Welfare Analysis When People Are Different

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Abstract

Inequality and poverty estimation (indeed, all welfare analysis) must deal with the fact that people are heterogeneous. Equivalence scales and indifference scales are tools that may be used for this. An equivalence scale gives the relative costs faced by different people; an indifference scale gives the relative cost of living for people in different types of households. Equivalence scales and indifference scales can be estimated using off-the-shelf household-level consumer expenditure data and standard econometric techniques for nonlinear equation systems. I offer a short introduction to the identification, estimation and use of equivalence scales and indifference scales, and argue that these are complementary tools in the analysis of inequality and poverty. The methods are illustrated with Canadian household expenditure data from the Surveys of Household Spending 2004-2009. Estimated equivalence scales for disability are presented, along with estimated household model parameters, and an analysis of consumption poverty.

1 Introduction

As inequality has been rising in rich countries since the 1980s, we have become more and more interested in meaningfully measuring it. If people in the population are identical, utility (or material well-being) is an identical function of consumption for all people. In this case, the measurement of inequality is straightforward: if one person has more consumption than another, that person is better off. Thus consumption inequality maps directly into

*I am grateful for the intellectual support of my friends and colleagues throughout my decades of work in this area. In my early work on equivalence scales and inequality, I would have gotten exactly nowhere without the mentorship of David Donaldson, Richard Blundell and Curtis Eaton and the generous financial support of the Social Sciences and Humanities Research Council of Canada. I am also thankful to the colleagues who gave me feedback on this paper: David Donaldson, Geoff Dunbar, Dave Freeman, John Knowles, Arthur Lewbel, Chris Muris and Sam Norris.

utility inequality. But, measuring inequality is difficult when the individuals that comprise a population are different from each other.

Suppose we observe two individuals with identical income (or consumption) levels, but we know that one of them is able-bodied and the other is disabled. Is this a state of equality or of inequality? It might be that the disabled person is worse off than the able-bodied person even though they have the same level of income. In terms of income, there is equality, but in terms of well-being (aka: utility), there is inequality.

People live in different economic environments, in particular, in different types of households. Some live in big households, where there are lots of scale economies deriving from shared or joint consumption, and some live alone. Some have lots of power to access resources in their household, some have little. People in these different types of households face different economic environments from those living alone. Suppose we observe 3 individuals: two live together with an income of \$90,000 and one lives alone with an income of \$50,000. Which of them are better and worse off? In per-capita terms, the household members look poorer than the person alone, but that ignores both scale economies in household consumption and the possibility that the two household members might have unequal access to household resources

Historically, economists and applied researchers investigating inequality and poverty used one tool to deal with both of these types of heterogeneity: equivalence scales. The *equivalence scale* is the scale to income that makes two consumers—who are different from each other—equally well off. Consider the able-bodied and disabled person. If the equivalence scale for the disabled person is 2 and that of the able-bodied person is 1, then the disabled person needs twice as much income as an able-bodied person to be equally well off. Equivalently, a disabled person with income x and an able-bodied person with income $x/2$ are equally well off.

Equivalence scales are thus inextricably bound up with interpersonal comparisons of utility. They tell you when 2 different types of people are equally well off. If you believe that the able and disabled person with equal incomes (mentioned above) are a picture of inequality, then such interpersonal comparisons of utility cannot be escaped. Real people differ from each other, and these differences must be accounted for in the measurement of inequality and poverty.

Equivalence scales may be reasonable tools to account for differences in utility functions across people. We can argue about whether or not different people are better off than each other. But, for households comprised of many people, what are we even talking about? People (might) have utility, but households don't have utility. They are not people; they are collections of people. In principle, each household member might have a different utility level. This means that some household members might be poor, even if other members of the same household are not.

Recent work on collective households has allowed us to separate these two kinds of heterogeneity, the facts that people are different from each other, and that people live in different types of households which constitute different economic environments. Chiappori (1988, 1992) introduced the efficient collective household model, wherein households are modeled as collections of individuals. Browning, Chiappori and Lewbel (2013) introduce a variant of this model, and the concept of the *indifference scale.* The indifference scale is the scale to income that gets an individual living in two different household types to the same indifference curve.

For example, if the indifference scale for one of the women in a household comprised of two women is 1.5 and the indifference scale for that same woman living alone is 1, then she requires a household income 50 per cent larger in the household to attain the same indifference curve that she would living alone. Here, there are no interpersonal comparisons of utility—it is the same woman, but in two economic environments. The indifference scale accounts for both the sharing or joint consumption of goods in the household, and the possibility that resources are shared unequally across household members.

Indifference scales do not need to be the same for every member of the household. If one member values shareable goods a lot, then scale economies deriving from sharing are more valuable to her, so her indifference scale in the household would be smaller. Or, if one household member has better access to household resources—perhaps because she has higher bargaining power—then her indifference scale would be smaller. In both cases, the smaller indifference scale means that less household income suffices to attain the same indifference curve that would be attained living alone.

Chiappori (2016) suggests that we can use indifference scales to deal with heterogeneity when we do welfare analysis, and that we can leave equivalence scales and equivalent incomes in the dustbin of failed economic tools. Essentially, he argues that since indifference scales take us far as we can go without invoking interpersonal comparisons of utility, we should stop there and leave the rest to policy-makers. In this paper, I argue that we should instead use indifference scales to deal with the fact that people live in heterogeneous economic environments and use equivalence scales to deal with the fact that people are themselves different from each other. That is, these tools are complements, not substitutes.

We can use indifference scales and equivalence scales in sequence to generate what I call *individual equivalent income.* The individual equivalent income of a person is the amount of money that, e.g., an able-bodied woman living alone requires to be as well off as the person, and it accounts for both heterogeneity across people and for heterogeneity in the types of households in which people live.

Suppose we have an economy with a single able-bodied woman with an income of $50,000, and a household comprised of an able-bodied woman and a disabled woman with a household income of $90,000. Suppose further that resources in the household are allocated equally, and that each woman in the household has an indifference scale of 1.5. We would use the indifference scale to convert the 2 household members into their as-if single equivalents, by dividing their household income by 1.5 and assigning $60,000 to each household member. We would then use the equivalence scale to further divide the disabled woman's income by 2 yielding $30,000—this accounts for the fact that a disabled person requires more consumption to attain any given utility level than does an able-bodied person. The final numbers would give the individual equivalent income needed by an able-bodied woman living alone to attain the utility levels of the real population: $50,000 for the single able-bodied woman; $60,000 for the able-bodied woman in the household; and $30,000 for the disabled woman in the household.[1] These numbers could be used directly to measure inequality or poverty.

There is a venerable and well-developed toolkit for the identification and estimation of equivalence scales from off-the-shelf micro-data on household consumption.[2] Such data sources typically contain many thousands of observations of data giving the allocation of household expenditure across many consumption goods. Statistical agencies collect this type of data to support the estimation of consumer price indices. We can use them to estimate equivalence scales. Such estimation must deal with interpersonal comparisons of utility head-on. I provide a brief introduction to these methods in Section 2.

The identification of indifference scales, household scale economies and the access of individual members to household resources (aka "resource shares") has been considered more recently. Strategies have been proposed to use the same kind of household-level consumption micro-data to identify and estimate these objects. I provide a brief introduction to some of these methods in Section 3.

In Sections 4 and 5, I use parametric consumer demand models to estimate equivalence scales and household models with repeated cross-sections of Canadian household expenditure micro-data. In this work, I estimate equivalence scales for disability, which is a novel use of these equivalence scale estimation methods. I also estimate a collective household model, and show how to use equivalence scales and indifference scales together in the measurement of poverty in Canada over 2004 to 2009.[3]

[1] It is just coincidence that the individual equivalent incomes within the household add up to household income.

[2] The first work on this was Engel (1895), and was followed by important innovations by Sydenstricker and King (1921), Rothbarth (1943), Prais and Houthakker (1955), and Barten (1964). Lewbel and Pendakur (2007) provide a short survey.

[3] I spent roughly half the State-of-the-Art lecture on incorporating unobserved heterogeneity into the

2 Equivalence Scales

2.1 Notation

Table 1 gives the notation to be used throughout.

symbol		meaning
i	$i = 1, ..., N$	indexes people
t	$t = 0, ..., T$	indexes types of people, 0 is the reference type
$t(i)$		the type of person i
h	$h = 1, ..., H$	indexes households
n_h		number of members in household h
p	$p = \{p^1, ..., p^J\}$	$J-$vector of prices of goods
x		budget (aka: income, consumption, total expenditure)
V_t	$V_t(p, x)$	indirect utility function of type t.
s_t	$S_t(p, x)$	equivalence scale of type t
Δ_t	$\Delta_t(p)$	exact equivalence scale, with elasticities $\delta_t(p) = \nabla_{\ln p} \ln \Delta_t(p)$
ϕ_t	$\phi_t(\cdot)$	type-specific monotonic transformation (of utility)
q	$q = \{q^1, ..., q^J\}$	J-vector of quantities of goods
w	$w = \{w^1, ..., w^J\}$	$w^j = p^j q^j / x$ is the budget-share commanded by good j
$\widetilde{p}$	$\widetilde{p} = A_h p$	shadow price vector; A_h is matrix giving scale economies
η	η_{ih}	resource share; can be different for each person i in household h
y_h		vector of observed household characteristics
z_i		vector of observed personal characteristics
$z = 0$	type 0	reference type, single childless able-bodied woman aged 50

2.2 What are Equivalence Scales and Why Are They Useful?

Suppose everyone lives alone. We come to multi-person households later when we consider collective household models. Let p be the $J-$vector of prices and x be the income[4] faced

estimation of consumer demand. That material is much more technically demanding, so I dropped it from this paper. If you are interested, please check out Lewbel and Pendakur (2009 and 2017) for my work in the area. Additionally, Hausman and Newey (2016) and Hoderlein et al (2010) have good reviews of the literature on this.

[4]Throughout this paper, I most often use the word "income" rather than the "budget" or "expenditure". This choice leads me to use "equivalent income", "indifferent income" and "individual equivalent income" rather than alternative phrases. But, I am not drawing any conceptual difference here between income, the budget, consumption, spending or total expenditure. Rather, I'm just trying to stick with the most easily recognized words. In a one period model, of course, all these are the same, because there is no savings to put a wedge between income and spending. In my own work on the measurement of inequality (e.g., Pendakur 1998; Pendakur 2002; Norris and Pendakur 2015), I have considered expenditure and equivalent expenditure in a multi-period context where saving and borrowing put wedges between income and total expenditure.

by a person. Let i index people in a population of people $i = 1, ..., N$. Let t index *types* of people, who have different observed characteristics. Each type is an observable class of people. For example, in data where age, sex and disability status are observed, a type could be "40 year old able-bodied man". Let $t(i)$ give the type of person i. We need to keep track of both types and individuals because identification is most easily understood if we compare the demand behaviour of groups of people of the same type.

Let $V_t(p, x)$ be the indirect utility function of a person of type t, giving the maximum utility attained when facing prices p and budget x. Define type 0 as a reference type: in this paper, the reference type is a single childless able-bodied woman aged 50.

