ELSEVIER

Journal of Econometrics 88 (1999) 1–40

JOURNAL OF Econometrics

Semiparametric estimates and tests of base-independent equivalence scales

Krishna Pendakur*

Department of Economics, Simon Fraser University, Burnaby, BC, Canada V5A 1S6

Received 1 October 1994; received in revised form 1 February 1998

Abstract

Previous papers estimate base-independent equivalence scales and test base-independence using strict parametric assumptions on Engel curves and equivalence scale functions. These parametric tests reject the hypothesis of base independence. I construct a semiparametric estimator of a household equivalence scale under the assumption of base independence without putting any further restrictions on the shape of household Engel curves. This estimator uses cross-equation restrictions on a system of estimated nonparametric engel curves to identify equivalence scale parameters. I test the hypothesis of base independence against a fully nonparametric alternative and find that preferences are consistent with the existence of a base-independent equivalence scale for some interhousehold comparisons.

JEL classification: C14; C43; D11; D12; D63

Keywords: Equivalence scales; Semiparametric estimation; Household demand

1. Introduction

Equivalence scales are used to compare the incomes and expenditures of different household types. A *Household Equivalence Scale* is a vector of numbers such that dividing household expenditure by the appropriate scale number gives us an equivalent expenditure for the reference household type. If preferences satisfy *base independence*, then the household equivalence scale does not vary across household income levels. This invariance implies a restriction on

* E-mail: pendakur@sfu.ca.

0304-4076/99/$ – see front matter
PII S0304-4076(98)00020-7

household preferences across household types and, therefore, implies restrictions on the shapes of expenditure share equations across household types. If preferences do not satisfy base independence, then the conventional and convenient use of base-independent equivalence scales is inappropriate.

Base-independent equivalence scales are useful. First, base-independent equivalence scales may allow governments to ensure that redistribution is fair by making sure that each member of the transfer target population ends up with the same level of well-being. An accurate equivalence scale may allow policy-makers to design transfer programmes that do not create incentives for programme participants to change their household type to increase their level of welfare.[1] Second, accurate equivalence scales permit social evaluation, such as the construction of inequality indices, on the basis of household data. Third, the restrictions on preferences imposed by base independence provide a convenient and integrable way of incorporating demographic information into the non-parametric estimation of engel curves.

Many econometric studies have estimated equivalence scales using fully parametric models. A few of these studies have tested whether or not preferences satisfy base independence, and these parametric tests have rejected base independence. In this paper, I estimate equivalence scales semiparametrically with a model that parameterises the equivalence scale function, but allows household expenditure share equations to be estimated non-parametrically. I also test the hypothesis that preferences are consistent with the existence of a base-independent equivalence scale, and find some support for this hypothesis in the data.

2. Equivalence scales and preferences

One can view an equivalence scale as a relationship between the expenditure functions of different types of households. Define the expenditure function, $\mathrm{E}(\boldsymbol{p}, u, \alpha)$, to give the minimum amount of expenditure necessary for a household of characteristics α to get utility level u at prices $\boldsymbol{p}$. The vector of household characteristics, α, may include information such as the numbers and ages of household members. The expenditures of each household type can be defined in relation to a *reference* household type, with characteristics α^{R}. Since an equivalence scale is just a ratio of expenditures at equal utility, I write an equivalence scale function, $D(\boldsymbol{p}, u, \alpha)$, as follows:

$$D(p, u, \alpha) = \frac{\mathrm{E}(p, u, \alpha)}{\mathrm{E}(p, u, \alpha^{\mathrm{R}})}. \tag{2.1}$$

[1] For example, if welfare benefits to single persons were high relative to benefits an accurate equivalence scale might indicate, there would be an economic incentive for couples on welfare to separate in order to better their standard of living.

Eq. (2.1) does not impose any restrictions on the expenditure functions of households; it is simply a ratio. However, Eq. (2.1) is not useful in practice because $D(\boldsymbol{p}, u, \alpha)$ varies with u, the utility level at which the expenditure comparison is made. To make the equivalence scale *base independent*, that is, invariant with respect to the utility level at which the expenditure comparison is made, some structure on preferences across household types is required.[2]

Lewbel (1989b) and Blackorby and Donaldson (1989, 1993) show that if there exists a *base-independent* equivalence scale function $\Delta(\boldsymbol{p}, \alpha)$ which varies with prices $\boldsymbol{p}$ and household characteristics α, then expenditure functions must be related by

$$\mathrm{E}(p, u, \alpha) = \mathrm{E}(p, u, \alpha^{\mathrm{R}})\Delta(p, \alpha) \tag{2.2}$$

Here, $\Delta(\boldsymbol{p}, \alpha)$ must not depend on u. Further, because both reference and nonreference expenditure functions are homogeneous of degree 1 in prices, the equivalence scale function must be homogeneous of degree zero in prices. Blackorby and Donaldson (1989, 1993) express this relationship in terms of the dual indirect utility functions, $V(\boldsymbol{p}, y, \alpha)$, which give the level of utility of a type α household with total expenditures of y at prices $\boldsymbol{p}$:

$$V(p, y, \alpha) = V\left(p, \frac{y}{\Delta(p, \alpha)}, \alpha^{\mathrm{R}}\right). \tag{2.3}$$

Defining $y/\Delta(\boldsymbol{p}, \alpha)$ as *equivalent expenditure*, Eq. (2.3) states that if two households facing the same prices have the same equivalent expenditure, then they are equally well off.

There are several difficulties associated with the economic implementation of equivalence scales presented in Eqs. (2.2) and (2.3). First, these specifications for expenditure and indirect utility functions are not directly based on *individual* utility; rather they are functions representing household level maximization and household utility, presumably based on some kind of aggregation of individual utilities within households.[3] Second, Eqs. (2.2) and (2.3) require an ordinal

[2] The structure required for base independence was explored independently by Lewbel (1989b) and Blackorby and Donaldson (1989, 1993), who referred to it as *Equivalence Scale Exactness*. This structure requires the expenditure function to be multiplicatively decomposable into two functions, one which depends only on prices and utility, and one which depends only on prices and household characteristics.

[3] Discussion of household equivalence scales necessitates some concept of household utility. For this research, I assume the existence of a household utility function over goods which is maximized subject to household expenditure constraints, prices and household characteristics. The aggregation of individual household members' utilities into a household utility function is potentially very complicated, and will not be discussed here (see Phipps (1996) for some discussion of these issues). However, if the household aggregation function is maximin, then equivalent expenditure will equate well-being across individuals as well as across households; I use this aggregation function to interpret estimated equivalence scales.

interhousehold comparison of utility.[4] Third, Eqs. (2.2) and (2.3) treat household characteristics, α, as exogenous instead of explicitly modelling how households choose their characteristics, including household size and number of children.[5] Fourth, even if the presence of children can be taken as exogenous, households may be able to adjust their intertemporal consumption paths in response to anticipated child-bearing.[6] Finally, many researchers (notably, Pollack and Wales, 1995) have suggested that it is unreasonable to expect equivalence scales to be valid over wide income/expenditure ranges. This last point requires that the first four difficulties have been solved (or overlooked), and that the main problem lies in whether or not equivalence scales defined by Eq. (2.1) are actually base independent as in Eqs. (2.2) and (2.3). It is this latter question to which the current paper is addressed. Are household preferences (as manifested in share equations) consistent with the existence of a base independent equivalence scale, or must equivalence scales vary with total expenditures?

Lewbel (1989b) and Blackorby and Donaldson (1989, 1993) do not derive the specific observational consequences of Eqs. (2.2) and (2.3) in the context of an unspecified reference demand system. Using Roy's Identity, Eq. (2.3) and the chain rule, I derive demand equations, $x_i(\boldsymbol{p}, y, \alpha)$, in terms of the demand equations of the reference household, $x_i(\boldsymbol{p}, y, \alpha^R)$, as follows:

$$\mathrm{x}_i(p, y, \alpha) = \Delta(p, \alpha) x_i\left(p, \frac{y}{\Delta(p, \alpha)}, \alpha^{\mathrm{R}}\right) + \frac{y}{\Delta(p, \alpha)} \frac{\partial \Delta(p, \alpha)}{\partial p_i}. \tag{2.4}$$

[4] Note that in Eq. (2.3), two indirect utility functions are related by an equality, which is not consistent with utility functions satisfying Ordinal Non-Comparability. Blackorby and Donaldson (1989, 1993) show that smaller information classes are needed, and define Income Ratio Comparability (IRC) as the information structure necessary to support Eq. (2.3). IRC requires that equality of utility be observable (as in Ordinal Full Comparability) at a single level of well being, and sustainable over income scalings. Once a single point of utility equality is found, the preference restrictions in Eq. (2.3) ensure that it will be maintained over all scalings of total expenditure. Blackorby and Donaldson (1993) also show that IRC is equivalent to base independence (which they refer to as Equivalence Scale Exactness).

[5] Many researchers, including Pollak and Wales (1979), Pollak (1991) and Blundell and Lewbel (1991), have suggested that exogenity of household characteristics is too demanding, and have thus suggested that demand data can at best reveal the price response of equivalence scales, but not equivalence scale sizes themselves (Pollak and Wales refer to 'conditional' versus 'unconditional' scales).

[6] Pashardes (1991) suggests that if families plan to have children, they may reduce their consumption in pre- and post- child-rearing periods in order to consume more when their child-rearing needs are greatest. He shows that equivalence scales estimated only from contemporaneous expenditures substantially under-estimate the costs of children due to this form of intertemporal utility smoothing.

Multiplying Eq. (2.4) by p_i/y gives Marshallian expenditure share equations, $w_i(\boldsymbol{p}, y, \alpha)$: *defining* $\eta_i(p,\alpha)$ *as the clasticity of* $\Delta(p,\alpha)$ *with respect to* p_i,

$$w_i(p, y, \alpha) = w_i\left(p, \frac{y}{\Delta(p, \alpha)}, \alpha^{\mathrm{R}}\right) + \eta_i(p, \alpha). \tag{2.5}$$

Under base independence, the Marshallian shares of a nonreference household are equal to the Marshallian shares of the reference household at the same equivalent expenditure plus the elasticity of the equivalence scale with respect to price. Eq. (2.5) shows that under base independence the shapes of Engel curves are linked across household types, but are not restricted to particular shapes. Fig. 1 shows this relationship. In $\{w_i(\boldsymbol{p}, y, \alpha), \log(y)\}$ space, share functions must be related by a horizontal and vertical shift. That is, household expenditure share functions for particular goods must have the same shape across household types. I call this restriction *shape invariance*, and note that base independence is sufficient for shape invariance, but not vice versa.

If we assume that the expenditure function for the reference household type, $\mathrm{E}(p, u, \alpha^{\mathrm{R}})$, satisfies the Slutsky conditions and that the equivalence scale function is symmetric and concave in prices then the Slutsky conditions must be satisfied for all household types.[7] Since Eqs. (2.4) and (2.5) are derived from the dual indirect utility function, this demand system is integrable for all household types. Thus base-independence provides a method for incorporating demographic information into nonparametric demand system estimation that satisfies integrability and leaves the shape of household engel curves unspecified.

