.186 \times 27)\ J.}{300\ K}$$

$$= -29.7 \ J/K$$

Total: $\Delta S = \Delta S_1 + \Delta S_2 + \Delta S_3$

$= 24.4 + 7.9 - 29.7$

$= 2.6 \ J/K$.

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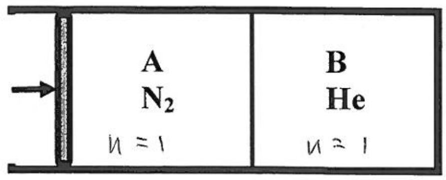
4(15marks). A cylinder at room temperature is divided into two parts by a fixed heat-conducting partition. Part A contains 1 mole of N_2 gas, and part B contains 1 mole of He gas (both can be considered as ideal gases). A student slowly pushes the piston at the end of part A. The work he does on the system is W=2J. Ignore the heat capacity of the cylinder, partition, and piston. The system is insulated from the surroundings so that no heat can be transferred into or out of the system.

a) Find the change in internal energy of the He gas in part B (ΔU_B=?).
b) Find the equation of process for the N_2 gas in part A.
c) Find the change in entropy of the system.



$N_2 : f=5$, $\quad He : f=3$.

$U_A = \frac{5}{2}RT$, $\quad U_B = \frac{3}{2}RT$

a) $U = U_A + U_B = \frac{5}{2}RT + \frac{3}{2}RT = 4RT$.

$W = \Delta U = 4R\Delta T$. $\quad \Delta T = W/4R$

$\Delta U_B = \frac{3}{2}R\Delta T = \frac{3}{2}R \cdot \frac{W}{4R} = \frac{3}{8}W = \frac{3}{4}J$

b). $T_A = T_B = T$. $\quad dW = -PdV$. $\quad P = P_A$, $dV = dV_A$.

$-PdV = 4RdT$. $\quad P = \frac{nRT}{V}$. $\quad n=1$ for A.

$-\frac{RT}{V}dV = 4RdT$.

$-\frac{dV}{V} = \frac{4dT}{T}$.

$-\ln(VD) = 4\ln T$. (D - constant).

$(VD)^{-1} = T^4$.

$\therefore VT^4 = const$. $\quad PV = RT$.

$P^4V^5 = const$.

c). $\Delta S = 0$ (adiabatic quasistatic).

also: $\Delta S = \Delta S_A + \Delta S_B = \int \frac{dQ_A}{T_A} + \int \frac{dQ_B}{T_B}$.

$= 0$. (Since. $T_A = T_B$. $dQ_A + dQ_B = dQ = 0$)