An *equivalence scale* $s_t = S_t(p, x)$ gives the scale to expenditure x that makes the utility level of a person of type t equal to that of a person of type 0 (the reference person). It is defined implicitly as the solution to

$$V_t(p, x) = V_0\left(p, \frac{x}{s_t}\right). \tag{1}$$

A person of type t with income x has the same utility level as a reference person (type 0) with income $x/S_t(p, x)$. We call $x/S_t(p, x)$ as the *equivalent income* of a person. By definition, if any two people of any two types have the same equivalent income, then they have the same utility level.[5]

2.2.1 Equivalence Scales are Not Identified from Consumer Behaviour Alone

Equivalence scales are inextricably linked to interpersonal comparisons of utilities. The definition of the equivalence scale—the value of s_t such that $V_t(p, x) = V_0(p, x/s_t)$—has equality of utilities across types of people built into it. Consider applying a type-specific monotonic transformation ϕ_t to the utility functions V_t in equation (1)[6]:

$$\phi_t\left(V_t(p, x)\right) = V_0\left(p, \frac{x}{s_t}\right). \tag{2}$$

Every choice of ϕ_t results in a new equivalence scale. But, we know that monotonic transformations of utility do not affect indifference curves (preferences, demand curves). So, a monotonic transformation of the utility function would change the equivalence scale, but would not change demand behaviour. Thus, if behaviour is consistent with one particular

[5] Why scale income x rather than prices p? Scales to prices that equate utility are called "Barten scales", and are due to Barten (1964). See also Lewbel and Pendakur (2017).

[6] Here, I do not apply a monotonic transformation to V_0. One could easily do so, but it makes the point harder to see.

equivalence scale function $S_t(p, x)$, it is also consistent with an infinite number of other equivalence scale functions, by suitable choice of the transformations ϕ_t. This is a key demerit of using demand behaviour to identify and estimate equivalence scales. They depend on cardinalizations of utility functions (the exact choice of ϕ_t), and cannot be determined from ordinal preferences alone.

2.2.2 Equivalence Scales and Preferences

Suppose two types, e.g., men and women, have different preferences from each other. Then, their utility functions must be different from each other. Consequently, the equivalence scale must be different from 1 at some price vector. This is an often overlooked point. The presence of preference heterogeneity implies that equivalence scales cannot be assumed to equal 1 at all price vectors. This means that inequality and poverty measurement in the presence of heterogeneity must face up to dealing with interpersonal comparisons of utility.

What if two types have different preferences, but we don't want to treat them differently in our analysis of inequality? Examples could include: men vs women; lazy versus motivated people; or vegetarians versus meat-eaters. These are on a continuum from ascribed to chosen characteristics. It is illegal in many circumstances to treat men differently from women. So, one might want to constrain the set of characteristics that equivalence scales depend upon. But, at present, there are no tricks in the toolbox to deal with this issue. The problem is that restrictions on the equivalence scales (such as independence of the equivalence scale from the sex of the person) imply restrictions on behaviour that may be false. Dealing with ethically relevant vs ethically irrelevant characteristics in equivalence scales would be a useful direction for future research.

2.2.3 Using Equivalence Scales

For now, suppose we knew the equivalence scale for each person and denoted it $s_i = S_{t(i)}(p, x_i)$. Here, s_i is the equivalence scale for a person of the type of person i (denoted $t(i)$), evaluated at the income x_i of person i.[7] Then, the dataset defined by incomes and utility functions

$$\left\{x_i, V_{t(i)}\right\}_{i=1}^{N}$$

has the same set of utilities as, and is therefore welfare-equivalent to, the dataset defined by

$$\left\{\frac{x_i}{s_i}, V_0\right\}_{i=1}^{N}.$$

[7] Here, prices are the same for all individuals. If prices differ across individuals, add i subscripts to prices and calculate $s_i = S_{t(i)}(p_i, x_i)$.

Any measurement of income inequality or of the poverty rate would have the same result regardless of which dataset were used.

For example, suppose we have a poverty threshold of $\overline{x}$ that applies to the reference person. We could compute utilities $u_i = V_{t(i)}(p, x_i)$ for the population, and count the number of individuals who have $u_i < \overline{u}$ where $\overline{u} = V_0(p, \overline{x})$. Or, equivalently, we could count the number of individuals who have $x_i^e < \overline{x}$. Thus equivalent income functions completely characterise interpersonal comparisons of utility—they turn a heterogeneous population into an as-if population of people with identical (reference) utility functions.

The measurement of income inequality or the poverty rate does not require knowledge of the utility function of any person—all you need is the equivalence scale. Knowing the equivalence scale function but not the utility function is like knowing everything about interpersonal comparisons of utility, but nothing about the cardinalization of utility. Information about that cardinalization is embodied in the reference utility function V_0. Measurement of *utility* inequality, or other welfare measures generally, requires knowledge of the reference utility function V_0 even if the equivalence scale is known.

2.3 Identification of Equivalence Scales

There are many ways to use data to identify equivalence scales. One approach is to conduct surveys asking people what they think about how much money is needed to equate utilities across types. For example, one may ask individuals how well-off they are, thus identifying their utility level, and find the scaling factors to their incomes that equate utilities across people. In terms of equation (1), we would assume that utility is observable, so that the indirect utility functions V_t and V_0 can be estimated directly as functions of prices and budgets. We then solve equation (1) directly for the equivalence scale. This is called the Leyden approach, and was used extensively from the 1970s onwards.[8]

More recently, researchers have asked people to fill in vignettes where they directly imagine how much money it would take to hold utility constant if a person's type changed. This is like assuming that the equivalence scale function S_t is observable and that we can just ask people about it. Kouvoulatianos et al (2003, 2005, 2009) use this approach, and find that people are indeed able to answer direct questions about the appropriate equivalence scales for different household types.

Blundell and Lewbel (1991) take a slightly different approach—they ask whether anything

[8]See especially, Goedhart et al (1977) and Van Praag et al (1980, 1984) for applications of this approach, which uses cardinally fully comparable utilities. Van Praag and Van der Sar (1988) provide a version using ordinally fully comparable utilities. Zaidi and Burchardt (2005) use this methodology to consider equivalence scales for disability.

about equivalence scales can be identified from consumer behaviour alone. They point out that, from standard ordinal demand analysis, we can identify how equivalence scales vary with prices. That is, although we cannot identify the level of an equivalence scale from consumer demand curves, we can identify how they respond to price changes.

Asking people directly what they think about utility and/or equivalence scales has merits: it is simple and understandable. But it has some demerits: equivalence scales may be poorly estimated if the utility measures are noisy. Further, reported utility functions and equivalence scales may not be consistent with behaviour. In particular, they may not vary with prices in the way implied by observed consumer demands.

2.3.1 Identification of Equivalence Scales From Consumer Behaviour Using Exactness Restrictions

The logic of equation (2) implies that equivalence scales cannot be identified solely from consumer demand decisions. Blackorby and Donaldson (1993, 1994) and Donaldson and Pendakur (2004, 2006) provide some solutions to this conundrum based on functional form restrictions. The idea is: if we assume nothing about the equivalence scale, it is not identified from behaviour, meaning that an infinite number of equivalence scale functions are consistent with behaviour. However, if we impose a priori functional structure on the equivalence scale, it may be that only one equivalence scale function is consistent with behaviour. Ideally, the structure imposed would be derived from economic theory. The functional form restriction invoked here is not derived from economic theory, but has a few merits, described below.

Consider the case where, for any given p, the equivalence scale is *exact*, or independent of income x. Then,

$$S_t(p, x) = \Delta_t(p). \tag{3}$$

In this case, at any given price vector, the equivalence scale depends only on type, and not does not differ between rich and poor people. Lewbel (1989) says that equivalence scales satisfying this condition are Independent of Base. Blackorby and Donaldson (1993) call this condition *equivalence-scale exactness* (ESE). I will use the latter name throughout this paper.[9]

There are several reasons to consider using a restriction like this. First, this restriction is used in almost all empirical analyses of poverty and inequality. For example, all OECD and

[9]The working paper versions of Lewbel (1989) and Blackorby and Donaldson (1993) were roughly coincident in time. Both papers considered the same independence restriction, and both worked out the cost function. Lewbel additionally worked out the corresponding demand functions, but not the identification condition; Blackorby and Donaldson worked out the identification result, but not the demand functions. Since I am using the identification result extensively in this paper, I use Blackorby and Donaldson's terminology. Also, David Donaldson's graduate public economics course is a big reason I became an economist.

World Bank studies of inequality use an equivalence scale that is exact. However, it is notable that such equivalence scales are used, incorrectly in my view, to account for differences across households and not differences across people.[10] The reasoning here is a bit tortured: we use exact equivalence scales in applied work, but these exact scales are not derived from or consistent with observed consumer behaviour. The idea is then that we can improve such applied work by using exact equivalence scales that are consistent with behaviour.

Second, if the equivalence scale were not exact, we would have to specify its value at every level of income. One might reasonably argue that even the exact equivalence scale $\Delta_t(p)$ must be specified for every price vector. However, as noted above, Blundell and Lewbel (1991) show that one can identify how the equivalence scales change when prices change from consumer behaviour alone. So, the only further information needed for an exact equivalence scale is its value for every type t at one price vector.

Third, exactness implies if two people are equally well-off, then if we proportionately increased the incomes of both, they would still be equally well-off. In this case, if public assistance rates for able-bodied and disabled people were to increase, we would want to increase them by the same proportion to maintain horizontal equity. Actual adjustments to public assistance rates are often made in such a proportionate fashion.

Fourth, exactness of the equivalence scale implies that the equivalence scale is identified from consumer demand behaviour. If equivalence scales satisfy the exactness restriction (3), then there is only one equivalence scale function $\Delta_t(p)$ that is consistent with behaviour. Suppose there really is a equivalence scale that is independent of income x, and so is the same for rich and poor. Then, of course the demand behaviour of households is consistent with that equivalence scale. But, by equation (2), we know that the demand behaviour of households is also consistent with an infinite number of other equivalence scales. However, Blackorby and Donaldson (1993) show that only one of this infinitude of equivalence scale functions is independent of income. Identification of equivalence scales from demand behaviour then comes down to finding the one equivalence scale independent of income that is consistent with observed behaviour.[11]

What does demand behaviour look like when there is an exact equivalence scale? Let $q_t(p, x)$ be the $J-$vector of Marshallian quantity demand functions giving the quantities purchased by a person of type t when facing a budget constraint defined by (p, x). Lewbel (1989) and Pendakur (1999) show that if equivalence scales satisfy (3), then quantity demand

[10] Braithwaite and Mont (2008) is an exception to this pattern: they consider equivalence scales for disability.

[11] Donaldson and Pendakur (2004, 2006) offer show that other functional forms work similarly. Kouvoulatianos et al (2004) offer Leiden-style vignette evidence to support the particular affine functional restriction of Donaldson and Pendakur (2006).

functions satisfy

$$q_t(p,x) = \Delta_t(p) q_0 \left(p, \frac{x}{\Delta_t(p)} \right) + x\delta_t(p),$$

where $\delta_t(p) = \nabla_{\ln p} \ln \Delta_i(p)$ is the vector of price-elasticities of the equivalence scale and q_0 is the vector of Marshallian quantity demand functions of the reference type.