Since the elasticity functions, $\eta_i(\boldsymbol{p}, \alpha)$, do not depend on total expenditure, it must be the case that for all households with the same equivalent expenditure,

[7] The expenditure function is increasing for all household types if and only if the marshallian budget shares are everywhere positive (i.e., if equivalence scale elasticities are not too large). The Hessian of the expenditure function given by Eq. (2.2) is:

$$\Delta^2\mathrm{E}(p,u,\alpha) = \Delta^2\mathrm{E}(p,u,\alpha^{\mathrm{R}})\Delta(p,\alpha) + \Delta\mathrm{E}(p,u,\alpha^{\mathrm{R}})\Delta\Delta(p,\alpha)' + \Delta\Delta(p,\alpha)\Delta\mathrm{E}(p,u,\alpha^{\mathrm{R}})' + \Delta^2\Delta(p,\alpha)\mathrm{E}(p,u,\alpha^{\mathrm{R}})$$

The Hessians of $E(\boldsymbol{p},u,\alpha^{\mathrm{R}})$ and $\Delta(\mathrm{p},\alpha)$ are negative semidefinite (NSD) and symmetric by assumption. Since $\Delta(p,\alpha)$ is homogeneous of degree zero in p, its gradient sums to zero, so that the sum of the middle two terms is symmetric and has a zero determinant. Thus, the Hessian of $E(\boldsymbol{p},u,\alpha)$ is symmetric and NSD for all household types.

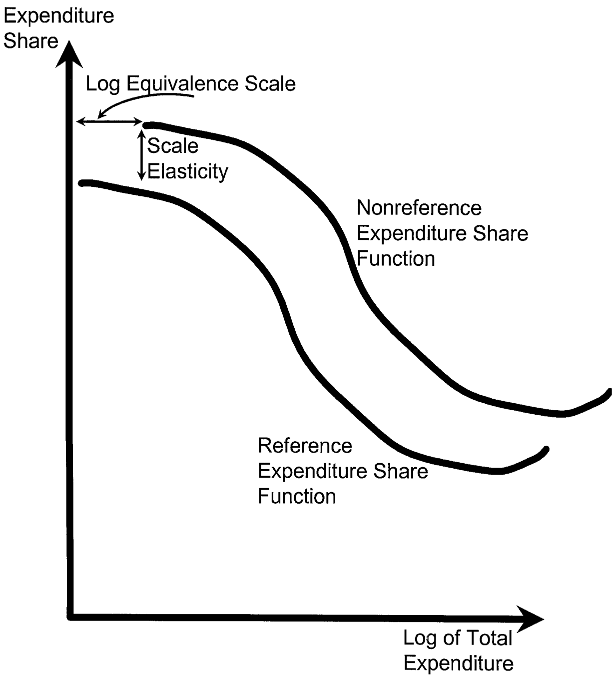

Fig. 1. Base independence and Preferences.

Marshallian demand shares of different household types respond identically to proportional changes in expenditure. Thus, shape invariance is a testable restriction on preferences.

Blackorby and Donaldson (1989, 1993) note that an equivalence scale is only uniquely identifiable from expenditure data if Marshallian share functions are nonlinear functions of the log of total expenditure. If share equations were linear, then a pair of share functions with slope b that are consistent with $\{\log \Delta^*, \eta^*\}$ would also be consistent with $\{\log \Delta^* + \lambda/b, \eta^* + \lambda\}$ for any choice of λ. With nonlinear share equations, however, unique equivalence scales are estimable from expenditure data under the assumption of base independence.

To estimate the (log of) equivalence scale sizes, one needs to estimate the horizontal shift in Fig. 1, and to estimate equivalence scale elasticities, one needs to estimate vertical shifts in Fig. 1. Hitherto, all estimates under (and tests of) base independence have specified parametric representations for the reference share functions, $w(\boldsymbol{p}, y, \alpha^{R})$, the equivalence scale function $\Delta(\boldsymbol{p}, \alpha)$, and (implicitly) its associated elasticities, $\eta(\boldsymbol{p}, \alpha)$.[8] Many attempts have been made to measure (equivalence scales in this fashion. Jorgenson and Slesnick (1987) and Nicol (1991) use an exactly aggregated Translog model for the reference share functions, $w(\boldsymbol{p}, y, \alpha^{R})$, and demographically shifted exactly aggregated Translogs for the nonreference share functions. They impose Barten exactness and generate unique equivalence scales from these restrictions that are related to prices in a complicated way.[9] Phipps (1991) and Pendakur (1994) use a Translog specification for reference preferences and a simple specification of the equivalence scale function (Phipps uses a Cobb–Douglas and Pendakur a Translog equivalence scale). Browning (1989), Nelson (1991), Blundell and Lewbel (1991) and Dickens et al. (1993) use the Almost Ideal (AI) demand system as a model of reference preferences and a Cobb–Douglas representation of the equivalence scale function.[10] Dickens et al. (1993) and Pashardes (1995) use extended (log-quadratic) AI demand systems for reference preferences and translog equivalence scale functions.

All of these models share the approach of specifying functional forms for *both* expenditure share functions *and* for the equivalence scale functions. Browning (1989), Blundell and Lewbel (1991)[11] , Dickens et al. (1993) and Pashardes (1995)

[8] Gozalo (1997) estimates a model with nonparametric engel curves and base-dependent equivalence scales based on Engel's method. Engel equivalence scales are based on the assumption that two households which have the same expenditure share for food are equally well off. Gozalo found that for most household types, Engel equivalence scales are not base independent (i.e., they vary substantially with total expenditures). Gozalo's approach differs from the current paper in two important ways: (1) Engel's method uses only food shares whereas I look at three different 2-equation systems and one 4-equation system; and (2) Gozalo estimates price-independent scales whereas I estimate price-dependent scales. In terms of Fig. 1, Gozalo examines whether shape invariance holds in the food share equation with $\eta = 0$.

[9] Jorgenson and Slesnick (1987) impose Exact Aggregation on preferences, which forces all expenditure share functions to be loglinear in total expenditure. As noted above (and shown in Blackorby and Donaldson, 1989, 1993), this means that unique identification of the equivalence scale function is coming from the additional restrictions in the model. In this case, identification of $\Delta(\boldsymbol{p}, \alpha)$ comes from making some of the parameters of $\Delta(\boldsymbol{p}, \alpha)$ also appear in $w(p, y, \alpha^{R})$. In particular, Translog parameters that measure the price responses of the reference share equations also appear in the equivalence scale for nonreference households.

[10] All three papers have difficulty identifying the parameters of the equivalence scale function. Blackorby and Donaldson (1989, 1993) show that this is due to the fact that the Almost Ideal demand system forces all expenditure share functions to be loglinear in total expenditures. Banks et al. (1994) suggests the use of an extended AI demand system wherein share functions are quadratic in the log of total expenditures. Banks et al. (1994) and Dickens et al. (1993) use extended AI systems and find that it fits the data much better than the loglinear AI model.

[11] Blundell and Lewbel (1991) reject base independence, but find that the departure from base independence is quite small, which may be consistent with results presented in this paper.

go on to test the *joint* hypotheses that these functional forms are viable, and reject the hypothesis of base independence in this context. The innovation in this paper is that the model specifies a functional form *only* for the equivalence scale function, and leaves the reference expenditure share functions unrestricted. I find that with unrestricted reference expenditure share functions, semiparametric estimates provide moderate support for the hypothesis of base independence, and generate somewhat different estimates of equivalence scale sizes than the parametric models.

3. Estimation of an equivalence scale

3.1. Nonparametric regression

The standard approach in the measurement of expenditure share equations has hitherto been to assume a particular form for the functions, and to estimate the parameters of that function by minimising some criterion function (either ML or GMM).[12] The idea of nonparametric regression is to let the data determine the shape of the function to be estimated. Given an underlying data generating function, $\xi_i = m(\psi_i) + \varepsilon_i(\varepsilon \sim \text{iid}(0))$, an estimated nonparametric regression curve, $\hat{m}(\psi)$, may be defined over the data points, $\{\psi_i, \xi_i\}$, as follows.[13]

$$\hat{f}(\psi) = \frac{1}{Nh}\sum_{i=1}^{N} K\left(\frac{\psi - \psi_i}{h}\right),$$

$$\hat{r}(\psi) = \frac{1}{Nh}\sum_{i=1}^{N} K\left(\frac{\psi - \psi_i}{h}\right)\xi_i.$$

[12] The Working-Leser (Working, 1943) model posits that household expenditure share equations are loglinear in total expenditure. The Exactly Aggregated Translog (Jorgenson and Slesnick, 1985) and Almost Ideal (Deaton and Muelbauer, 1980) models also have loglinear shares at any price vector. The unrestricted Translog (Jorgenson et al., 1960) and Integrable Quadratic Almost Ideal (Banks et al., 1994) model both feature nonlinear share equations. Parametric representations, however, rarely allow complications above the second moment (whatever is used as the independent variable).

[13] This paper uses only kernel-based nonparametric estimation. Although other types of non-parametric regression, such as spline smoothing and running median estimation are available, semiparametric estimators are most well-developed in the context of kernel estimation.

where $K(\cdot)$ is defined as a weakly positive Kernel function, and h as nonzero bandwidth. Define

$$\hat{\mathrm{m}}(\psi) = \frac{\hat{r}(\psi)}{\hat{f}(\psi)} = \frac{\sum_{i=1}^{N} K\left(\frac{\psi - \psi_i}{h}\right)\xi_i}{\sum_{i=1}^{N} K\left(\frac{\psi - \psi_i}{h}\right)}. \tag{3.1}$$

In Eq. (3.1), the choice of the kernel function, $K(\cdot)$, has a minimal affect on the estimate of $\hat{m}(x)$ (see Härdle, 1993 Chapter 4.5), so I choose the gaussian kernel for $\mathrm{K}(\cdot)$.[14,15] I use *cross-validation*[16] to find optimal bandwidth for each estimated nonparametric regression. Finally, to avoid boundary bias, I trim the top and bottom 2.5% of estimated nonparametric regression functions used in the semiparametric estimation.

3.2. The data

I use data from the 1990 Canadian Family Expenditure Survey (Statistics Canada, 1992) to estimate expenditure share equations for food, recreation and clothing for four household types. The four household types used are: (i) single adults; (ii) adult couples; (iii) adult couples with one child less then ten years old; and (iv) adult couples with two children less than ten years old. Because the age of children has been shown to be an important determinant of expenditure shares (see Browning, 1992), I focus on households with young children (under ten years old). Further I exclude all households with members aged 65 or more, because the expenditure patterns of seniors also seem to differ from those of the non elderly. Only households where all members are full-year members are used. I use only households residing in the fourteen largest Census Metropolitan

[14] A kernel function is a distribution that is used to smooth out irregularity in the data. Effectively, kernel smoothing is the convolution of a regular distribution, the kernel, with an irregular distribution, that of the actual data. The gaussian kernel is defined as: $K(u) = \exp(-u^2/2)/(2\pi)^{1/2}$.

[15] Please note that I suppress the dependence of the bandwidth, h, on the sample size N.

