Phys 344.

Tutorial 2.

Assignment #2.

1.36, 1.38, 1.41,

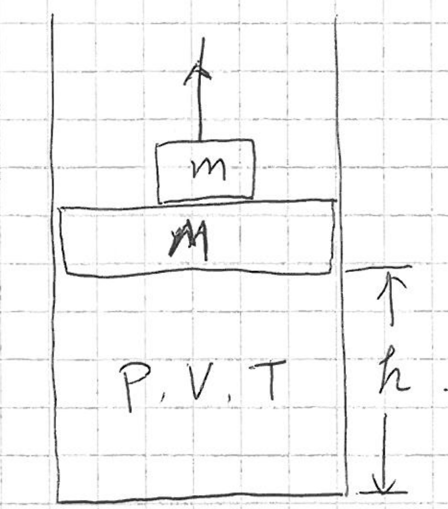
1.45, 1.47, 1.50.

example. Carrington p. 40.

ideal monatomic gas. $f = 3$.

$PV = nRT = NkT$.

$U = \frac{3}{2}NkT = \frac{3}{2}nRT$.



$T_i = 300\ \text{K}$

$A = 1 \times 10^{-3}\ m^2$.

$h_i = 1\ m,\quad V_i = Ah_i = 1 \times 10^{-3}\ m^3$

$Mg = 150\ N.\quad P_i = \frac{Mg}{A} = 150\ kPa.$

$mg = 100\ N.\quad P_f = \frac{Mg + mg}{A} = 250\ kPa$

$\left(Nk = \frac{PV}{T} = \frac{1.5 \times 10^5 \times 10^{-3}}{300} = 0.5\ J/K \right)$

Find: $h_f = ?\quad W = ?\quad Q = ?$

a) quasistatic isothermal process (Not isobaric!)

(because of the force exerted by the experimenter)

$P_f V_f = P_i V_i$.

$V_f = \frac{P_i V_i}{P_f} = \frac{1.50 \times 10^5 \times 1 \times 10^{-3}}{2.50 \times 10^5} = 6 \times 10^{-4}\ m^3$.

$h_f = \frac{V_f}{A} = \frac{6 \times 10^{-4}\ m^3}{1 \times 10^{-3}\ m^2} = 0.6\ m$.

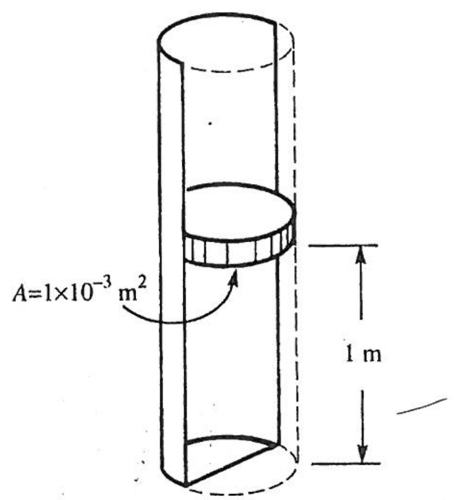


Fig. 3.14 Right circular cylinder fitted with leak-free frictionless piston.

Example

A right circular cylinder containing an ideal monatomic gas is fitted with a frictionless leak-free piston. The axis of the cylinder is vertical (Fig. 3.14) and the system is initially in equilibrium at a temperature of 300 K. The weight of the piston is 150 N and its cross-sectional area is 1×10^{-3} m^2. Initially the height of the gas column is 1 m. The external pressure on the exterior of the cylinder and piston is zero.

An experimenter places a mass weighing 100 N on top of the piston. This is done in four different ways in separate experiments. The starting condition of the apparatus is the same in each case. For each experiment determine the final height of the piston, the work done on the gas, and the heat input.

(a) The experimenter gently lowers the added mass so that the piston slowly approaches its new equilibrium state while the gas temperature remains at 300 K throughout the process.
(b) The added mass is released by the experimenter on to the top of the piston when it is in its initial position. Eventually the piston reaches a new equilibrium state at 300 K.
(c) The added mass is lowered gently as in experiment (a), but in this experiment the cylinder and piston are insulated to prevent heat transfer into or out of the gas.
(d) The added mass is released as in (b), but the system is insulated against heat transfer as in (c).