It is a little easier to see the relationship in budget-share equations which give the fraction of the budget commanded by each good:

$$w_t^j(p,x) = \frac{p^j q^j(p,x)}{x}.$$

Let $w_t(p,x) = \left(w_t^1(p,x), ..., w_t^J(p,x) \right)$ be the $J-$vector of budget share equations. They satisfy

$$w_t(p,x) = w_0 \left(p, \frac{x}{\Delta_t(p)} \right) + \delta_t(p). \tag{4}$$

Pendakur (1999) refers to this relationship across the budget-share equations of different household types under the assumption of equivalence-scale exactness as "shape-invariance".

Consider budget share functions at a single price vector $\overline{p}$, sometimes called the "Engel curve" vector, and write them as $\overline{w}_t(x) = w_t(\overline{p}, x)$. Here, we drop p as an argument not because budget shares are independent of prices, but rather because we are evaluating them over x at a fixed price vector $\overline{p}$. Then, we have:

$$\overline{w}_t(x) = \overline{w}_0 \left(\frac{x}{\overline{\Delta}_t} \right) + \overline{\delta}_t. \tag{5}$$

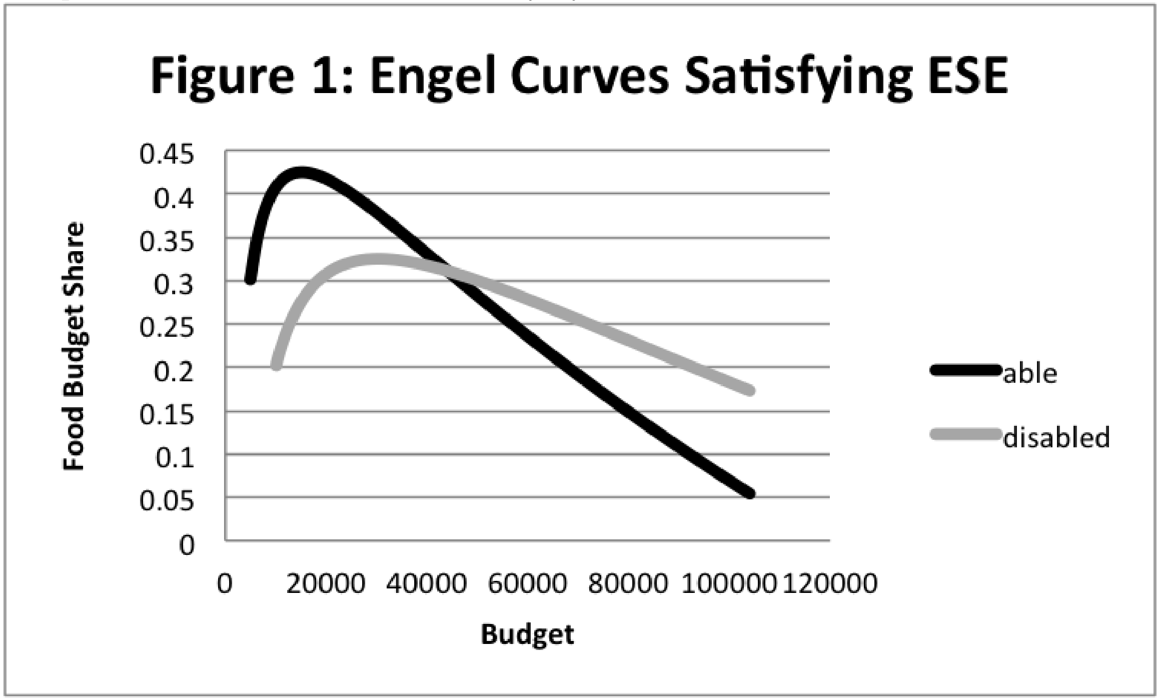
Here, there are just two type-specific parameters, $\overline{\Delta}_t = \Delta_t(\overline{p})$ and $\overline{\delta}_t = \delta_t(\overline{p})$, and one vector-function, $\overline{w}_0(x)$, that describe the all the Engel curves of all the types.

In this paper, I will use the word *identified* in a specific way. I say that a parameter in a model is identified if there is only one value of the parameter consistent with any observed data set generated by the model. That is, I start by assuming that the model is true, and ask whether or not the the observed data are sufficient to uniquely pin down the value of the parameter. If they are, then the parameter is identified; if they are not, the parameter is not identified.

Except for one pathological case, described below, the equivalence scale given ESE is identified from data on budget-share (or quantity-demand) functions. Suppose we have enough data to draw out budget-share functions for each of two types, able (type 0) and disabled (type 1), at a fixed price vector $\overline{p}$.[12] Figure 1 gives a picture of two budget share

[12] That is, suppose we have data on many observations of each of two types of people at many different

equations that satisfy (5) at the fixed price vector $\overline{p}$ (so that the only variation is across income x and the two types, 0 and 1). These two functions have the same shape, except that one is a stretched version of the other, and they differ by an additive constant. The stretch in the x direction equals the equivalence scale $\overline{\Delta}_t$ and the additive term equals its price elasticity $\overline{\delta}_t$. In the figure, the equivalence scale is equal to 2, so the Engel curve of the disable person has is stretched horizontally by a factor of 2.



So, identification from consumer behaviour comes down to drawing out budget share functions, and finding the stretches that relate them across household types. There is a case where exact equivalence scales are *not* identified from behaviour. Blackorby and Donaldson (1993) show that if budget share functions are affine (aka: linear) in the log of the budget x, then there are infinite number of exact equivalence scales consistent with behaviour. Pendakur (1999) gives some graphical intuition for this result. Lewbel (2010) provides algebraic intuition for this result. Thus, nonlinearity is essential to the identification of exact equivalence scales.

Taken together with Blackorby and Donaldson (1993), identification of $\Delta_t(p)$ from behaviour is possible in the following sense. Suppose that equivalence scale really is independent of expenditure as in (3). Then, budget shares must satisfy (4). In this case, we can still run utilities through monotonic transforms as in (2), thus generating an infinite number of equivalent expenditure functions consistent with behaviour. However, only one of these equivalent expenditure functions is proportional to expenditure, and it is the one recovered

income levels facing a single price vector $\overline{p}$. We use these data to draw out budget-share functions.

by using the observed budget share functions $w_t(p,x)$ to solve (4) for the equivalence scale $\Delta_t(p)$.[13]

Estimation of equivalence scales may thus use standard microdata on the allocation of spending across goods for people of different types and concorded data on the prices of those goods. In Canada, the Surveys of Household Spending (previously the Family Expenditure Surveys) and consumer price index provide data of this sort. Such data are widely available because they are used to construct consumer price indices, and are consequently needed by central banks worldwide. Further, more recent World Bank data are publicly available for a growing set of developing countries.[14]

Given microdata on the allocation of spending across goods, the prices of those goods, and the types of people making consumption decisions, equivalence scales may be estimated by fitting equations like (4) or (5) to the observed data. Typically, this involves choosing a functional form for the reference budget-share system $w_0(p,x)$ and for the equivalence scale function $\Delta_t(p)$ and estimating model parameters (including equivalence scales) by nonlinear seemingly unrelated regression or by GMM.[15] We will discuss the details of parametric estimation in section 4.

Before moving on to collective household models, let us return to the point made at the beginning of this section: if utility were observable in some dataset, we could estimate utility functions and then solve for equivalence scales directly via equation (1). Suppose there were data that had *both* utility or well-being data *and* consumer demand data. Then, we could use the data on utilities to cardinalise the utility function, and the data on demands to take care of price effects. Such data would allow us to escape the low precision and definitional fuzziness of the Leyden approach and would relax the need for functional form restrictions supporting identification. To my knowledge, the only paper in this vein is Allessie et al

[13]If the equivalence scale is independent of income, then the equivalent income function is proportional to income: $x_t^e = X_t(p,x) = x/s_t(p)$. This restriction is sufficient to identify the equivalence scale and equivalent incomes. There are several other functional form restrictions that support identification. Suppose that, for every p, the equivalent income function is additive (Blackorby and Donaldson 1994), proportional (Blackorby and Donaldson 1993), affine (Donaldson and Pendakur 2006) or log-affine (Donaldson and Pendakur 2004) in x. Then, it is identified from behaviour. In particular, although it is still the case that every choice of ϕ_i yields a different equivalent income function $X_t(p,x)$, only one of these equivalent income functions is additive/proportional/affine/log-affine. The pathological cases where identification fails are if quantity demands are linear in budgets (for additive equivalent income), budget shares are affine in the log of the budget (for proportional or log-affine equivalent income), or if either condition holds (for affine equivalent income).

[14]For example, the World Bank Living Standards Measurement Study website has links to more than 100 publicly available datasets for more than 20 countries.

[15]Pendakur (1999) shows how to estimate (5) semi-parametrically, that is, without specifying the functional form of $\overline{w}_0$. Blundell, Duncan and Kristensen (2011) modernise these methods. But, essentially the intuition is that identification from behaviour proceeds by finding the scalings of expenditure that best match Engel curves across household types.

(2006). So, it would certainly be nice to augment standard surveys on consumer spending with a few questions on well-being.

3 Collective Household Models

In the previous section, I considered how to model the differences in costs across people, but where those people all live alone. In this section, I turn to how to model people who may live in different types of households. I introduce the efficient collective household model of Browning, Chiappori and Lewbel (2013), hereafter BCL. This is a model that explicitly recognizes that there are scale economies in household consumption and that the individuals that comprise the household may have unequal access to household resources.

3.1 The collective household model of Browning, Chiappori and Lewbel

The model of BCL has its roots in the household models of Becker (1965, 1981) and Chiappori (1988, 1992).[16] Unlike Becker, Chiappori does not assume that the household pursues the objectives of a single household member (the "dictator"), but instead that the household pursues a mix of the members' objectives. Specifically, he assumes that the household achieves an efficient allocation in its purchases of pure public and pure private goods. In contrast, BCL do not require goods to be purely public or purely private, but instead permit goods to be fully or partly shared or not shared at all. Like earlier results in general equilibrium theory, BCL show that as a result of the Pareto efficiency of the household's resource allocation process, maximizing the household's objective function is observationally equivalent to a decentralised allocation. In this decentralised allocation, each household member demands a vector of consumption quantities given their preferences and a personal budget constraint, and the household purchases the sum of these demanded quantities (adjusted for economies of scale).

One can thus picture the household as a machine that makes budget constraints for its members. Each person's budget constraint is characterised by a shadow budget and a shadow price vector. They are "shadow" budgets and prices because they govern each person's consumption demands but they are not observed and do not equal the observed household budget or market prices.[17]

[16] Bourguignon and Chiappori (1994), Browning, Bourguignon, Chiappori, and Lechene (1994), and Browning and Chiappori (1998) also fall into this group of papers. Vermeulen (2002) is a review of the earlier literature. Dunbar, Lewbel and Pendakur (2013) provide a review of more recent papers in their introduction.

[17] This view of the household does not account for the fact that some people enter into households, and

Let i index people, and let the household consist of n_h people. Denote the set of household members in household h as $i\epsilon h$, and let x_h denote the household income (budget). Each person i gets a shadow budget, and these shadow budgets must add up to the full household budget. Person i's share of the household budget in household h is called their *resource share*, denoted η_{ih}, so each person's shadow budget is $\eta_{ih}x_h$. Resource shares sum to 1 in each household h. They may be unequal across people in the household, reflecting the fact that the within-household distribution of consumption may be unequal. Resource shares may depend both characteristics of the people comprising the household, and the characteristics of the household itself. Resource shares may also generally depend on the market prices of goods and the household budget.