[16] Choosing bandwidth by cross-validation involves computing a new 'leave-out' regression curve, $\bar{m}(\Psi)$, such that each element of $\bar{m}(\Psi_i)$ uses all the data except the single datapoint (Ψ_i, ξ_i) exactly at that point. Then, the bandwidth that minimises the integrated squared error between $\bar{m}(\Psi)$ and the data is the optimal bandwidth. Cross-validation thus creates an estimate of how good the nonparametric regression curve is at predicting out of sample data, and chooses the bandwidth that is best in this sense. I note that although cross-validation yields a bandwidth that is optimal in a pointwise sense, it is not necessarily optimal for semiparametric applications (such as estimating semiparametric shifts or testing shape invariance).

Areas of Canada to minimise the effects of home production. All estimates are made without price variability,[17] and the dependence of equivalence scales and elasticities on prices will henceforward be dropped. The data come with weights that reflect the sampling frame, and the weights are incorporated into the estimation of all estimated nonparametric regression curves and semi-parametric estimators.[18]

To begin, I estimate three different two-share systems in which there is only a single independent equation for each of four household types. I examine the

Table 1
Descriptive statistics[a]

	Household type			
	Singles	Couples 0	Couples 1	Couples 2
Number of observations	638	753	350	310
Range of log consumption	7.84 9.92	8.48 10.31	8.78 10.49	8.94 10.51
Average log consumption	8.91	9.45	9.69	9.72
Range of consumption ($)	2540 20,333	4817 30,031	6503 35,954	7631 36,680
Average consumption ($)	8426	14,100	17,472	18,108
Average food share	0.28	0.31	0.32	0.35
Average clothing share	0.17	0.18	0.17	0.16
Average Recreation Share	0.35	0.32	0.27	0.24

[a] Statistics are for the middle 95% of the weighted data. Consumption refers to total expenditure on nondurable consumption.

[17] Because the data are from a single time and place, there is no price variation. Thus, the form of the equivalence scale cannot be uniquely determined with respect to its price arguments. However, because the elasticity of the equivalence scale will be estimated directly, the elasticity will be determined at a single point. The estimates will therefore be consistent with a Cobb–Douglas equivalence scale as follows:

$$\ln(\Delta(\boldsymbol{p}, \alpha)) = \ln(S(\alpha)) + \sum \eta_i \ln(p_i)$$

where $S(\alpha)$ is a scale function and prices are normalized so that $\sum \eta_i \ln(p_i) = 0$ at the point of estimation. Thus, for all estimation to follow, $\Delta(\alpha)$ can be taken as $S(\alpha)$,

[18] Statistics Canada oversamples lower population areas, and the weights downweight these areas. The nonparametric regression estimator with data $\{\Psi_i, \xi_i, \Omega_i\}$ where Ω_i are weights is:

$$\hat{m}(\Psi) = \frac{\Sigma_i K((\Psi - \Psi_i)/h)\Omega_i \xi_i}{\Sigma_i K((\Psi - \Psi_i)/h)\Omega i}$$

shares of nondurable consumption spent on (i) food purchased in stores, (ii) clothing, and (iii) recreation, including restaurant meals. Nondurable consumption is defined to include all food (including restaurant purchases), clothing, recreation (not including recreational vehicles), household operation and personal care. In Section 4, I estimate all three independent equations as a system of equations with cross equation restrictions.

Table 1 offers some summary statistics about the data for the four household types used in this paper. Adult couples with no children are denoted 'Couples 0', couples with one child, 'Couples 1', and couples with two children, 'Couples 2'. Descriptive statistics are for the middle 95% of the data. Note from Table 1 that these three types of expenditure account for about three quarters of nondurable consumption for all four household types.

Table 2 shows results from estimated OLS log-quadratic regressions for the three separate share equations for four household types. The log-quadratic regressions are a good starting point for two reasons. First, the shape-invariance restrictions given in Eq. (2.5) have a simple interpretation in the case where true share equations are log-quadratic: for any share equation, the coefficients on the square of log expenditure must be the same for all household types.[19] Second, identification under base independence requires that the coefficient on the square of log expenditure must be nonzero. If the second order term is zero, then the quadratic model collapses to a model with linear Engel curves, and demand data cannot uniquely identify the equivalence scale under base independence.

Examination of Table 2 suggests two things. First, for all three types of expenditure, while the point estimates of second-order terms look fairly different across household types, they are quite imprecisely estimated and so may not be significantly different across household type. Thus, these simple OLS log-quadratic regressions may not reject the hypothesis of base independence (see Table 4). Second, many of the second-order terms are not significantly different from zero. In particular, the clothing equation is nearly log-linear for all

[19] If two log-quadratic share functions have the same second-order term, then they can be made to coincide with horizontal and vertical translations in $\{w, \ln y\}$ space. In particular, consider two household types, α^{R} (the reference household) and α^{N} (a nonreference household), with share equations for some good (suppressing the dependence of the share equations on $\mathbf{p}$) given by

$$\mathrm{w}(y, \alpha^{R}) = a^{R} + b^{R}\ln(y) + c\ln(y)^{2},$$

$$w(y, \alpha^{N}) = a^{N} + b^{N}\ln(y) + c\ln(y)^{2}.$$

These share equations are consistent with base independence, with equivalence scale and elasticity given by

$$\ln(\Delta(\alpha^{N})) = (b^{R} - b^{N})/2c, \eta(\alpha^{N}) = a^{N} - a^{R} + b^{R}\ln(\Delta(\alpha^{N})) + c\ln(\Delta(\alpha^{N}))\cdot 2$$

Table 2
Estimates under log-quadratic model, by household type[a]

Expenditure Share	Household type			
	Singles	Couples 0	Couples 1	Couples 2
Food purchased from stores				
Constant	8.61	4.22	5.69	7.02
Std Error	*1.59*	*2.11*	*2.71*	*3.47*
Log expenditure	− 1.69	− 0.63	− 0.96	− 1.17
Std error	*0.36*	*0.45*	*0.55*	*0.72*
Log expenditure squared	0.084	0.023	0.042	0.050
Std error	*0.021*	*0.024*	*0.028*	*0.037*
R-squared	0.331	0.361	0.240	0.396
Std Error of regression	0.114	0.108	0.095	0.093
Restaurant food and recreation				
Constant	− 5.67	− 6.63	− 4.66	− 7.50
Std Error	*2.16*	*2.13*	*2.66*	*3.29*
Log expenditure	1.20	1.31	0.94	1.51
Std Error	*0.48*	*0.45*	*0.55*	*0.67*
Log expenditure squared	− 0.059	− 0.060	− 0.045	− 0.073
Std Error	*0.027*	*0.024*	*0.029*	*0.034*
R-squared	0.182	0.254	0.111	0.131
Std Error of regression	0.151	0.117	0.087	0.090
Clothing				
Constant	− 1.69	1.87	2.46	− 1.68
Std Error	*1.31*	*1.53*	*2.04*	*2.60*
Log expenditure	0.36	− 0.43	− 0.54	0.31
Std Error	*0.29*	*0.32*	*0.42*	*0.53*
Log expenditure squared	− 0.017	0.026	0.032	− 0.012
Std Error	*0.016*	*0.016*	*0.022*	*0.027*
R-squared	0.089	0.105	0.078	0.117
Std Error of regression	0.091	0.082	0.069	0.131

[a] Each share equation uses total expenditures on nondurable consumption as the denominator. Nondurable consumption is: Food purchased from stores, restaurant food, recreation, clothing, household operation and personal care. No cross-equation restrictions are imposed.

household types, suggesting that identification of equivalence scale sizes and elasticities from the clothing equations will be questionable.[20]

Fig. 2 shows the food share data, log-quadratic regressions and nonparametric regressions (cross-validated bandwidths and other details are available in

[20] Blundell et al. (1993) and Banks et al. (1997) find clothing shares to be quite nonlinear in British expenditure data. There are at least two possible sources for this difference. First, I use a measure of nondurable consumption which excludes alcohol, tobacco, shelter, and transportation while Blundell et al. (1993) use a much broader measure of total expenditure. Second, I have far fewer observations than Blundell et al. and Banks et al., suggesting that noise in the data may be a factor.

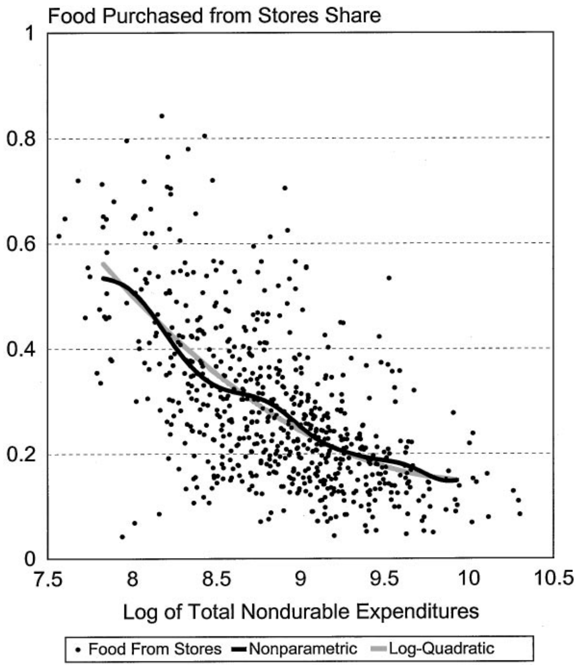

Fig. 2. Food expenditure shares, single adults.

Appendix B) for single adults. The regression curves have a great deal of variance around them in both the parametric and nonparametric cases. Fig. 3 shows log-quadratic and nonparametric estimates of food share equations for single adults, with the 90% confidence band (calculated pointwise at each decile of the data by monte carlo simulation) shown as a dotted line for the nonparametric regression. At first glance, it seems that the log-quadratic and nonparametric estimated regression curves are only trivially different from each other. However, in estimates and tests of base independence, the critical question is whether or not the log-quadratic and nonparametric estimates differ across households. I find that in this regard, the parametric and nonparametric approaches yield quite different results.

Fig. 4 shows estimated log-quadratic regression curves for recreation shares for childless couples and couples with two children. From Table 2, one can see that the coefficients on the square of log expenditures are very close across these two household types. Not surprisingly, the log-quadratic estimates appear to have the same degree of curvature (which fully defines their shape), but it seems like they are mapping different parts of the log-quadratic curve. In terms of the model of interhousehold preferences given in Eqs. (2.2) and (2.3), that means that

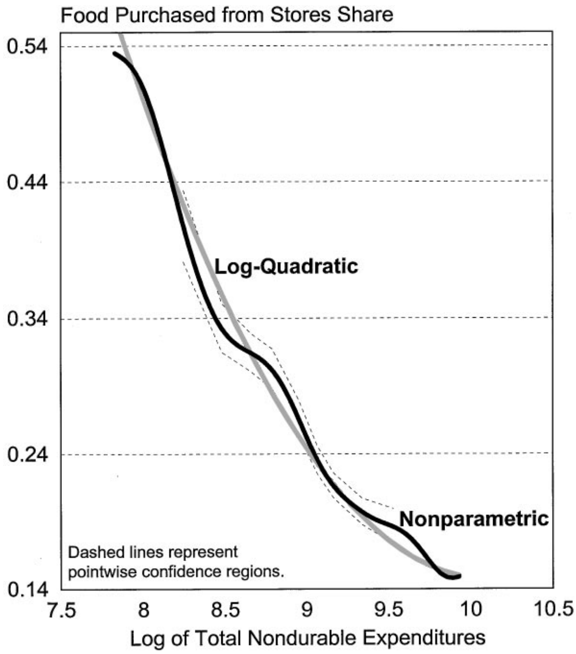

Fig. 3. Log-quadratic and nonparametric estimates, food shares, single adults.

the childless couples are spread over a different part of the equivalent income distribution than the couples with two children.