Solution (a)

For this example I will follow the above solution strategy in some detail. The numbers (1)–(5) refer to the steps suggested.

$$W = -\int p\,dV = -NkT \cdot \ln \frac{V_f}{V_i}.$$

$$= -P_i V_i \cdot \ln \frac{V_f}{V_i}$$

$$= -1.5\times10^5 \times 1\times10^{-3} \times \ln \frac{6\times10^{-4}}{1\times10^{-3}}$$

$$= 76.6\ \mathrm{J}$$

$$Q = -W \quad (\text{since } \Delta U = 0).$$

$$= -76.62\ \mathrm{J}.$$

b) Non-quasistatic process.

The initial and final states are the same as part a)

$\therefore\ h_f = 0.6\ m. \quad (h_i = 1.0\ m).$

work done on gas = Potential energy loss of the piston M and mass m.

$$W = (m+M)g(h_i - h_f)$$

$$= 250 \times 0.4$$

$$= 100\ \mathrm{J}.$$

$$Q = -W = -100\ \mathrm{J}. \quad (\text{since } T_f = T_i).$$

i.e. $\Delta U = 0$.

c). quasistatic adiabatic process.

$$P_i V_i^{\gamma} = P_f V_f^{\gamma}, \qquad \gamma = \frac{2+f}{f} = \frac{5}{3}$$

$$\left(\frac{V_f}{V_i}\right)^{\gamma} = \frac{P_i}{P_f}.$$

$$V_f = V_i \cdot \left(\frac{P_i}{P_f}\right)^{1/\gamma} = 1\times10^{-3} \times \left(\frac{150}{250}\right)^{3/5} = 7.36\times10^{-4}\ m^3.$$

$$h_f = \frac{V_f}{A} = 0.736\ m.$$

$$T_f = \frac{P_f V_f}{Nk} = \frac{2.5\times10^5 \times 7.36\times10^{-4}}{0.5} = 368\ K$$

$$\Delta U = \frac{3}{2} Nk\Delta T = \frac{3}{2}\times 0.5 \times (368-300) = 51\ J.$$

$$Q = 0.$$

$$W = \Delta U = 51\ J.$$

d) non-quasistatic adiabatic process: $Q = 0$.

but: $W = (m+M)g\cdot(h_i - h_f)$ ①

$\Delta U = \frac{3}{2} Nk (T_f - T_i) = W$ ②

$P_f V_f = NkT_f \Rightarrow P_f A h_f = Nk\,T_f$ ③

Solve {①, ②, ③} for h_f, T_f and ΔU.

$$\frac{3}{2}Nk\left(\frac{P_f A}{Nk}\cdot h_f - T_i\right) = (m+M)g\,(h_i - h_f).$$

$$\frac{3}{2}P_f A\,h_f - \frac{3}{2}NkT_i = (m+M)gh_i - (m+M)g\,h_f.$$

$$\left[\frac{3}{2}P_f A + (m+M)g\right]h_f = (m+M)gh_i + \frac{3}{2}NkT_i.$$

$$h_f = \frac{(m+M)gh_i + \frac{3}{2}NkT_i}{\frac{3}{2}P_f A + (m+M)g}$$

$$= \frac{250\times 1 + \frac{3}{2}\times 0.5\times 300}{\frac{3}{2}\times 2.5\times 10^5\times 10^{-3} + 250}.$$

$$= \frac{475}{625}.$$

$$= 0.76\ \text{m}.$$

$$T_f = \frac{P_f A\,h_f}{Nk} = \frac{2.5\times 10^5\times 10^{-3}\times 0.76}{0.5} = 380\ \text{K}.$$

$$\Delta U = \frac{3}{2}Nk\,\Delta T = \frac{3}{2}\times 0.5\times(380-300) = 60\ \text{J}.$$

$$Q = 0$$

$$W = \Delta U = 60\ \text{J}.$$

input work = gain in energy of the system.