Shadow prices are weakly lower than market prices because goods may be shareable. Shadow prices for goods, denoted $\widetilde{p}$, must be the same for all household members. (If they were not the same, then there would be gains from trade across household members, a violation of the assumption of efficiency.) Although BCL allow shadow prices to be any function of market prices, let us consider the case where shadow prices are proportional to market prices, so that $\widetilde{p} = A_h p$ where A_h is diagonal. This case is convenient because, given a linear market budget constraint, it results in a linear shadow budget constraint for each household member. The matrix A_h may depend on the characteristics of the household, but not on market prices or the household budget (if it did, the shadow budget constraint would not be linear).[18]

The matrix A_h has some bounds. The more a good is shared, the lower is its shadow price. Shadow prices may in principle be as low as p/n_h for a good that is so shareable that each household member can consume the purchased quantity. In this case, the diagonal element of A_h corresponding to the good equals $1/n_h$. For goods that are not shared, the shadow price equals the market price, and the element of A_h corresponding to the good equals 1. Define a *private good* to be a good that is not shared, and an *assignable good* as one where we can observe which household member consumes the good. Private assignable goods are very useful for identification, as we see below.

can exit them, voluntarily. In a lovely paper, Cherchye, De Rock and Vermeulen (2017) show that such restrictions can be be very informative. In particular, they show that the voluntary nature of household formation implies that the budget constraints generated by the household must be "better" than feasible alternatives offered by living alone or living in other households.

[18]BCL show that an analytical solution for household demand functions exists if the transformation between market and shadow prices is affine: $\widetilde{p} = Ap + a$. Here, A need not be diagonal, and a gives a vector of "overheads". In the case where $a = 0$, the BCL demand functions are exactly as in equation (7) below. I focus on the case where A is diagonal because additional results regarding private assignable goods are simpler.

3.2 Indifference scales and indifferent incomes

In this model, the resource share of a person and the shadow price vector together define the budget constraint faced by any household member. Imagining changing their household type now becomes a consumer surplus exercise. If the household is a machine that generates budget constraints, then changing household types is changing the budget constraint, and we can evaluate that for a person just as we'd evaluate any other change in the budget constraint.

Let the *indifference scale* r_{ih} be the scale to expenditure that equates utility between a person living in a household h, with resource share η_{ih} and facing shadow prices Ap, and that same person living alone, with resource share 1 and facing prices p.[19] It is defined as the solution to

$$V_{t(i)}\left(A_h p, \eta_{ih} x_h\right) = V_{t(i)}\left(p, \frac{x_h}{r_{ih}}\right). \quad (6)$$

Indifferent income is then equal to household income divided by the indifference scale, x_h/r_{ih}. Indifferent income answers the question: how much money do I have to give a person if they were living alone so that they are as well off as if they were living in household h?

Suppose that for a person i indirect utility were homothetic with $V = x_i/\gamma(p)$, where γ is increasing in prices. Then that person's indifference scale is relatively easy to calculate:

$$r_{ih} = \frac{\gamma(A_h p)}{\gamma(p)} \frac{1}{\eta_{ih}}.$$

It has a component driven by scale economies, $\gamma(A_h p)/\gamma(p)$. Recall that the elements of A_h are small for highly shareable goods, and 1 for shareable goods. Thus, $A_h p$ is smaller than p. Since γ is increasing in prices, $\gamma(A_h p) < \gamma(p)$, so $\gamma(A_h p)/\gamma(p)$ is less than 1. The more shareable are goods, the smaller is $\gamma(A_h p)/\gamma(p)$, and the smaller is the indifference scale. A small indifference scale implies that it takes a lot of income as a single to reach the utility level in the household.

The indifference scale also has a component driven by the resource share equal to $1/\eta_{ih}$. This component is strictly bigger than 1. The greater is the resource share, the smaller is the indifference scale, and the larger is the income level as a single required to reach the utility

[19] BCL actually define the indifference scale in a more general way. The define r_{ih} as the solution to $\phi_{t(i)}\left(V_{t(i)}\left(A_h p, \eta_{ih} x_h\right)\right) = V_{t(i)}\left(p, \frac{x_h}{r_{ih}}\right)$, where $\phi_{t(i)}$ is a type-specific monotonic transformation of utility. This means that the indifference scale is the scale to income that gets the consumer to the same *indifference curve* in the household as when living alone. It only corresponds to the same *utility level* when $\phi_{t(i)}$ is the identity function. The reason they define it in this way is to use only ordinal properties of utility functions—that is, indifference curves—to identify interesting objects. However, in order to engage in welfare analysis such as the inequality or poverty measurement, we have to take a stand on what $\phi_{t(i)}$ is. Throughout this paper, I assume it is the identity function.

level in the household. In real life, utilities are not homothetic, so the calculation is more complex, but the homothetic version captures the big stuff: scale economies and resource shares drive the indifference scale.

Indifference scales and indifferent incomes thus account for the fact that people live in heterogeneous households. We can convert a population of individuals in heterogeneous households into a population of as-if individuals living as singles by assigning indifferent income x_h/r_{ih} to each person. Welfare anaysis may treat all these as-if individuals as though they faced the market price vector p.

Indifference scales and indifferent incomes do not use interpersonal comparisons of well-being. They only compare across budget constraints for the same type of person. Thus, they are completely identified from knowledge of indifference curves, and do not suffer from the dependence on monotone transformations of utility that hinders our use of equivalence scales and equivalent incomes.

3.3 Individual Equivalent Incomes

Chiappori (2016) argued that since equivalence scales carry the baggage of interpersonal comparisons of utility and indifference scales do not, we should use indifference scales and dispense with equivalence scales. Unfortunately, although indifference scales allow us to convert people living in households to as-if versions of themselves living alone, those as-if single individuals are still heterogeneous. That is, indifference scales solve the problem of accounting for scale economies and resource access in households, but do not solve the problem of how to account for individual heterogeneity. For example, after we have converted our able-bodied person and disabled person into as-if singles, we must still account for the fact that the disabled person has greater needs than does the able person. Thus, indifference scales and equivalence scales are complements, not substitutes.

The *individual equivalent income* of a person living in a household h, denoted x^e_{ih}, is the amount of income that a reference person living alone would need to attain the same utility level.[20] It accounts for the scale economies and resource sharing in households and for heterogeneity across people. It solves

$$V_{t(i)}(A_h p, \eta_{ih} x_h) = V_0(p, x^e_{ih}).$$

Note that for single-member households, $A_h = I_J$ and $\eta_{ih} = 1$. Inverting (1) and (6), we

[20]Donaldson and Pendakur (2004, 2006) use x^e_i to denote equivalent income. Here, I extend the notation to include the subscript h and to accomodate the variation across households in which individuals might find themselves.

have that individual equivalent income is given by

$$x_{ih}^{e} = \frac{x_h}{s_i r_{ih}}$$

where, as noted above, $s_i = S_{t(i)}(p_i, x_i)$ is the equivalence scale for person i and r_{ih} is the resource share for person i in household h. By definition, $r_{ih} = 1$ for all people who live alone, and $s_i = 1$ for all people of the reference type. So, for reference type people living alone, we have $x_{ih}^{e} = x_h = x_i$.

We can use the vector of x_{ih}^{e} for every person in the population to estimate inequality or poverty. Note that the unit of analysis is the person, not the household, so we do not run into the issues raised by Ebert and Moyes (2001) as to how to make them comparable.[21] Further, within a household, the value of individual equivalent income may differ across people, because resource shares may differ across people. Thus, measuring inequality in this fashion accounts for intra-household inequality, inter-household inequality and individual heterogeneity.

3.4 Identification of Household Model Parameters from Consumer Behaviour

BCL show that given this simple model—where households reach an efficient allocation, have scale economies described by the diagonal matrix A_h and individuals have resource shares η_{ih}—household demand decisions are simply the sum of individual demand decisions, appropriately weighted to account for scale economies. Let Q_h be the $J-$vector of commodity demands for a household h with individuals $i\epsilon h$. Our notation of individual utilities and demands above had individuals coming in different types, indexed by t. Let $q_{t(i)}(p, x)$ be the quantity demand vector-function of individual i who is of type t. Then, we have

$$Q_h(p, x) = A \sum_{i \varepsilon h} q_{t(i)}(A_h p, \eta_{ih} x_h), \tag{7}$$

or, in budget-share form,

$$W_h(p, x) = \sum_{i \varepsilon h} \eta_{ih} w_{t(i)}(A_h p, \eta_{ih} x_h), \tag{8}$$

where W_h is the vector of household-level budget share functions. In equation (7), each individual chooses quantities given their shadow budget constraint, defined by the shadow

[21] Ebert and Moyes propose weighting household data by the equivalence scale of the household. Here, since households do not have equivalence scales (only people do), we give every person a weight of 1.

price vector $A_h p$ and their shadow budget $\eta_{ih} x_h$. The household considers the sum of these quantities, $\sum_{i \varepsilon h} q_{t(i)}(.)$, as what its members want to consume.

In (7), the term A_h multiplying the sum accounts for scale economies. Since the household has access to scale economies defined by A_h, it only needs to purchase A_h times that sum of quantities. For a non-shareable good, the element of A_h corresponding to that good equals 1. Thus, the household purchases the sum of individual demands of such goods. In contrast, for a shareable good, the element of A_h corresponding to that good is less than 1. For example, it might equal 0.5. In that case, the household need purchase only half the sum of individual demands to satisfy the demands of all the individuals in the household. The fact that the market purchase is less than the sum of the experienced consumption flows attained by all the individuals is how scale economies are manifested in the household.

BCL give some conditions under which one can identify the model parameters η_{ih} and A_h from consumer demand data. They show that if both the quantity demand functions of households and the quantity demand functions of each type of person are observed, then the parameters of the household model are identified. For example, if the quantity demand functions $q_t(p, x)$ can be observed by observing the consumption behaviour of all types as singles (not living in collective households), then the observed household demand functions $Q_h(p, x)$ and individual demand functions $q_{t(i)}(p, x)$ for all $i \epsilon h$ provide enough information to "back out" the matrix A_h and the scalars η_{ih}.

Note that the scalars η_{ih} show up in all J household budget share equations Q_h. There are $n_h - 1$ such scalars to identify. Thus, if there are many more goods than people in the household, so that J is much bigger than n_h, then there is a lot over overidentification for the resource shares η_{ih}. This implies that we could quite precisely identify resource shares in real data sets, because we often have many goods observed and few people comprising any one household. In contrast, the matrix A has a scalar to identify for each good (because each good has its own scale economies). So, adding more goods adds more parameters to identify.[22]

The identification strategy proposed by BCL is nice because it is general—it doesn't rest on further assumptions about the model. But it may be unsatisfying because it requires that single people have the same preferences as people in households, so that when we observe singles' demand functions $q_{t(i)}(p, x)$, we can just plug them into the BCL formulation for household demands (7). It does not allow for the possibility that, e.g., a person's preferences

[22]Further, the matrix A multiplies observed prices p, so if there is not much price variation in the data, we may have trouble getting precise estimates of A. In real data sets in rich countries, there is typically not very much relative price variation over time, so this can be a problem.

might change when they get married or join households.[23,24]

3.5 Identification of Resource Shares from Engel Curves

Lewbel and Pendakur (2008) and Dunbar, Lewbel and Pendakur (2013, 2017: hereafter DLP 2013 and DLP 2017) propose some strategies for identifying parameters in household models from Engel curve data (that is, data without price variation). As the scale economy parameters A multiply prices in the structural model, identification of these parameters would typically come from observed price variation. In the absence of price variation, then, we do not try to identify A, but instead focus on identification of the resource shares η_{ih} of each person i in a household h.