Fig. 5 shows estimated nonparametric regression curves for recreation shares for childless couples and couples with two children. These regression curves do not seem to have the same shapes in the areas of estimation. However, it is not obvious whether or not the differences between the shapes of nonparametric regression curves constitute big differences and, given the large amount of variance in the data, whether they constitute statistically significant differences. The next subsection will formalize a measure of difference between nonparametric regression curves, and construct a semiparametric estimator of equivalence scale sizes and elasticities.

While maximum likelihood techniques allow the easy testing of the log-quadratic curves for shape invariance, these techniques use the global properties of the distributions to find the parametric fits for equivalence scales and elasticities. This means that standard maximum likelihood estimation might identify equivalence scale sizes from household data that do not have overlapping equivalent incomes, which may be an unsatisfactory way of estimating equivalence scales. Because nonparametric regression curves are purely local,

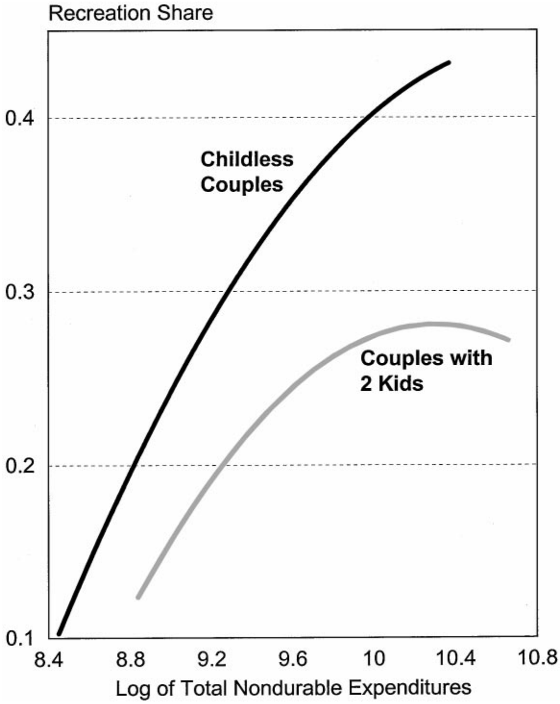

Fig. 4. Log-quadratic estimates, Recreation shares. childless and couples with 2 kids.

semiparametric comparison of nonparametric curves must be based on local properties of the distributions, and thus equivalence scales can only be identified from data with overlapping levels of equivalent income.

3.3. A semiparametric equivalence scale estimator

The basic idea of the semiparametric approach to estimating an equivalence scale is to find the pair $\{\log \Delta(\alpha), \eta(\alpha)\}$[21] that is able to most nearly fit the estimated nonparametric expenditure share equations of two household types. The search algorithm will be a simple gridsearch across a fairly wide span of $\{\log \Delta(\alpha), \eta(\alpha)\}$, which seeks the minimum value of a Loss Function that measures the distance between the reference share function and the shifted nonreference share function. Härdle and Marron (1990) suggest a simple Loss Function equal to the integrated squared distance between the reference

[21] The dependence of $\Delta(\boldsymbol{p}, \alpha)$ and $\eta(p, \alpha)$ on prices is dropped. See footnote 15.

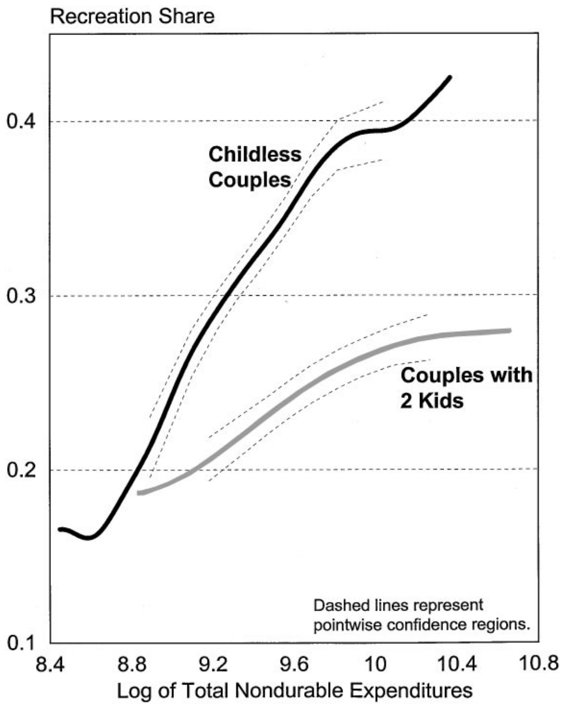

Fig. 5. Nonparametric estimates, recreation shares, childless couples and couples with 2 kids.

function and the transformed function for the fixed design case.[22] However, because this approach does not use any information about the relative densities of the data in the two estimated regression equations, it is not appropriate for the random design case. Pinkse and Robinson (1995) suggest a slightly more complex Loss Function for the random design case. Under base independence, reference and nonreference true expenditure share regression functions must be related by Eq. (2.5). Denoting the log of total expenditure as ψ, the log of the equivalence scale function as $\delta(\alpha)$, and the true expenditure share functions as

[22] The term 'fixed design' refers to the process generating the independent variable. If the independent variable is nonstochastic and appears only at fixed intervals (as in much experimental research), then the design is fixed. If the independent variable is randomly distributed, then it is called 'random design'. Budget data are suited to random design models because the independent variable, total expenditure, is a random variable.

$m(\psi, \alpha)$, base independence implies that for the reference household type, α^{R} , and a nonreference type, α^{N}, the following must hold:

$$m(\psi, \alpha^{N}) = m(\psi - \delta(\alpha^{N}), \alpha^{R}) + \eta(\alpha^{N}). \tag{3.2}$$

Noting that $m(\psi, \alpha^{N}) = r(\psi, \alpha^{N})/f(\psi, \alpha^{N})$ and $m(\psi, \alpha^{R}) = r(\psi, \alpha^{R})/f(\psi, \alpha^{R})$ (see Eq. (3.1)), then multiplication of Eq. (3.2) by $f(\psi, \alpha^{N})f(\psi - \delta(\alpha^{N}), \alpha^{R})$ implies the following relationship:

$$\begin{aligned} f(\psi - \delta(\alpha^{N}), \alpha^{R})r(\psi,\alpha^{N}) = {} & f(\psi, \alpha^{N})r(\psi - \delta(\alpha^{N}), \alpha^{R}) \\ & + \eta(\alpha^{N})f(\psi, \alpha^{N})f(\psi - \delta(\alpha^{N}), \alpha^{R}). \end{aligned} \tag{3.3}$$

Pinkse and Robinson (1995) suggest the use of Eq. (3.3) to find the best semiparametric fit for the parameters, $\delta(a^{N})$ and $\eta(\alpha^{N})$. In particular, they show that the solution which minimises the integrated squared difference between the left- and right-hand sides of Eq. (3.3) using estimated nonparametric regression curves is a $\sqrt{N}$-consistent estimator of the true parameters in the random design semiparametric model.[23] Thus, I define and minimise a loss function, $L(\delta(\alpha), \eta(\alpha))$, over estimated regression curves for household types, α^{N} and α^{R}, as follows:

$$\begin{aligned} L(\delta(\alpha^{N}), \eta(\alpha^{N})) = \int_{\psi_{hi}}^{\psi_{lo}} & [\eta(\alpha^{N})\hat{f}(\psi,\alpha^{N})\hat{f}(\psi - \delta(\alpha^{N}),\alpha^{R}) + \hat{f}(\psi,\alpha^{N}) \\ & \times \hat{r}(\psi - \delta(\alpha^{N}), \alpha^{R}) - \hat{f}(\psi - \delta(\alpha^{N}),\alpha^{R})\hat{r}(\psi,\alpha^{N})]^{2}\partial\psi \end{aligned} \tag{3.4}$$

3.4. Single equation estimates

Table 3a and Table 3b show estimates for equivalence scale sizes for each pairwise comparison between our four household types. I report results for three share equations (using nondurable consumption as the denominator): (i) food

[23] There are several regularity assumptions required for Pinkse and Robinson's result. $\sqrt{N}$-convergence requires that the expenditure shares are mutually independent across household type, that the independent variables (the log of total expenditures) are iid draws, that the translations δ and η are in a bounded and open set, that the engel curves are sufficiently differentiable, that both sample sizes increase at the same rate and that both nonparametric estimators employ the same kernel. These assumptions seem reasonable for the present application. In particular, the expenditure shares are not assumed to be homoskedastic across the log of total expenditure, nor are they assumed to have the same densities across household type. However, Pinske and Robinson also require that bandwidths go to zero at faster than pointwise optimal rates, which may not be as reasonable for the present application. Pinkse and Robinson also show, without the use of higher-order kernels, that if the two nonparametric estimators have different bandwidths (and those bandwidths converge at the same rate), then the estimated parameters converge at a rate $N^{-2/5}$.

purchased from stores; (ii) recreation and restaurant food; and (iii) clothing. Table 3a gives the results imposing log-quadratic Engel curves, and Table 3b gives results where the Engel curves are estimated nonparametrically, and the equivalence scale sizes and elasticities are estimated semiparametrically. The log-quadratic model is estimated using a maximum likelihood model with log-quadratic share equations restricted so that the second order terms are equal.[24] The results in Table 3b are based on the gridsearch minimisation of $L(\delta(\alpha), \eta(\alpha))$,[25,26] and standard errors for the semiparametric model are estimated using monte carlo simulations.[27] Equivalence scales are expressed treating the top household type as the reference (see Eq. (2.2)). Details are in Appendix B.

The results in Table 3a suggest that under the log-quadratic model estimated equivalence scales are not very stable across the different expenditure share equations, ranging for the comparison of childless couples versus couples with

[24] For the reference household, α^{R}, and the nonreference household type, α^{N}, the econometric model for estimating expenditure share equations, $w(\psi, \alpha)$, is given by:

$$\mathrm{w}(\psi, \alpha^{R}) = a^{R} + b^{R}\psi + c^{R}\psi^{2} + \varepsilon^{R}, \quad \varepsilon^{R} \sim N(0, \sigma_{R}^{2})$$

$$w(\psi, \alpha^{N}) = a^{N} + b^{N}\psi + c^{N} + \psi^{2} + \varepsilon^{N}, \quad \varepsilon^{N} \sim N(0, \sigma_{N}^{2})$$

The shape invariance restriction is $c^{N} = c^{R}$. Errors are heteroskedastic across household type.