Resource shares are interesting even without knowledge of scale economies. First, resource shares provide a measure of consumption within the household: higher resource shares mean higher consumption. Second, they speak to inequality within the household: if resource shares are very unequal, there is a lot of inequality within the household. Third, resource shares may respond to policy variables in the context of poverty reduction. If we can find policy variables that shift resource shares upwards for disadvantaged individuals, then their poverty rates may decrease.

DLP 2013 and DLP 2017 focus on the case where we do not observe singles' demand functions. There are at least two reasons to stress this case. First, people in households may not have the same utility functions as people living alone. That is, household formation may change both the budget constraints people face *and* the preferences people have. Most importantly, single people might be different from married people. Second, some people cannot be observed living alone. Children and severely disabled people cannot live alone, and so cannot reveal their preferences even if they wanted to.

Identification of resource shares without the use of data on singles comes at a cost. First, we lose the ability to identify the scale economies available to households relative to singles.[25] Second, both DLP 2013 and 2017 impose restrictions on preferences. To achieve identification of resource shares, DLP 2013 impose a preference similarity restriction on the people who comprise the household to replace the BCL restriction that singles and people in

[23] In an approach which deals with heterogeneity across people, we can allow for the possibility that utility functions differ between people who are single versus people who live in multi-person household. For example, one could capture that difference in utility functions in the equivalence scale. Here, we would include a dummy variable in z_i indicating that the person is in a multi-person household.

[24] A related issue is that those who choose to get married may have different preferences from those who choose not to get married. This would induce selection bias, a distinct problem from preference change.

[25] Without data on the behaviour of singles, one cannot hope to identify the scale economy differences between singles and multi-member households. However, it is possible to identify the difference in scale economies between different types (sizes) of multi-member households.

couples have the same preferences. DLP 2017 avoid the preference similarity restriction and instead assume the existence of variables, called distribution factors, that change resource shares but do not affect preferences.

A second restriction imposed by DLP 2013 and DLP 2017 is that there is an observed private assignable good for every household member. A private good is one that is not shared, so its element of A equals 1. An assignable good is one where we know who consumes it. In those papers, clothing is used as the private assignable good, because in their Malawian household expenditure data, there are separate variables for spending on men's clothing, women's clothing and children's clothing.

These papers and Lewbel and Pendakur (2008) also impose the restriction that the resource share η_{ih} is independent of household expenditure, that is, it does not change as the household has a larger and larger budget. An implication of this restriction is that variation in household demands over income—the Engel curve—is driven solely by changes in individual demands and not by changes in resource shares. Thus, this restriction goes part of the way to allowing identication of resource shares from household Engel curves. Two papers have checked this restriction, and found that it is satisfied in real data (Menon, Pendakur and Perali 2013; Cherchye, De Rock, Lewbel and Vermeulen 2015).

3.5.1 BCL Demands for Assignable Goods

Let each of the n_h individuals in the household have a private assignable good, and let the first n_h goods of the vectors Q_h and $q_{t(i)}$ be those goods. Thus, Q_h^i and $q_{t(i)}^i$ are the scalar-valued household demand and individual demand, respectively, for person i's assignable good. For private assignable goods, their element of A_h equals 1, and so BCL's household demand equations for the private assignable goods simplify to

$$Q_h^i(p, x) = q_{t(i)}^i(A_h p, \eta_{ih} x_h). \tag{9}$$

This equation is simpler than equation (7): the matrix A only shows up inside the function $q_{t(i)}^i$ and only person i's demand function matters to the household demand for their assignable good.

To transform these equations into budget shares, we multiply by p^i and divide by x_h, yielding

$$W_h^i(p, x) = \eta_{ih} w_{t(i)}^i(A_h p, \eta_{ih} x_h). \tag{10}$$

where W_h^i is the household budget-share function for the assignable good of person i, and $w_{t(i)}^i$ is the budget-share function of person i (who is of type t) for their assignable good.

3.5.2 Preferences Similar Across People (SAP)

By the same reasoning as BCL, if we observe the individual assignable-good budget share functions $w^i_{t(i)}$, for example by observing single individuals in each type, then we can fully identify the model from the observed functions $\{w^i_{t(i)}, W^i_h\}$ for the n_h assignable goods. But, what can we do if we do not observe the individual budget-share functions $w^i_{t(i)}$? Dunbar, Lewbel and Pendakur (2013, 2017) address this question by focusing attention on Engel curves.

The Engel curves for assignable goods for these households evaluated at market prices $\overline{p}$, $\overline{W}^i_h$, may be written as

$$\overline{W}^i_h(x) = \eta_{ih}\widetilde{w}^i_{t(i)}(\eta_{ih}x_h), \tag{11}$$

where $\widetilde{w}^i_{t(i)}$ is the individual budget-share function for their assignable good evaluated at shadow prices $\widetilde{p} = Ap$. Suppose we did not observe the functions $\widetilde{w}^i_{t(i)}$ but did know that they were identical for all the people in the household (e.g., if people had identical utility functions within the household). Then, we could use the n_h observed functions W^i_h to identify the one unobserved function $\widetilde{w}$ plus the $n_h - 1$ unobserved constants η_{ih}. However, there is no variation to identify the matrix A—it is swallowed up by the function $\widetilde{w}$.

DLP 2013 show that the spirit of this identification holds if the individual budget-share functions for assignable goods are similar to each other (as opposed to identical to each other). In particular, they call preferences for assignables as similar across people (SAP) if they satisfy $\widetilde{w}^i_{t(i)}(x) = \widetilde{w}(x/G_{t(i)}) + g_{t(i)}$ for a function $\widetilde{w}$ and some constants G_t and g_t. Look familiar? This is exactly the shape-invariance restriction (5), but just applied to the assignable good of each person. They show that if preferences satisfy SAP, then just the Engel curves for the assignable goods of all the people in a single household type are sufficient to identify the resource shares of each person in the household.

Given SAP, household Engel curves for assignable goods satisfy

$$\overline{W}^i_h(x) = \eta_{ih}\widetilde{w}\left(\frac{\eta_{ih}x_h}{G_{t(i)}}\right) + \eta_{ih}g_{t(i)}, \tag{12}$$

so that the resource share scales both the budget share (outside the parentheses) and the budget (inside the parentheses). To see how identification works, consider the semi-elasticity with respect to the budget x of the household budget-share function for the assignable good of person i, $\overline{W}^i_h(x)$, as x limits to 0. This semi-elasticity is observable from household behaviour.

Evaluating the limiting derivative, we have $\lim_{x\to 0}\partial\overline{W}^i_h(x)/\partial\ln x = \eta_{ih}\kappa$, where κ is a constant equal to the limiting semi-elasticity of $\widetilde{w}(x)$: $\kappa = \lim_{x\to 0}\partial\widetilde{w}(x)/\partial\ln x$. We can evaluate this limiting semi-elasticity for each household member, yielding observable constants $\lambda_{ih} = \lim_{x \to 0} \partial \overline{W}_h^i(x)/\partial \ln x$ for each person i in household h. Resource shares satisfy

$$\eta_{ih}\kappa = \lambda_{ih}$$

for all i in h. Since resource shares sum to 1, we have the additional restriction that

$$\sum_{i \varepsilon h} \eta_{ih} = 1.$$

Together, these provide enough information to exactly identify the $n_h - 1$ resource shares in the household (and the constant κ).

The ratios of semi-elasticitiesof household budget-share functions as x gets close to 0 give the ratios of resource shares η_{ih}. A similar property holds at other values of x, but it is much messier due to presence of η_{ih} inside the parentheses.

The intuition is: if preferences are similar across people, then the ratios of slopes of household budget-share equations for each person's assignable are closely related to the ratios of their resource shares. Bigger budget-responses for your assignable good means you have a bigger resource share.

Equivalence-scale exactness (3), implies shape invariance (5), and in turn implies SAP (12). So, ESE implies SAP, but SAP is much weaker than ESE. In particular, SAP restricts only the shape of one budget-share equation, that of the assignable good. Further, SAP does not imply anything about interpersonal comparisons of utility. However, if one is already using equivalence scales to deal with heterogeneity across people, then SAP is implied without further structure, and resource shares are identified from household-level demands for assignable goods.

3.5.3 Identification with Distribution Factors (IDF)

DLP 2017 propose an alternative restriction to use with assignable goods Engel curve (11). Rather than imposing a similarity restriction on preferences, they propose the use of distribution factors to enable identification of resource shares. *Distribution factors* are defined as variables which affect resource shares but which do not affect preferences. They are important variables in the collective household literature for two reasons. First, they are closely related to individual's relative bargaining power with the household, and so are important components of marriage market models and are also found in other literatures associated with household formation, stability, and function. Second, distribution factors, such as the local supply of education or the local availability of nursing, may be policy variables, providing governments with the ability to affect the within-household distribution of resources. DLP 2017 show that in addition to these useful features, distribution factors generally provide identification of the levels of resource shares, without restrictions on the preferences of household members, or requiring that the preferences of household members can be observed via singles' behaviour.

When we considered identification of resource shares given SAP, we used assignable goods budget share functions (11) for a single household type h, and sought to identify the resource shares, η_{ih}, of each person i in the household. Identification using distribution factors (IDF) instead uses comparisons across D different household types where resource shares may vary, but where preferences of people, $\widetilde{w}^i_{t(i)}$, are fixed. Since prices are already held fixed in Engel curve analysis, the only way to vary resource shares without changing preferences is via distribution factors.

Specifically, whereas SAP used equation (11) for people $i = 1, ..., n_h$, IDF uses the same equation, but for people $i = 1, ..., n_h$ and for households of types $h = 1, ..., D$. Here again is equation (11):

$$\overline{W}^i_h(x) = \eta_{ih}\widetilde{w}^i_{t(i)}(\eta_{ih}x_h).$$

Now, we consider n_g assignable goods equations (indexed by i) for each of D household types (indexed by h), yielding $n_h D$ observed household Engel curves functions $\overline{W}^i_h(x)$. But, the function $\widetilde{w}^i_{t(i)}(x)$ does not depend on household type: it is the same for all D household types. There are n_g such functions to identify. Additionally, we have to identify resource shares, which do depend on household type. There are $(n_h - 1)D$ values of resource shares η_{ih}, because for every household type, the last resource share is given by the restriction that they sum to 1. Thus, if we have $D \geq n_h$, so that there are more support points for distribution factors than there are people in the household, then we have more observed household Engel curves than unobserved individual demand functions and resource share parameters. This is the order condition for identification, and it can be easily verified in the data. The rank condition is more complicated, and described in DLP 2017.

4 Estimation

4.1 Demand System for Estimating Exact Equivalence Scales

First, consider the estimation of equivalence scales for individuals. Instead of estimating separately by type (as in Figure 1), we parameterise types as varying by a vector of observed demographic characteristics z. Let type 0 have $z = 0$. Two individuals with the same value z have the same type, and therefore the same utility function. For this paper, we use 3 demographic characteristics: indicators that the person is male and disabled, age less 50. Thus, the reference type (0) is female, able-bodied, 50 years old.

A large literature in demand analysis suggests that budget share equations are usefully modeled with quadratic structures.[26] Banks, Blundell and Lewbel (1998) provide the popular[27] quadratic almost ideal (QAI) demand system, which I use for the reference indirect utility V_0 and budget-share system $w_0(p, x)$:

$$V_0(p, x) = \left(\left(\frac{\ln \frac{x}{\gamma(p)}}{\exp(b' \ln p)} \right)^{-1} - q' \ln p \right)^{-1} \tag{13}$$

where

$$\ln \gamma(p) = c' \ln p + \frac{1}{2} \ln p' C \ln p.$$

The implied budget-share system is given by

$$w_0(p, x) = c + C \ln p + b \ln \frac{x}{\gamma(p)} + q \left(\ln \frac{x}{\gamma(p)} \right)^2 / \exp(b' \ln p). \tag{14}$$

Next, specify the equivalence scale function so that it has simple price elasticities:

$$\ln \Delta(p, z) = d' z + \ln p' D z \tag{15}$$

with

$$\delta(p, z) = \nabla_{\ln p} \ln \Delta(p, z) = D z. \tag{16}$$

Substituting (14), (15) and (16) into (4), substituting observed values p_i, x_i, z_i and adding a conditionally mean-independent error term ε_i yields

$$w(p_i, x_i, z_i) = c + D z_i + C \ln p_i + b \ln \left(\frac{x_i}{\gamma(p_i)} - d' z_i + \ln p_i' D z_i \right) + q \frac{\left(\ln \frac{x_i}{\gamma(p_i)} - d' z_i + \ln p_i' D z_i \right)^2}{\exp(b' \ln p_i)} + \varepsilon_i. \tag{17}$$

Here, the model parameters are $\{c, C, b, q\}$ which characterise the reference (type 0) demand system and $\{d, D\}$ which characterise the equivalence scale. The variables are $\{p, x, z\}$.

Although the model is nonlinear, the identification of parameters from data variation is relatively clear: the constant in each equation gives c, variation in z gives D, variation in $\ln p$ gives C, variation in $\ln x$ gives b, and variation in $(\ln x)^2$ gives q. The only tricky part is

[26] Pollak and Wales (1992) is a good early survey of parametric consumer demand methodology; Deaton (1998) is a survey, with special emphasis on development economics; Lewbel (2008) is a survey of work on Engel curves.

[27] 1500 google-scholar citations as of 10 June 2017.

the identification of d. Multiplying out the quadratic term reveals a $\ln \frac{x_i}{\gamma(p_i)} \cdot d'z_i$ term, which suggests that variation in the interaction of $\ln x$ and z provides identifying information for d. Like Pendakur (2001), below I estimate this model by nonlinear seemingly unrelated regression. Alternatively, one could use GMM with instruments $\{1, \ln p_i, \ln x_i, z_i, (\ln x_i)^2, x_i z_i\}$.

4.2 Demand System for Estimating Collective Household Model

Now, consider a collective household. Suppose one uses the identification strategy of BCL, where data on singles are used to identify each person's preferences, and data on couples are used to identify collective household model parameters (A_h and η_{ih}). Then estimation amounts to choosing either the quantity demand system (7) or budget share system (8), substituting an off-the-shelf demand system for individual quantities $q_i(p, x)$ or individual budget shares $w_i(p, x)$, and estimating the $3(J-1)$ equations corresponding to single men's demands ($J-1$ equations, where $A_h = 1$ and $\eta_{ih} = 1$), single women's demands ($J-1$ equations, where $A_h = 1$ and $\eta_{ih} = 1$) and household demands (A_h and η_{ih} as estimated parameters).

Because I want to use both equivalence scales and indifference scales, I use the demand system above, which supports identified exact equivalence scales. For singles, the budget share equations are as above. For households, substitute (14), (15) and (16) into (4). Then, substitute those into the BCL model (8), substitute observed values p_h, x_h, z_i, and add an error term ε_h to get:

$$W_h = c + C \ln A_h \ln p_h +$$

$$\sum_{i \varepsilon h} \eta_{ih} \left(Dz_i + b \left(\ln \frac{\eta_{ih} x_i}{\gamma(p_i)} - d'z_i + \ln A_h \ln p_h' Dz_i \right) + q \frac{\left(\ln \frac{\eta_{ih} x_i}{\gamma(p_i)} - d'z_i + \ln A_h \ln p_h' Dz_i \right)^2}{\exp(b' \ln A_h p_h)} \right) + \varepsilon_i \tag{18}$$

The diagonal matrix A_h, which gives the scale economies for each good, may be written as a linear index of household characteristics. However, in the empirical example below, I have only two-person households, so I write each diagonal element of A_h^j as a good-specific constant. For goods that are a priori known to be non-shareable, I set $A_h^j = 1$.[28]

Let y_h give a vector of household-level distribution factors. Let z_h be the list of all characteristics of all household members. Resource shares may in general depend on individual

[28] Recall that each element A_h is weakly less than 1, and note that A_h is always logged in equation (18). So, each element of $\ln A_h$ must be less than 0. In the empirical work below, I specify each free element of $\ln A_h$ as the negative of a squared parameter.

characteristics of all household members, z_h, on household-level distribution factors, y_h, and on the prices of goods, p. However, because the resource share is assumed independent of the household budget, it must be homogeneous of degree 0 in p.

In general, the resource shares η_{ih} of each person may be written as logit functions of linear indices of z_h and y_h. This form keeps all the resource shares between 0 and 1, and maintains the restriction that they add up to 1. In the empirical example below, I have only two-person households, so that there is only one free resource share η_{1h} (the other being given by $\eta_{2h} = 1 - \eta_{1h}$). So, I have

$$\eta_{1h} = \frac{\exp\left(f_0 + f_z' z_h + f_y' y_h + f_p' \ln p\right)}{1 + \exp\left(f_0 + f_z' z_h + f_y' y_h + f_p' \ln p\right)},$$

and

$$\eta_{2h} = \frac{1}{1 + \exp\left(f_0 + f_z' z_h + f_y' y_h + f_p' \ln p\right)},$$

with $\iota' f_p = 0$.[29]

Since there are many identifying restrictions available for BCL, we may approach estimation of this model in many ways. For example, following BCL 2013 and 2017, we may assume the existence of an assignable good. This assignable good must be the same good for each person. In the empirical work below, I use clothing, yielding men's clothing and women's clothing as the 2 assignable goods in a 2 person male-female household. For such a good, BCL household budget share functions are given by

$$w^i(p_i, x_i, z_i) = \eta_{ih}\left(c + Dz_i + C \ln A_h \ln p_h + b\left(\ln \frac{\eta_{ih} x_i}{\gamma(p_i)} - d' z_i + \ln A_h \ln p_h' D z_i\right)\right) + \eta_{ih} q \frac{\left(\ln \frac{\eta_{ih} x_i}{\gamma(p_i)} - d' z_i + \ln A_h \ln p_i' D z_i\right)^2}{\exp(b' \ln A_h \ln p_i)} + \varepsilon_i. \quad (19)$$

Here, since we have already assumed ESE, we get SAP for free, and the resource shares are identified for each Engel curve, without the use of single's data (BCL 2013). Here, only the clothing elements of c, D, b, q are identified.

In our empirical example below, we use both these strategies together, using data on both singles' and couples' demands for all goods, and using assignable clothing demands for extra identifying power (aka precision).

This model may be estimated by nonlinear seemingly unrelated regressions, or by GMM with instruments $\{1, \ln p_h, \ln x_h, y_h, z_h, (\ln x_h)^2, x_h y_h, x_h z_h, \ln n_h \ln p_h\}$. Here, the instruments $x_h y_h, x_h z_h$ provide identifying information for the parameters f^i in the resource shares η_{ih}.

[29] This restriction implies that resource shares are homogeneous of degree zero in prices.

Whereas the equivalence-scale function $\Delta(p, z)$ is essentially a parameter of the model, the indifference scale must be solved for using (6). The indirect utility function for a person with characteristics z is given by combining (15) and (13):

$$V(p, x, z) = \left(\left(\frac{\ln \frac{x}{\gamma(p)\Delta(p,z)}}{\exp(b' \ln p)} \right)^{-1} - q' \ln p \right)^{-1}.$$

We compute the indifference scale by solving (6) with this indirect utility function. The indifference scale is then given solving $V(A_h p_h, \eta_{ih} x_h, z_i) = V(p, x/r_{ih}, z_i)$ for r_{ih}, and is given by:

$$\ln r_{ih} = \ln \frac{x}{\gamma(p)\Delta(p, z)} - \exp(b' \ln p) \left(\left(\frac{\ln \frac{\eta_{ih} x}{\gamma(Ap)\Delta(Ap,z)}}{\exp(b' \ln p + b' \ln A)} \right)^{-1} - q' \ln A \right)^{-1}.$$

Note that V is homothetic if $b = q = 0$. In this case, $\ln r_{ih} = \ln \frac{x}{\gamma(p)\Delta(p,z)} - \ln \frac{\eta_{ih} x}{\gamma(Ap)\Delta(Ap,z)}$, so $r_{ih} = \frac{\gamma(Ap)\Delta(Ap,z)}{\gamma(p)\Delta(p,z)} \frac{1}{\eta_{ih}}$ (as in the homothetic example above).

Individual equivalent income is given by

$$x^e_{ih} = \frac{x_h}{r_{ih} \Delta_i}$$

where $\Delta_i = \Delta(p, z_i)$.

5 An Illustration with Canadian Data

I bring these models to Canadian data as follows. The consumption data are drawn from the Surveys of Household Spending 2004 to 2009, and are described in detail in Norris and Pendakur (2013 and 2015). I estimate using singles data and household data, where the collective households are comprised of childless male-female households, and the single-adult households are comprised of single childless men or women. In collective households, the woman is person 1.

I use a very small demand system with just $J = 4$ goods: food at home; food outside home (e.g., restaurants); clothing; rent. Total nondurable expenditure is defined as the sum of these 4 expenditure categories, and it is normalized to have a mean of 1 for singles in Ontario in 2006. Budget shares are defined as the fraction of total nondurable expenditures spent on each good. Clothing is divided into men's clothing and women's clothing. For single women, women's clothing expenditures are observed, and men's clothing expenditures are set to 0; for single men, the opposite. In models with just singles, there are 3 equations. In models with singles and couples together, there are 4 equations, because collective households consume both types of clothing.

Sample selection is done in the following order. Households in rural areas and in cities with less than 100,000 residents are dropped. Households with members aged less than 25 or greater than 59 are dropped. Only rental-tenure households where rent is observed and not reduced (e.g., subsidized) are retained in the sample. Finally, for each sample year, households in the top and bottom percentile of the total nondurable expenditure distribution for singles and couples, respectively, are dropped. There remain a total of 4568 observations, comprised of 1553 single women, 1891 single men, and 1124 female-male married couple households.

The types depend on observed characteristics z_i: an indicators that the person is male; an indicator that the person is disabled; and the age of the person less 50 divided by 20. Disability status is the answer to the question: "Does any member of this household/Do you have any difficulty hearing, seeing, communicating, walking, climbing stairs, bending, learning or doing any similar activities?".