[25] In practice the grid search can be concentrated over $\eta(\alpha^{N})$, because for every $\bar{\delta}(\alpha^{N})$ value in the search space for $\delta(\alpha^{N})$, there is a unique value of $\bar{\eta}(\alpha^{N})$ which minimises the loss function, as follows:

$$\bar{\eta}(\alpha^{N},\bar{\delta}) = \int_{\psi_{lo}}^{\psi_{hi}} \hat{f}(\psi - \bar{\delta},\alpha^{R})\hat{r}(\psi, \alpha^{N}) - \hat{f}(\psi,\alpha^{N})\hat{r}(\psi - \bar{\delta}, \alpha^{R})\partial\psi \Big/ \int_{\psi_{lo}}^{\psi_{hi}} \hat{f}(\psi - \bar{\delta}, \alpha^{R})\hat{f}(\psi, \alpha^{N})\partial\psi.$$

[26] Härdle and Marron (1990) found that in the fixed design case, the choices of ψ_{hi} and ψ_{lo}, the limits of integration, were very important. However, in the random design case, because information is used about the relative densities of data, changing the limits of integration did not affect estimates very much at all. For all semiparametric estimation, I use nonparametric regression curves from the middle 95% of each data set.

[27] I estimate small-sample distributions for estimated δ, η and $L(\delta, \eta)$ via monte carlo simulations. Under shape invariance, the underlying true regression curves are related by (constant) horizontal and vertical shifts, δ and η. I construct bootrap samples maintaining this restriction, and simulate the distribution of the semiparametric estimates of δ, η and $L(\delta, \eta)$. Although Pinkse and Robinson (1995) show that these random variables are asymptotically normal, the small sample simulations suggest that while δ and η have roughly symmetric distributions, the distribution of $L(\delta, \eta)$ is right-hand skewed. Thus, standard errors for $L(\delta, \eta)$ should be interpreted with caution: standard errors have no obvious meaning when the distribution is non-normal. Details, including percentiles of the bootstrap distribution, are in Appendix B.

Table 3a
Estimates under base-independence, single equations, log-quadratic model[a]

Log-quadratic Expenditure share equation	Singles versus Couples 0	Singles versus Couples 1	Singles versus Couples 2	Couples 0 versus Couples 1	Couples 0 versus Couples 2	Couples 1 versus Couples 2
Food Purchased From Stores						
2nd-order term	0.029	0.025	0.026	0.025	0.042	0.017
Std Error	*0.008*	*0.010*	*0.010*	*0.018*	*0.018*	*0.026*
Log equivalence scale	**0.96**	**0.17**	**1.24**	**−0.83**	**0.28**	**1.55**
Std Error	*0.23*	*0.39*	*0.33*	*0.86*	*0.21*	*2.27*
Equivalence Scale	2.62	1.19	3.47	0.44	1.33	4.70
Restaurant food and recreation						
2nd-order term	− 0.025	− 0.019	− 0.021	− 0.049	− 0.075	− 0.047
Std Error	*0.010*	*0.011*	*0.012*	*0.019*	*0.019*	*0.024*
Log equivalence scale	**0.91**	**−0.44**	**−0.76**	**−0.78**	**−0.29**	**0.16**
Std Error	0.32	0.84	1.04	0.43	0.19	0.21
Equivalence scale	2.50	0.64	0.47	0.46	0.75	1.18
Clothing						
2nd-order term	0.000	0.006	0.002	0.030	0.018	0.024
Std Error	*0.006*	*0.007*	*0.008*	*0.014*	*0.014*	*0.019*
Log equivalence scale	**8.17**	**1.90**	**−1.89**	**0.21**	**0.13**	**−0.03**
Std Error	*119.69*	*1.60*	*12.93*	*0.22*	*0.40*	*0.33*
Equivalence scale	3541.77	6.69	0.15	1.23	1.13	0.97

[a] Each share equation uses total expenditures on nondurable consumption as the denominator. Estimates and standard errors are computed via Maximum Likelihood. Nondurable consumption is defined as food purchased from stores, restaurant food, recreation, clothing, household operation and personal care.

one child from 0.44 in the food equation to 1.23 in the clothing equation. Further, the equivalence scale estimates do not satisfy even basic a priori expectations, such as increasingness with respect to household size. For example, according to the food and recreation equations, couples with one child require only half as much expenditure as couples with no children to get the same level of utility. Finally, although the coefficients on ψ^2 are significantly different from zero at the 10% level for most comparisons in the food and recreation equations, the coefficients on ψ^2 are insignificantly different from zero for all but one comparison in the clothing equation. That is, the clothing equations under shape invariance are nearly log-linear and thus cannot identify

Table 3b
Estimates under base independence, single equations, semiparametric model[a]

Semiparametric						
Expenditure share equation	Singles versus Couples 0	Singles versus Couples 1	Singles versus Couples 2	Couples 0 versus Couples 1	Couples 0 versus Couples 2	Couples 1 versus Couples 2
Food Purchased From Stores						
Log equivalence scale	**0.68**	**0.87**	**1.01**	**0.61**	**0.86**	**0.21**
Std Error	*0.05*	*0.07*	*0.08*	*0.15*	*0.15*	*0.11*
Equivalence scale	1.97	2.39	2.75	1.84	2.36	1.23
Restaurant food and recreation						
Log equivalence scale	**0.80**	**0.66**	**0.90**	**0.75**	**0.86**	**0.25**
Std Error	*0.12*	*0.13*	*0.12*	*0.15*	*0.15*	*0.10*
Equivalence scale	2.23	1.93	2.46	2.12	2.36	1.28
Clothing						
Log equivalence scale	**0.63**	**1.04**	**1.22**	**0.45**	**0.63**	**0.29**
Std Error	*0.11*	*0.16*	*0.14*	*0.13*	*0.12*	*0.13*
Equivalence scale	1.88	2.83	3.39	1.57	1.88	1.34

[a] Each share equation uses total expenditures on nondurable consumption as the denominator. Standard errors are calculated via monte carlo simulation. Nondurable consumption is defined as food purchased from stores, restaurant food, recreation, clothing, household operation and personal care.

equivalence scale parameters, leading to very large confidence intervals for some equivalence scale estimates.[28]

[28] Table 3a reports results from ML estimates and thus may not be directly comparable with the results in Table 3b which reports results from the simulated distributions of estimated parameters. However, simulating the distributions of the coefficients in Table 3a does not change any of the test results shown in Table 4 or Table 5. To illustrate, the following table shows simulated standard errors analogous to those given in Table 2; they are slightly tighter, but not enough to change the analysis.

Simulated standard errors for 2nd order terms

Household type	Singles	Couple 0	Couples 1	Couples 2
Food	0.0193	0.0201	0.0320	0.0347
Restaurant food and recreation	0.0257	0.0228	0.0293	0.0330
Clothing	0.0157	0.0155	0.0234	0.0254

Table 3b shows results from the semiparametric model. Here, almost all of the estimated scale sizes increase with household size and are similar (though not identical) across different expenditure share equations. The simulated[29] standard errors for the log equivalence scale and elasticities are also somewhat smaller than those estimated using the log-quadratic model.

Depending on which expenditure share equation is considered,[30] the equivalence scale sizes for comparisons with single adults average approximately 2.0, 2.4 and 2.8 for couples without children, couples with one child and couples with two children, respectively.

3.5. Tests of base independence

Having estimated equivalence scales imposing base independence on preferences, it is natural to wonder whether or not preferences are in fact consistent with base independence. The null hypothesis of shape invariance (which is necessary for base independence) requires that δ and η are constants, and the alternative is that they vary with ψ. In the log-quadratic model, this null hypothesis is violated if the second-order terms are significantly different across household types. In the semiparametric model, I ask whether or not the Loss Function, $L(\delta, \eta)$, is very big at its minimised value compared to loss functions minimised over simulated data with base independence holding by construction.

Table 4 shows tests of base independence for four household types and three expenditure share equations under the log-quadratic and semiparametric

[29] To estimate the small-sample distributions of semiparametric estimates computed on N1 and N2 real data points, I use the percentile-t method as follows.

Draw two bootstrap samples of sizes N1 and N2. Shift one of them by the estimated log scale size and elasticity. Compute semiparametric estimates. Repeat 1000 to generate simulated distributions for semiparametric estimates.

I use two methods to draw the bootstrap sample. In method I, I draw with replacement pairs of {log total expenditure, expenditure share} from the original data for each bootstrap sample. When I test on multiple equations, I draw quadruples of {log total expenditure, food share, clothing share, recreation share}. This is appropriate for the random design model because the independent variable (log of total expenditure) is a random variable, too. However, Härdle and Marron (1993) show that such 'naïve' bootstrapping may lead to overly conservative testing due to the bias in nonparametric regression (especially when the errors are heteroskedastic), and suggest that one should instead replicate the conditional distribution of the errors. I allow for this by using a second bootstrap method. In method II, I use the draw of the independent variable from the original data, and draw only the error term on the dependent variable for each bootstrap sample. I draw from the distribution of errors conditional on the log of total expenditure (i.e., the joint pdf divided by the marginal density). In the text, I report statistics from method I. Results for method II are reported in Appendix B. It turns out that method II tends to reject slightly less often.

[30] In fact, base independence requires that δ be the same no matter which two equation demand system is used to estimate it. I explore this restriction in the Section 4.

Table 4
Tests of base independence, single equations[a]

Expenditure share equation	Singles versus Couples 0	Singles versus Couples 1	Singles versus Couples 2	Couples 0 versus Couples 1	Couples 0 versus Couples 2	Couples 1 versus Couples 2
			Log-quadratic[b]			
Food purchased from stores						
2nd order term difference	0.062	0.043	0.034	– 0.019	– 0.027	– 0.008
Std Error of difference	*0.032*	*0.035*	*0.043*	*0.037*	*0.043*	*0.046*
P-value	5%	22%	42%	61%	53%	86%
Restaurant food and recreation						
2nd-order term difference	0.001	– 0.014	0.014	– 0.016	0.013	0.029
Std Error of difference	*0.036*	*0.040*	*0.044*	*0.037*	*0.041*	*0.044*
P-value	97%	72%	74%	67%	75%	52%
Clothing						
2nd-order term difference	– 0.043	– 0.048	– 0.004	– 0.005	0.039	0.044
Std Error of difference	*0.023*	*0.027*	*0.032*	*0.027*	*0.031*	*0.034*
P-value	0%	7%	89%	85%	21%	20%
			Semiparametric[c]			
Food purchased from stores						
Integrated loss	1.485	2.308	1.427	3.864	1.629	2.049
E[*Simulated loss*]	*1.192*	*0.865*	*0.941*	*0.833*	*0.659*	*1.221*
P-value	26%	6%	20%	1%	9%	19%
Restaurant food and recreation						
Integrated loss	0.745	5.012	7.495	17.600	15.710	0.747
E[*simulated Loss*]	*1.548*	*2.234*	*0.351*	*1.873*	*0.252*	*0.302*
P-value	72%	6%	0%	0%	0%	17%
Clothing						
Integrated loss	0.150	0.608	0.036	0.406	0.022	0.330
E[*Simulated Loss*]	*0.265*	*1.147*	*0.317*	*1.163*	*0.302*	*0.399*
P-value	58%	69%	90%	81%	94%	41%