These characteristics z enter the parametric equivalence scale function (15). The reference person with $z = 0$ is a 45 year old able-bodied woman, and their budget share functions are given by the QAI functional form (14). Thus, the budget share functions of singles are given by (17).

Table 1 gives descriptive statistics for singles on budget shares, log total nondurable expenditures, log-prices and demographic characteristics.

Table 1: Descriptive Statistics—Singles

N=3444	mean	mean	sd	min	max
budget shares	food in	0.239	0.103	0.000	0.886
	food out	0.089	0.093	0.000	0.576
	clothing	0.072	0.067	0.000	0.616
	rent	0.599	0.133	0.000	0.986
log nondurable consumption		-0.201	0.401	-2.482	0.808
log prices	food in	0.042	0.057	-0.049	0.189
	food out	-0.007	0.021	-0.053	0.036
	clothing	0.013	0.025	-0.041	0.050
	rent	-0.072	0.149	-0.411	0.146
demographics	male	0.549	0.498	0.000	1.000
	disability	0.207	0.405	0.000	1.000
	(age-45)/20	-0.441	0.587	-1.250	0.500

Households' budget share functions, given by (8), depend on singles' preferences defined above, and scale economies and resource shares. Scale economies are determined by the diagonal matrix A_h, where the diagonals of A_h are given by the constants A^{jj} (with no subscript h because there is just one type of collective household). I set those constants to 1 for both clothing and food out, the idea being that these are a priori not shareable goods. Thus, there are only two elements of A_h to estimate: the scale economy parameters for food at home and for rent.

Resource shares depend on a constant all the demographic characteristics of all household members. In a two-person household, resource shares must be symmetric in the sense that if I permute household members, the resource shares should permute along with them. The easiest way to achieve this is to have resource shares depend on the difference in z across household members. Since person 1 is a woman in every collective household, the difference in the male dummy is a constant, and so is excluded. We are left with the difference between the woman's and man's disability status and the difference between the woman's and the man's age.

Table 2: Descriptive Statistics–Couples

N=1124		mean	sd	min	max
budget shares	food in	0.278	0.105	0.000	0.688
	food out	0.091	0.078	0.000	0.449
	womens clothing	0.067	0.054	0.000	0.368
	mens clothing	0.045	0.041	0.000	0.377
	rent	0.519	0.118	0.119	0.860
log nondurable consumption		0.181	0.365	-1.562	1.405
log prices	food in	0.043	0.059	-0.049	0.189
	food out	-0.006	0.021	-0.053	0.036
	clothing	0.013	0.025	-0.041	0.050
	rent	-0.060	0.149	-0.411	0.146
demographics	woman's disability	0.109	0.312	0.000	1.000
	woman's age	-0.624	0.574	-1.250	0.500
	man's disability	0.117	0.321	0.000	1.000
	man's age	-0.516	0.594	-1.250	0.500
	disability difference	-0.007	0.382	-1.000	1.000
	age difference	-0.108	0.278	-1.000	1.000
distribution	relative income	-0.099	0.247	-0.500	0.500
factors	relative education	0.039	0.442	-1.400	1.400

Resource shares also depend on a pair of observed distribution factors y_h: the relative wage of the woman vs the man, the relative education level of the woman vs the man. In other research (e.g., Cherchye, De Rock and Vermeulen 2015), these two variables have been shown to be positively correlated with womens' resource shares. Finally, the resource share depends on logged relative prices, $\ln p_j/p_J$.

Table 2 gives descriptive statistics for couples on budget shares, log total nondurable expenditures, log-prices and demographic characteristics and their differences, and distribution factors.

We first estimate equivalence scales with just singles data (reported in Table 1) via nonlinear seemingly unrelated regression applied to the parametric budget share functions given by equation (17). This equation system has 4 equations, but the fact that budget shares add up to 1 implies that we can estimate just 3 of the 4 equations (food in, food out, and clothing).

We then add the observations of households (reported in Table 2) and estimate equations (17) and (18) together. For observations of singles, we use equation (17) to define their budget share equations, with the other person's clothing share set to 0. For observations of couples, we use equation (18), which has an equation for each person's clothing share.

Table 3 presents estimates of the parameters in the reference demand system for both the singles-only model and the all-data model. Standard errors are robust to heteroskedasticity. Hausman z-tests are provided on a parameter-by-parameter basis.

Table 3: Estimates of Reference Demand System Parameters

Parameter			singles only est	std err	all households est	std err	hausman z test
c	food in		0.227	0.004	0.228	0.003	-0.537
	food out		0.068	0.004	0.067	0.003	0.429
	clothing		0.087	0.002	0.086	0.002	0.588
b	food in		0.170	0.047	0.208	0.036	-1.300
	food out		0.204	0.041	0.191	0.033	0.522
	clothing		0.145	0.029	0.108	0.022	1.902
q	food in		-0.187	0.049	-0.224	0.038	1.184
	food out		-0.137	0.043	-0.133	0.034	-0.180
	clothing		-0.103	0.031	-0.081	0.023	-1.111
C	food in	food in	0.097	0.034	0.074	0.016	0.777
	food in	food out	0.077	0.046	0.094	0.016	-0.371
	food in	clothing	-0.048	0.024	-0.027	0.015	-1.079
	food out	food out	-0.262	0.095	-0.227	0.043	-0.409
	food out	clothing	0.169	0.048	0.116	0.029	1.378
	clothing	clothing	-0.069	0.027	-0.059	0.018	-0.487

Since nondurable consumption, x, is normalized to equal 1 for the average household in Ontario in 2006, that household has $\ln x = 0$. At $\ln x = 0$, the predicted budget shares for the reference household are equal to c. Inspection of these parameters suggest that predicted budget shares are reasonable, and are very similar for the model with data on singles alone and the model that incorporates couples data. In particular, the estimates of c are similar across the two models, and the Hausman tests don't detect a statistically significant difference.[30]

Evaluated at $\ln x = 0$, the response of budget shares to a proportionate increase in the budget is given by b. These parameters suggest that the Engel curves for food in, food out and clothing all slope upwards at $\ln x = 0$. Because budget shares sum to 1, this implies that rent shares slope downwards at $\ln x = 0$. It is moderately strange to see food in shares rising with the budget, but simple scatter plots of the data suggest that for this subset of households, the data support this finding.

[30]The Hausman test is appropriate when we have a consistent estimator and an efficient estimator to compare. In this context, the singles-only data provide a consistent estimator that is inefficient due to leaving out useful data on couples. The all-data estimates provide an efficient estimator that is also consistent if the household model is true, but may be biased if the household model is false. The joint test that all the parameters are the same across the two models does not fail the Hausman test—that is, they look quite similar. However, the joint test that all the parameters in the equivalence scale, shown in Table 4, does fail the Hausman test. I am inclined to take this as saying that the household model is "not that false".

Estimation of exact equivalence scales requires that budget shares not be linear in the log of nondurable consumption, $\ln x$. In this context, that requires that q be nonzero in at least one element. A glance at the estimated standard errors suggests that all three elements of q are nonzero.

A final check on the estimated reference demand system is to ask whether or not the estimates are consistent with negativity of the Slutsky matrix (aka: concavity of the cost function, quasi-concavity of the indirect utility function). One implication of negativity is that compensated demand functions decrease in own-prices. Evaluated at $\ln x = 0$, these budget share functions all satisfy this condition.[31]

Table 4 presents estimates of equivalence scale parameters d and D. Again, we provide parameter-by-parameter Hausman z-statistics to assess whether or not adding couples data and the collective household model affects estimated equivalence scale parameters.

Table 4: Estimates of Equivalence Scale Parameters

			singles only		all households		hausman
Parameter			est	std err	est	std err	z test
d	man		-0.250	0.136	-0.211	0.128	-0.855
	disability		0.368	0.189	0.291	0.156	0.722
	(age-50)/20		0.094	0.116	-0.011	0.088	1.394
D	food in	man	0.019	0.004	0.025	0.004	-4.571
		disability	-0.010	0.005	-0.006	0.004	-1.356
		(age-50)/20	0.019	0.003	0.021	0.002	-0.933
	food out	man	0.017	0.010	0.023	0.009	-0.948
		disability	0.016	0.014	0.010	0.011	0.568
		(age-50)/20	-0.017	0.009	-0.023	0.006	1.012
	clothing	man	-0.036	0.007	-0.087	0.004	10.009
		disability	0.003	0.009	-0.002	0.005	0.624
		(age-50)/20	-0.019	0.006	-0.016	0.003	-0.636

Tthis formulation of the equivalence scale is easy to evaluate when prices equal 1 as they do for households in Ontario 2006: it is equal to $\exp(d'z)$. There are a couple of cool things to note. First, this is the first demand-based estimate of an exact equivalence scale for disability that I have seen.[32] Second, the estimated equivalence scale for men is not equal

[31] More precisely, the own-price Slutsky term for budget-share functions is given by $\partial w^j / \partial \ln p^j + \left(w^j\right)^2 - w^j$. Evaluated at $\ln x = 0$, this object is given by $C^{jj} + \left(c^j\right)^2 - c^j$. The value of this is statistically significantly negative for all goods.

[32] Braithwaite and Mont (2008) provide a nice survey of how to think about the interaction of disability and poverty, and use some equivalence scales. However, their equivalence scales are not based on, or consistent with, demand behaviour.

to 1.

The estimated value of the equivalence scale for a disabled woman in comparison to an able-bodied woman is $\exp(0.368) = 1.45$. This implies that a disabled woman with an income of $\$50,000$ has an equivalent income of $\$34,500$ and therefore is as well off as an able-bodied woman with an income of $\$34,500$.

In Table 1, I show that roughly 20 per cent of the sample reports disability. This means that even if income were distributed perfectly equally among all people, the disabled people would be worse off than the able-bodied people, and that there would therefore be meaningful inequality in the distribution of well-being. Using equivalent incomes instead of nominal incomes allows us to account for this. Of course, this comes at a cost: I had to impose a functional form restriction (exactness) on the equivalence scale in order to achieve identification of equivalence scales from consumer demand behaviour.

A second statistically significant element of d is the estimated coefficient on the male indicator. This is statistically signficantly negative, and indicates that the equivalence scale for an able-bodied 50 year old man is 0.78. This means that women need more money than men to be equally well off. In particular, if a 50 year old able-bodied single man and single woman each have incomes of $\$50,000$, the woman's equivalent income is $\$50,000$ but the man's is $\$64,000$ indicating a situation of inequality. But, do we really want to call that economy as one characterised by inequality?

This is a troubling finding, but it cannot be easily escaped. Men have different preferences from women, and they must therefore have an equivalence scale different from 1 when facing some price vector(s). The equivalence scale methodology does not have an obvious way to force an equivalence scale to be independent of a priori welfare-irrelevant characteristics. Developing such analytical tools would be a useful endeavour.

Table 5 presents estimated parameters of the collective household model. I report the two free parameters of A_h—the elements for food in and for rent–and the coefficients on parameters that enter the linear index inside the logit function for the woman's resource share η_{1h}.

Table 5: Estimates of Collective Household Model Parameters				
			all households	
Parameter			est	std err
A	food in		0.626	0.083
	rent		0.589	0.044
η	constant (woman)		0.194	0.049
	disability diff		0.038	0.055
	(age-50)/20 diff		0.115	0.065
	y_1, income share of woman - 0.50		0.245	0.078
	y_2, education difference		-0.046	0.040
	$\ln p_{foodin} - \ln p_{rent}$		1.064	0.672
	$\ln p_{foodout} - \ln p_{rent}$		-0.950	1.463
	$\ln p_{clothing} - \ln p_{rent}$		0.061	0.750
η	$z_1 = 0, 0, 0$	$z_2 = 1, 0, 0$	0.548	0.012
	$z_1 = 0, 0, 0.5$	$z_2 = 1, 0, 0$	0.562	0.016
	$z_1 = 0, 0, 0$, $y_1 = 0.75$	$z_2 = 1, 0, 0$	0.563	0.014

The estimated values of A_h suggest substantial scale economies in these two-person households. Food at home is quite shareable, with a scale economy of 0.626. Rent (shelter) is even more shareable with a scale economy of 0.589. Food out and clothing are assumed to have no economies of scale, so their elements of A_h equal 1. Recall the homothetic expression for the scale economy component of the indifference scale, $\gamma(Ap)/\gamma(p)$. Evaluated at $p = 1$, a first-order approximation of this is given by $\exp(c' \ln A)$, which is equal to 0.64 (with a standard error of 0.05). This means that, due to economies of scale in consumption, people in two-person households need roughly 64 per cent as much money to buy the same effective consumption of people living alone. However, there is still the issue of who in the household gets the larger share of that consumption, which leads us to estimated resource shares.

The woman's resource share η_{1h} is equal to $\exp(\mu_h) / (1 + \exp(\mu_h))$ where μ_h is a linear index in observed variables whose coefficients are displayed in Table 5. The estimated value of the constant term is 0.194. In the bottom three rows of the table, we evaluate η_{1h} at various values of its arguments. The first of these rows gives the estimated value of η_{1h} for married couple households where neither person has a disability, both are aged 50, all distribution factors are 0 and all prices equal 1. The estimated value is $\exp(0.194) / (1 + \exp(0.194)) = 0.548$, because the constant term is the only parameter entering the index. That the woman's resource share exceeds half is striking, but is consistent with other estimates of resource shares using Canadian data, in particular, with BCL's estimates.

Neither the relative age or disability status of the married couple, nor the prices of commodities, have a statistically significant effect on the resource share. However, the relative ages and relative incomes of the women and men in these couples do matter. The second row from the bottom gives the estimated value of η_{1h} for a couple where neither partner is disabled and where the woman is 10 years older than the man. This woman has a resource share of 0.562, which is 1.4 percentage points higher than the household considered above where the spouses are the same age. The third row from the bottom gives the estimated value of η_{1h} for a couple where neither partner is disabled and both are the same age, but where the woman's income constitutes 75 per cent of household income. This woman has a resource share of 0.563, which is 1.5 percentage points higher than the first type.

Although these differences in resource shares due to individual characteristics and distribution factors are statistically significant, they are not very large in economic terms. If we simulate η_{1h} for the full sample of 1124 households, we find an average of 0.56 with a standard deviation of 0.03. So, resource shares do not induce a lot of within-household inequality.

Finally, we come to using equivalence scales and indifference scales to estimate the distribution of individual equivalent income. To measure consumption inequality when commodity prices vary over time, we must use price deflators.[33] The correct price deflator that turns individual equivalent income into real individual income is the price deflator for a reference type person who is single. Let the base price vector (to which real values refer) be 1_J, the price vector faced by residents of Ontario in 2006. The price index is given by solving $V_0(p, x) = V_0(1_J, x/\pi)$ for the price index π. Reference indirect utility V_0 is given by equation (13), and it simplifies greatly at a price vector of 1, with $V_0(1_J, x) = \ln x$.[34] The price index π is therefore given by

$$\ln \pi(p, x) = \ln x - \left(\left(\frac{\ln \frac{x}{\gamma(p)}}{\exp(b' \ln p)} \right)^{-1} - q' \ln p \right)^{-1}$$

I will discuss both nominal measures and real measures, where real measures equal nominal measures divided by the price index above.

Table 6 presents means of logs of equivalence scales, indifference scales, equivalent incomes, indifferent incomes and individual equivalent incomes, in nominal and real terms, for the singles and couples in our sample. Note that in all cases "income" is equal to nondurable consumption as defined above. In both Table 6 and Table 7, the unit of analysis is the person, so households are split into their consituent members. Thus, lines denoted couples indicate the 2248 people living in the 1124 2-person households, lines denoted singles indicate the

[33]For a much lengthier discussion of these issues, see Pendakur 2002.

[34]Another way to say this is that the log real individual equivalent income equals reference indirect utility evaluated at individual equivalent income, as defined in equation (13): $\ln x^e_{ih} - \ln \pi_{ih} = V_0(p, x^e_{ih})$.

3444 single individuals, and lines denoted all indicate all 5692 people in the sample of 4568 households.

Table 6: Means of logs

		2004	2005	2006	2007	2008	2009
indifference scales	couples	0.274	0.278	0.274	0.281	0.277	0.285
equivalence scales	singles	-0.051	-0.052	-0.055	-0.049	-0.066	-0.039
	couples	-0.069	-0.068	-0.066	-0.067	-0.070	-0.065
income	all	-0.140	-0.156	-0.047	0.002	0.033	0.070
indifferent income	couples	-0.153	-0.180	-0.109	-0.052	-0.051	0.005
individual equivalent income	singles	-0.264	-0.266	-0.129	-0.093	-0.023	-0.045
	couples	-0.084	-0.111	-0.043	0.014	0.019	0.070
	all	-0.191	-0.206	-0.095	-0.051	-0.007	0.002
price deflator	all	-0.014	-0.006	0.007	0.033	0.040	0.096
real individual equivalent income	singles	-0.137	-0.154	-0.030	-0.018	0.040	-0.031
	couples	-0.236	-0.271	-0.215	-0.188	-0.184	-0.182
	all	-0.177	-0.200	-0.102	-0.084	-0.047	-0.093

There are a few notable observations from the table of means. First, the log of the indifference scale for the members of couples is on average about 0.28, meaning that the indifference scale is roughly $\exp(0.28) = 1.32$. This means that a person in a 2 person household may be expected to be about as well off as that same person living alone with the household income divided by 1.32. If there were no scale economies available to the household, then this number would be closer to 2. So, scale economies in household consumption are pretty important.

Equivalence scales for the individuals living alone and living in households are computed at market prices because the indifference scale does the work of accounting for the scale economies in households. Equivalence scales are on average larger for people living alone than for people living in households, indicating that households are comprised of people with lower equivalence scales and thus lower costs. For example, 21 per cent of single individuals report a disability, but only 11 per cent of the members of 2-person households report a disability. Neither equivalence scales nor indifference scales change very much over time on average.

Income, that is nondurable consumption expenditures on our four goods (food in, food out, clothing and rent), is normalized so that log-income is 0 for the average single person in Ontario in 2006. Nominal income rises by about 21 log points over the period. Indifferent income for members of 2-person households also rises over time, but only by about 15 log points. This is because the nominal incomes of singles grew faster than those of households.

Nominal individual equivalent income, which accounts for scale economies, the within-household distribution of income, and individual heterogeneity, rose by 22 log points for singles and 15 log points for couples. This gave an overall increase of 19 log points for all people (in the sample). However, prices rose by 11 log points, eating up much of the increase in nominal individual equivalent income.

The bottom panel gives the over-time pattern in average log real individual equivalent income. It rose by about 10 log points for singles and about 5 log points for people in couples.

Table 7 presents standard deviations of logs of equivalence scales, indifference scales, equivalent incomes, indifferent incomes and individual equivalent incomes, in nominal and real terms, for the singles and couples in our sample.

Table 7: standard deviations of logs

		2004	2005	2006	2007	2008	2009
indifference scales	couples	0.115	0.126	0.124	0.128	0.137	0.153
equivalence scales	singles	0.166	0.158	0.164	0.157	0.158	0.158
	couples	0.136	0.137	0.146	0.141	0.135	0.142
income	all	0.460	0.468	0.382	0.401	0.394	0.403
indifferent income	couples	0.358	0.416	0.335	0.378	0.346	0.368
individual equivalent income	singles	0.488	0.480	0.413	0.415	0.431	0.419
	couples	0.374	0.430	0.350	0.400	0.355	0.370
	all	0.454	0.467	0.392	0.413	0.403	0.403
price deflator	all	0.223	0.217	0.216	0.217	0.211	0.215
real individual equivalent income	singles	0.564	0.550	0.492	0.496	0.502	0.497
	couples	0.408	0.478	0.396	0.452	0.387	0.412
	all	0.509	0.526	0.465	0.487	0.473	0.470

Consider the first three rows of Table 7. The typical analysis of inequality would use equivalence scales to convert households into individuals, ignoring the fact that households are not people and thus don't have utility functions. It would then treat all these individuals as identical. In this analysis, we instead use a model of the household which generates indifference scales that turn people that live in multi-person households into people that live alone. We then deal with heterogeneity among people with equivalence scales. The fact that the standard deviation of log indifference scales is of roughly the same magnitude as that of log equivalence scales suggests that household scale economies are roughly as important as individual heterogeneity when it comes to measuring inequality. Ignoring individual heterogeneity is thus a bad idea.

A second feature of the indifference scales and equivalence scales shown in Table 7 is that the standard deviation of both log indifference scales and log equivalence scales rose over time. These increases in variance contribute to the change in economic inequality over time.

The standard deviation of log income (remember: nondurable consumption on our 4 goods) declined over time, by roughly 6 basis points.[35] (For comparison, the standard deviation of log equivalent nondurable consumption in the USA rose by 10 basis points between 1990 and 2010, see Attanasio, Hurst and Pistaferri 2013). However, the standard deviation of log nominal indifference income for people in couples rose by 1 basis point. Thus, patterns were quite different for singles than for people in couples.

Turning to nominal individual equivalent income, we see that for singles the standard deviation of the logs fell by 7 basis points, but for couples only by half a basis point, leading a decrease in of 5 basis points for all people. Overall, the use of indifference scales and equivalence scales turned a decline of 6 basis points (for nominal income) into a decline of 5 basis points for nominal individual equivalent income.

These patterns are slightly different in the real measures shown in the bottom panel. The standard deviation of log real individual equivalent income fell by 7 basis points for singles and rose by half a basis point for people in couples. But, since the over-time growth in average log real individual equivalent income of singles was greater than that of people in couples, the result is a decline of only 4 basis points in the standard devation of logs for all people.

6 Conclusions

In this paper, I show how to deal with heterogeneity among people, and heterogeneity in the economic environments in which they might find themselves, in the measurement of inequality, poverty and social welfare. Specifically, I sketch two models. First, I present the microeconomic and econometric theory of equivalence scales for heterogeneity among individuals, focusing on the revealed preference approach to identifying exact equivalence scales. Second, I present the collective household model of Browning, Chiappori and Lewbel (2013), and some related work, focusing on a revealed preference approach to identifying scale economies in household consumption and the allocation of resources among household members. I illustrate these concepts and tools with an application to Canadian data.

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[35] A "basis point" is equal to 0.01.

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