[a] Each share equation uses total expenditures on nondurable consumption as the denominator. Nondurable consumption is defined as food purchased from stores, restaurant food, recreation, clothing, household operation and personal care.
[b] Estimates and standard errors are computed via Maximum Likelihood.
[c] Standard errors are calculated via monte carlo simulation.

econometric models. The top panel shows the differences in coefficient estimates on second-order terms in the log-quadratic regressions, the standard error of this difference, and the *p*-value for chi-square test statistics on the hypothesis that the difference is zero. Out of eighteen comparisons, shape invariance may be rejected at the 10% level in only three cases: childless couples versus single adults in the food and clothing equations, and couples with one child versus singles in the clothing equation. In all other comparisons, the second-order terms in the type-specific log-quadratic equations are insignificantly different from each other.[31]

The bottom panel of Table 4 shows the minimised value of the estimated loss function, $L(\delta, \eta)$, its expected value under the null hypothesis of base independence and the probability that loss function values in the simulated distribution are larger than the observed $L(\delta, \eta)$. The results in the semiparametric model are quite different from those of the log-quadratic model. Shape invariance may be rejected at the 10% level in seven out of the eighteen comparisons. It is notable, though, that the two models do not reject for the same comparisons. For example, the log-quadratic model rejects comparisons between childless singles and childless couples, while the semiparametric model does not. On the other hand, the semiparametric model rejects four out of six comparisons in the recreation equation while the log-quadratic model rejects none.

Fig. 6 shows the log-quadratic estimates of clothing share equations for childless couples and single adults. Fig. 7 shows the nonparametric estimates of clothing share equations for the same pair of household types. Examination of Fig. 6 reveals that the log-quadratic approximations do not have the same

[31] Recall from Table 2 that the log-quadratic estimates for food and clothing share equations were nearly log-linear for a few household types. When share equations are log-linear, it is not possible to uniquely identify equivalence scales from demand data (see Blackorby and Donaldson, 1993), but it is still possible to test base independence. If share equations are log-linear, then shape invariance requires that they have the same slopes across household types. The following table shows *t*-statistics for tests of shape invariance under log-linear share equations:

	Couples 0 Singles	Couples 1 Singles	Couples 2 Singles	Couples 1 Couples 0	Couples 2 Couples 0	Couples 2 Couples 1
Food	0.60	− 2.04	− 0.29	− 2.57$^{\parallel}$	− 0.81$^{\parallel}$	1.56$^{\parallel}$
Recreation	− 0.54	3.27	4.26	4.16	5.26	1.01
Clothing	− 0.76	− 0.53$^{\parallel}$	− 1.17$^{\parallel}$	0.12$^{\parallel}$	− 0.54$^{\parallel}$	− 0.58$^{\parallel}$

$^{\parallel}$ Denotes comparisons where both share equations have 2nd order terms that are insignificant at the 10% significance level (from Table 2). Using these loglinear tests of shape invariance when 2nd order terms are insignificant does not change the number of rejections of shape invariance in the parametric models.

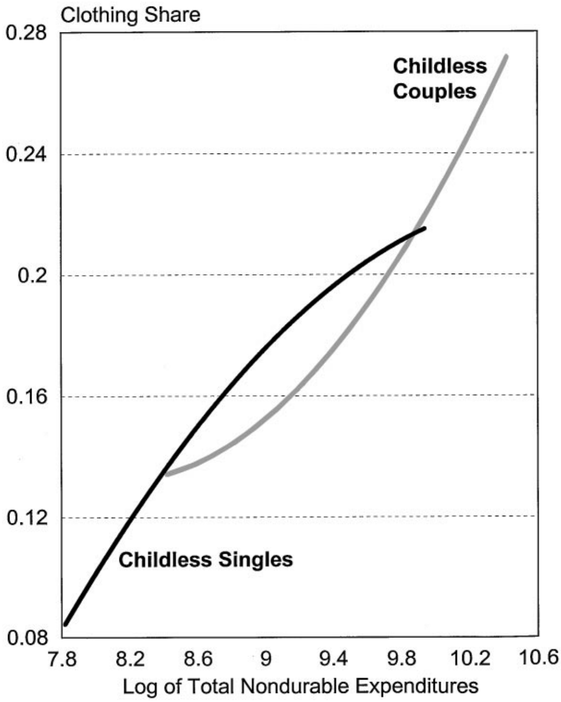

Fig. 6. Log-quadratic estimates, clothing shares, childless singles and childless couples.

shape,[32] and the results in Table 4 confirm this. Fig. 7 presents a different view of the same data. The nonparametric regression curves appear to have very similar shapes, but those shapes may not be log-quadratic. The results in Table 4 confirm that under the semiparametric model, the shape invariance restriction cannot be rejected for this comparison. Thus, the imposition of log-quadratic restrictions causes the rejection of the hypothesis of shape invariance in the comparison of clothing share equations for childless households. Indeed, because the nonparametric regression curves *could* map log-quadratic

[32] I note that the 2nd order term in the clothing share equation is insignificant for single adults and only barely significant (at the 10% level) for adult couples. However, under base independence, if one household type's share equation is log-linear, then all household types must have log-linear share equations. Thus, if single adults have log-linear clothing demands, then base-independence is rejected by the marginal significance of the second-order term in the clothing equation for adult couples.

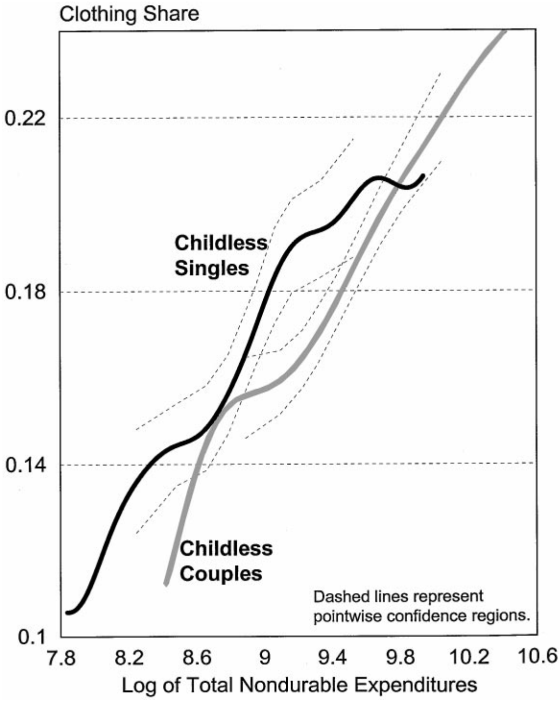

Fig. 7. Nonparametric estimates, clothing shares, childless singles and childless couples.

Engel shares if the data demanded it, the three rejections under the log-quadratic model could all be interpreted as false rejections.[33]

In looking at estimates of shift parameters under shape invariance in single equations, I have not considered two important implications of base independence implied by Eq. (2.5). First, base independence implies that the equivalence scale (the horizontal shift in Fig. 1) must be the same in all expenditure share equations. Second, base independence implies that the equivalence scales must be consistent across all pairwise comparisons. For example, under base independence,

[33] I note that the nonparametric estimates shown in Fig. 7 are not very different from the log-quadratic estimates shown in Fig. 6. Indeed a substantial body of research has suggested that log-quadratic models do a good job of representing empirical engel curves (see especially Banks et al., 1997). However, for this application, we are interested in whether or not shape invariance holds in the data, not whether or not household engel curves are log-quadratic. Thus the fact that semiparametric tests yield different results from parametric tests is interesting even if the parametric model does a good job of approximating household engel curves.

the log scale for couples with one child versus singles must equal the log scale for couples with one child versus childless couples *plus* the log scale for childless couples versus singles. In the next section, I address these issues by estimating equivalence scales and testing base independence under fully parametric and semiparametric models with restrictions of scale constancy across equations and consistency across pairwise comparisons.

4. Cross equation restrictions

In Table 3b, I present semiparametric estimates of equivalence scale sizes by expenditure share equation, and these estimates are allowed to differ across equations. However, Eq. (2.5) implies that while the equivalence scale price elasticities may be different for each commodity, the equivalence scale size must be the same across the equations. This stricter hypothesis may also be tested under the log-quadratic model or the semiparametric model. In the log-quadratic case, this amounts to restricting δ to be the same across equations, and estimating the equation system by maximum likelihood. In the semiparametric case, one uses information from all three equations to estimate the equivalence scale size and three price elasticities. Under the null hypothesis that shape invariance is true and the shift parameter δ is the same across equations, the solution that minimises the weighted sum of the integrated loss for the three equations is a consistent estimator of the true parameters (Pinkse and Robinson, 1995).[34]

Table 5 shows estimates under shape invariance and tests of shape invariance for both the parametric and semiparametric approaches with the imposition of scale constancy across equations. The likelihood ratio test statistic for the fully parametric model is distributed χ^2 with 5 degrees of freedom and has a 10% critical value of 9.24. The fully parametric model rejects shape invariance at the 10% level for every pairwise comparison because the cross-equation restrictions are binding. Given the differences in scale estimates across equations in Table 3a (log-quadratic estimates), this is not too surprising. Further, some of the estimates in the fully parametric model contradict intuition; for example, a couple with one child needs about half the expenditure of a couple with no children.

The results in Table 5 under the semiparametric model are rather different. Although the model rejects shape invariance for four out of six comparisons, it does not reject for the comparison between childless singles and couples and for

[34] The choice of weights does not matter under the null, and in practise, I found that while the choice of weight affected the estimates of scale sizes and elasticities a little bit, it did not affect the tests of shape invariance/base independence very much at all. I report results with equal weights.

Table 5
Estimates and tests, system estimation[a]

	Singles versus Couples 0	Singles versus Couples 1	Singles versus Couples 2	Couples 0 versus Couples 1	Couples 0 versus Couples 2	Couples 1 versus Couples 2
	Log-quadratic[b]					
Log equivalence scale	**0.62**	**0.39**	**0.34**	**−0.71**	**0.75**	**−0.35**
Std Err	*0.11*	*0.16*	*0.60*	*0.36*	*0.35*	*0.22*
Equivalence scale	1.86	1.47	1.40	0.49	2.11	0.71
Likelihood ratio test	12.29	10.09	18.72	10.40	10.45	11.26
P-value	3%	7%	0%	6%	6%	5%
	Semiparametric[c]					
Log equivalence scale	**0.68**	**0.77**	**0.96**	**0.75**	**0.93**	**0.23**
Std Err	*0.12*	*0.05*	*0.14*	*0.09*	*0.17*	*0.15*
Equivalence scale	1.97	2.16	2.61	2.12	2.53	1.26
Loss*	2.69	11.61	9.69	20.80	14.39	2.95
$P[Loss > Loss^*]$	62%	1%	0%	0%	0%	36%

[a] Each share equation uses total expenditures on nondurable consumption as the denominator. Nondurable consumption is defined as food purchased from stores, restaurant food, recreation, clothing, household operation and personal care. The left-out equation is household operation and personal care. Interhousehold cross-equation restrictions are imposed under base independence.
[b]Estimates, standard errors and *P*-values are computed via maximum likelihood.
[c]Standard errors and *P*-values are calculated via monte carlo simulation. Loss* is observed integrated loss from semiparametric system estimation.

the comparison between couples with one child and couples with two children. For these comparisons, the equivalence scale sizes are 1.97 and 1.26, respectively. Thus, the fully parametric model leads to a false rejection of base independence (by rejecting shape invariance) for two important comparisons.

For both the log-quadratic and the semiparametric models, where the equivalence scales are forced to be equal across share equations, the requirement that the estimated scales in pairwise comparisons be consistent with each other is a moot point. In the log-quadratic model, all six comparisons reject, so consistency is trivially satisfied, and in the semiparametric model, only two comparisons satisfy shape invariance and these two comparisons do not overlap, so again consistency is satisfied.

Why is base independence rejected for comparisons of childless households with households with children? One important reason may be that there exist some goods that are only purchased by households with children. If there are such 'child goods', then childless households would have Engel curves for these goods that are horizontal lines representing zero purchases at any total

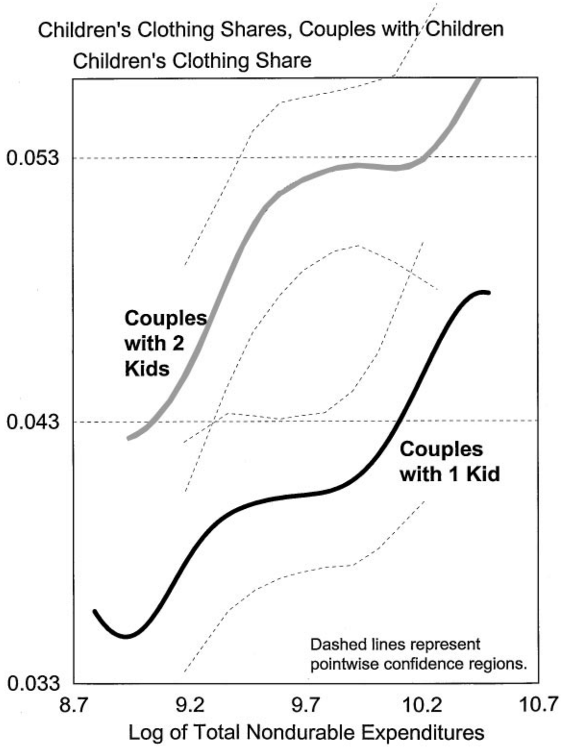

Fig. 8. Nonparametric estimates, children's clothing shares, couples with children.

expenditure level. Under shape invariance, households with children would have to have expenditure shares for child goods that were also flat, though they could be nonzero. This means that if households with children have income-dependent expenditure shares for child goods (nonflat Engel curves), then shape invariance could not hold, and therefore base independence could not hold.

If income-dependent Engel curves for 'child goods' are breaking shape invariance and base independence, then this should be observable in the demand data. Unfortunately, the family expenditure surveys do not ask sufficiently detailed expenditure questions to allow us to pursue the question of child goods very far. The data on clothing, however, are separable by age. Fig. 8 shows nonparametric regression curves (with bandwidth 0.50) for children's clothing for households with one and two children. The estimated regression curves for children's clothing for these two household types are decidedly upward sloping, which means that they cannot have the same shape as the (flat) children's clothing Engel curve for childless couples. Thus, it may be that income-dependent Engel curves for child goods are part of what is causing the rejection of base independence in comparisons of childless households with couples with children. Further, since the Engel curves in Fig. 8 seem to have similar shapes, shape invariance may hold over comparisons of households with children.

5. Discussion

Table 6 shows equivalence scale sizes from semiparametric estimates based on semiparametric estimates in Table 5, five other parametric estimates and estimates from three government and international agencies.

The semiparametric estimates of equivalence scale sizes presented in Table 5 are broadly in the same range as those from other econometric studies. Compared with the parametric estimates of equivalence scales, the semiparametric estimates of the equivalence scale sizes for couples with one child and couples with two children look fairly moderate. The semiparametric estimates suggest that there may be greater scale economies available to couples with two children than some parametric estimates (especially those using exactly aggregated translog forms) would lead one to believe. On the other hand, the three agency equivalence scales for couples versus single adults are much smaller than the semiparametric estimates (and parametric estimates). If the semiparametric

Table 6
Estimates of equivalence scale sizes[a]

Author; Type/Agency	Singles versus Couples 0	Couples 0 versus Couples 1	Couples 1 versus Couples 2
Pendakur; semiparametric system (Table 5)	1.97	rejected	1.26
Pashardes (1995); log-quadratic AI	1.56	1.16	1.14
Dickens et al. (1991); Extended AI	N/A	1.20	1.20
Blundell and Lewbel (1991); AI	N/A	1.16	N/A
Jorgenson and Slesnick (1985–1991); exactly aggregated TL	1.92	1.30	1.33
Nicol (1994); exactly aggregated TL	2.50	1.51	1.34
Statistics Canada Low Income Cutoff Ratios (1992)	1.34	1.26	1.15
US Poverty Line Ratios (1992)	1.28	1.23	1.28
OECD Equivalence Scale (1992)	1.70	1.29	1.24

[a] Sources:Pashardes Table 2; Dickens, et al. Table 1 (children aged 6-9); Blundell and Lewbel Table 2, Singles and Couples 2 are not estimated, I use children aged 6–10. Jorgenson and Slesnick Table 1, white household head, urban resident; Nicol Table 4, urban resident; Statistics Canada Catalogue # 11-210; US Poverty lines from United States 1994 (Green Book); OECD Scale from Phipps 1993.

equivalence scale estimates are right, then these agency scales attribute scale economies that are too large to couples relative to single adults.

Equivalence scales can be estimated semiparametrically without putting any restrictions on the shape of household Engel curves other than the minimum required for base independence. Further, the semiparametric methodology leads to different results from parametric approaches. Thus, parametric restrictions on the shapes of Engel curves may be binding, may affect the estimates of equivalence scales, and may affect testing of base independence. In my view, this suggests that we should abandon parametric approaches to equivalence scale estimation that put strict limitations on the shapes of expenditure share equations (such as log-linearity/exact aggregation) in favour of either parametric approaches that more realistically model the shapes[35] of household Engel curves or semiparametric approaches.

Semiparametric tests of shape invariance do not reject the hypothesis, and thus do not reject base independence, for comparisons among childless households and comparisons among households with children. Semiparametric tests do, however, reject shape invariance (and therefore base independence) for all comparisons of childless households with households with children. If base independence holds for some inter-household comparisons but not for others, then econometric analysis which tests only the hypothesis of base independence across *all* household types may lead researchers to overly general and substantively inaccurate conclusions. Parametric tests of shape invariance shown above, as well as those in Blundell and Lewbel (1994), Dickens et al. (1993) and Pashardes (1995), reject base independence against a more general parametric alternative. In particular, although the parametric tests shown above reject when the cross-equation restrictions of base independence are imposed, semiparametric tests under cross-equation restrictions do not reject base independence. Thus, in comparison to semiparametric tests, parametric tests may lead to false rejections of shape invariance and base independence.

6. Conclusions

Semiparametric estimation of equivalence scales under shape invariance (with cross-equation restrictions) yields an equivalence scale size of 1.97 for the comparison of childless couples with childless single adults, and an equivalence

[35] Banks et al. (1994) find some evidence that log-quadratic restrictions on expenditure share equations may not do too much violence to the data, and significantly ease the estimation of complex demand systems in comparison with higher-order specifications.

scale size of 1.26 for the comparison of couples with two children with couples with one child. Shape invariance and its sufficient condition of base independence are not rejected for these two comparisons. However, semiparametric tests of base independence do reject the hypothesis for all comparisons between childless households and households with children. That base independence is rejected for some comparisons but not others underscores the importance of separating these comparisons in econometric testing. Fully parametric tests of shape invariance reject in every comparison. That fully parametric tests reject shape invariance while semiparametric tests do not reject shape invariance suggests that the parametric restrictions on household engel curves embodied in fully parametric approaches may lead to false rejections of base independence.

7. For further reading

Blackorby and Donaldson, 1991; Blackorby and Donaldson, 1994; Härdle and Kelly, 1987; Lewbel, 1989a; Lewbel, 1991; Phipps, 1990; U.S. House of Representatives et al., 1994.

Acknowledgements

I acknowledge the Social Sciences and Humanities Research Council of Canada for financial support. I would like to thank my friends and colleagues for their thoughts and ideas on this work: Paul Beaudry, Charles Blackorby, Richard Blundell, David Donaldson, Greg Dow, Curt Eaton, Jane Friesen, David Green, Joris Pinkse, Clyde Reed and John Wald. I also acknowledge comments from three anonymous referees. Any mistakes that remain are of course my own.

Appendix A.

See Table 7

Table 7
Estimates Under the Log-Quadratic Model[a]

	Singles	Couples 0	Singles	Couples 1	Singles	Couples 2	Couples 0	Couples 1	Couples 0	Couples 2	Couples 1	Couples 2
						Food Purchased From Stores						
Unrestricted												
Constant	8.61	4.22	8.61	5.69	8.61	7.02	4.22	5.69	4.22	7.02	5.69	7.02
Std Error	*1.59*	*2.11*	*1.60*	*2.71*	*1.59*	*3.47*	*2.08*	*2.64*	*2.06*	*3.38*	*2.62*	*3.39*
Log expenditure	−1.69	−0.63	−1.69	−0.96	−1.69	−1.17	−0.63	−0.96	−0.63	−1.17	−0.96	−1.17
Std Error	*0.36*	*0.45*	*0.37*	*0.55*	*0.36*	*0.72*	*0.45*	*0.54*	*0.44*	*0.70*	*0.54*	*0.70*
Log expenditure squared	0.084	0.023	0.084	0.042	0.084	0.050	0.023	0.042	0.023	0.050	0.042	0.050
Std Error	*0.021*	*0.024*	*0.021*	*0.028*	*0.021*	*0.037*	*0.024*	*0.028*	*0.024*	*0.036*	*0.028*	*0.036*
R-Squared	0.361		0.352		0.403		0.324		0.370		0.332	
Log likelihood	1010.917		751.583		729.452		878.171		856.789		597.924	
Restricted												
Constant	6.40		7.73		7.96		4.72		4.88		6.06	
Std Error	*1.15*		*1.34*		*1.35*		*1.59*		*1.60*		*2.21*	
Log expenditure	−1.19		−1.49		−1.54		−0.77		−0.74		−1.03	
Std Error	*0.26*		*0.30*		*0.30*		*0.34*		*0.34*		*0.46*	
Log expenditure squared	0.056		0.073		0.076		0.028		0.030		0.046	
Std Error	*0.015*		*0.017*		*0.017*		*0.018*		0.018		0.024	
Log equivalence scale	**0.64**		**0.53**		**0.92**		**−0.60**		**0.30**		**0.59**	
Std Error	*0.13*		*0.14*		*0.12*		*0.61*		*0.29*		*0.35*	
Equivalence scale	1.90		1.69		2.51		0.55		1.35		1.80	
Vertical shift (Eta)	0.008		0.087		0.049		0.163		0.035		−0.073	
Std Error	*0.027*		*0.023*		*0.025*		*0.107*		*0.060*		0.063	
Cov(Ln Delta, Eta)	*−0.003*		*−0.003*		*−0.003*		*−0.065*		*−0.018*		*−0.022*	
R-squared	0.359		0.351		0.403		0.324		0.369		0.332	
Log likelihood	1008.670		750.971		729.082		878.053		856.561		597.909	
Wald test *p-value*	3.7443	*0.053*	1.4619	*0.227*	0.6532	*0.419*	0.2695	*0.604*	0.3957	*0.529*	0.0326	*0.857*
LRT *p*-value	4.4930	*0.034*	1.2244	*0.268*	0.7403	*0.390*	0.2372	*0.626*	0.4548	*0.500*	0.0302	*0.862*

Restaurant Food and Recreation

Unrestricted												
Constant	− 5.67	− 6.63	− 5.67	− 4.66	− 5.67	− 7.50	− 6.63	− 4.66	− 6.63	− 7.50	− 4.66	− 7.50
Std Error	*2.16*	*2.13*	*2.18*	*2.66*	*2.16*	*3.29*	*2.10*	*2.59*	*2.08*	*3.20*	*2.58*	*3.22*
Log expenditure	1.20	1.31	1.20	0.94	1.20	1.51	1.31	0.94	1.31	1.51	0.94	1.51
Std Error	*0.48*	*0.45*	*0.49*	*0.55*	*0.49*	*0.67*	*0.45*	*0.54*	*0.44*	*0.66*	*0.54*	*0.66*
Log expenditure squared	− 0.059	− 0.060	− 0.059	− 0.045	− 0.059	− 0.073	− 0.060	− 0.045	− 0.060	− 0.073	− 0.045	− 0.073
Std Error	*0.027*	*0.024*	*0.027*	*0.029*	*0.027*	*0.034*	*0.024*	*0.028*	*0.023*	*0.033*	*0.028*	*0.034*
R-squared	0.225		0.234		0.260		0.252		0.278		0.122	
Log Likelihood	809.593		632.721		582.212		866.719		817.163		641.070	
Restricted												
Constant	− 5.73		− 5.21		− 6.07		− 6.12		− 6.99		− 5.81	
Std Error	*1.34*		*1.52*		*1.59*		*1.60*		*1.67*		*2.06*	
Log expenditure	1.22		1.10		1.29		1.20		1.38		1.18	
Std Error	*0.30*		*0.34*		*0.36*		*0.34*		*0.35*		*0.43*	
Log expenditure squared	− 0.060		− 0.053		− 0.064		− 0.055		− 0.064		− 0.057	
Std Error	*0.017*		*0.019*		*0.020*		*0.018*		*0.019*		*0.022*	
Log equivalence scale	**0.66**		**0.04**		**0.29**		**−0.60**		**−0.36**		**0.13**	
Std Error	*0.15*		*0.33*		*0.24*		*0.32*		*0.24*		*0.17*	
Equivalence scale	1.93		1.04		1.34		0.55		0.70		1.14	
Vertical shift (Eta)	− 0.016		− 0.172		− 0.172		− 0.155		− 0.154		− 0.013	
Std Error	*0.025*		*0.039*		*0.029*		*0.039*		*0.030*		*0.016*	
Cov(Ln Delta,Eta)	*0.004*		*0.012*		*0.007*		*0.012*		*0.007*		*0.002*	
R-Squared	0.225		0.234		0.260		0.252		0.278		0.122	
Log likelihood	809.592		632.656		582.154		866.631		817.112		640.866	
Wald Test *P-value*	0.0017	*0.967*	0.1270	*0.722*	0.1084	*0.742*	0.1826	*0.669*	0.1001	*0.752*	0.4288	*0.513*
LRT *P-value*	0.0019	*0.965*	0.1296	*0.719*	0.1159	*0.734*	0.1756	*0.675*	0.1020	*0.749*	0.4080	*0.523*

Clothing

Unrestricted												
Constant	− 1.69	1.87	− 1.69	2.46	− 1.69	− 1.68	1.87	2.46	1.87	− 1.68	2.46	− 1.68
Std Error	*1.31*	*1.53*	*1.33*	*2.04*	*1.31*	*2.60*	*1.51*	*1.99*	*1.50*	*2.53*	*1.98*	*2.54*
Log expenditure	0.36	− 0.43	0.36	− 0.54	0.36	0.31	− 0.43	− 0.54	− 0.43	0.31	− 0.54	0.31
Std Error	*0.29*	*0.32*	*0.30*	*0.42*	*0.29*	*0.53*	*0.32*	*0.41*	*0.32*	*0.52*	*0.41*	*0.52*
Log expenditure squared	− 0.017	0.026	− 0.017	0.032	− 0.017	− 0.012	0.026	0.032	0.026	− 0.012	0.032	− 0.012

Table 7 Continued

	Singles	Couples 0	Singles	Couples 1	Singles	Couples 2	Couples 0	Couples 1	Couples 0	Couples 2	Couples 1	Couples 2
Std Error	*0.016*	*0.016*	*0.017*	*0.022*	*0.016*	*0.027*	*0.017*	*0.021*	*0.017*	*0.026*	*0.021*	*0.026*
R- squared	0.101		0.093		0.095		0.109		0.119		0.121	
Log likelihood	1372.263		1006.529		966.223		1205.124		1165.805		800.277	
Restricted												
Constant	− 0.05		− 0.57		− 1.60		2.01		0.91		0.56	
Std Error	*0.88*		*1.03*		*1.04*		*1.16*		*1.19*		*1.61*	
Log expenditure	− 0.01		0.10		0.34		− 0.23		− 0.46		− 0.15	
Std Error	*0.20*		*0.23*		*0.23*		*0.25*		*0.25*		*0.33*	
Log expenditure squared	0.004		− 0.002		− 0.015		0.028		0.016		0.011	
Std Error	*0.011*		*0.013*		*0.013*		*0.013*		*0.013*		*0.017*	
Log equivalence scale	**−0.24**		**1.86**		**1.09**		**0.26**		**0.30**		**−0.01**	
Std Error	*2.46*		*6.62*		*0.48*		*0.23*		*0.42*		*0.68*	
Equivalence scale	0.79		6.43		2.96		1.30		1.35		0.99	
Vertical shift (Eta)	− 0.036		0.074		0.013		− 0.009		− 0.017		− 0.011	
Std Error	*0.162*		*0.429*		*0.033*		*0.017*		*0.029*		*0.047*	
Cov(Ln Delta,Eta)	*0.398*		*2.842*		*0.015*		*0.004*		*0.012*		0.032	
R-Squared	0.099		0.091		0.095		0.109		0.117		0.119	
Log Likelihood	1370.380		1005.101		966.213		1205.108		1164.956		799.469	
Wald Test *P-Value*	3.4400	*0.064*	3.1086	*0.078*	0.0181	*0.893*	0.0366	*0.848*	1.5479	*0.213*	1.6867	*0.194*
LRT *P-value*	3.7664	*0.052*	2.8563	*0.091*	0.0203	*0.887*	0.0333	*0.855*	1.6979	*0.193*	1.6164	*0.204*
					ALL (System Estimation)							
Unrestricted												
(Log Expend)^2–Food	0.09	0.02	0.09	0.01	0.07	0.04	0.02	0.01	0.02	0.04	0.01	0.04
Std Error	*0.02*	*0.02*	*0.02*	*0.04*	*0.02*	*0.04*	*0.02*	*0.03*	*0.02*	*0.04*	*0.03*	*0.04*
(Log Expend)^2 − -Recreation	− 0.07	− 0.07	− 0.07	− 0.02	− 0.05	− 0.09	− 0.07	− 0.02	− 0.07	− 0.09	− 0.02	− 0.09
Std Error	*0.02*	*0.03*	*0.02*	*0.05*	*0.02*	*0.05*	*0.03*	*0.03*	*0.03*	*0.04*	*0.03*	*0.04*
(Log Expend)^2–Clothing	− 0.015	0.037	− 0.015	0.033	− 0.018	− 0.010	0.037	0.033	0.037	− 0.010	0.033	− 0.010
Std Error	*0.015*	*0.017*	*0.015*	*0.029*	*0.014*	*0.032*	*0.017*	*0.024*	*0.017*	*0.029*	0.024	*0.029*

Log Likelihood	4091.952	2868.781	2652.773	3547.060	3385.661	2254.960
Restricted						
(Log Expend)^2–Food	0.0548	0.0625	0.0064	0.020058	0.0033	0.0393
Std Error	*0.015*	*0.016*	*0.008*	*0.010*	*0.008*	*0.023*
(Log Expend)^2–Recr	– 0.072	– 0.075	– 0.036	– 0.051	– 0.051	– 0.041
Std Error	*0.017*	*0.020*	*0.017*	*0.018*	*0.016*	*0.022*
(Log Expend)^2–Cloth	0.006	0.002	– 0.001	0.008	0.006	0.012
Std Error	*0.011*	*0.011*	*0.006*	*0.007*	*0.006*	*0.015*
Elasticity (*Food*)	0.006	0.106	– 0.286	0.185	– 0.250	0.025
Std Error	*0.02*	*0.03*	*0.11*	*0.07*	*0.07*	*0.04*
Elasticity (*Recreation*)	– 0.02	– 0.13	0.24	– 0.17	0.20	0.00
Std Error	*0.02*	*0.02*	*0.07*	*0.04*	*0.04*	*0.02*
Elasticity (*Clothing*)	0.021	– 0.022	0.078	– 0.081	0.093	– 0.015
Std Error	*0.009*	*0.012*	*0.041*	*0.028*	*0.027*	*0.017*
Log equivalence scale	**0.62**	**0.39**	**0.34**	**−0.71**	**0.75**	**−0.35**
Std Error	*0.112*	*0.157*	*0.604*	*0.363*	*0.348*	*0.221*
Log Likelihood	4085.804	2863.736	2643.411	3541.860	3380.436	2249.328
LRT *P-Value*	12.2944 *0.031*	10.0898 *0.073*	18.7244 *0.002*	10.4000 *0.065*	10.4494 *0.063*	11.2629 *0.046*

[a] Each share equation uses total expenditures on nondurable consumption as the denominator. Estimates and standard errors are computed via ML. Nondurable Consumption is defined as food purchased from stores, restaurant food, recreation, clothing, household operation and personal care. Interhousehold cross-equation restrictions under base independence imposed in system estimation of all equations.

Appendix B.

See Table 8

Table 8
Estimates under semiparametric model[a]

		Singles	Couples 0	Singles	Couples 1	Singles	Couples 2	Couples 0	Couples 1	Couples 0	Couples 2	Couples 1	Couples 2
							Food purchased from stores						
Wavelength		0.34	0.34	0.34	0.44	0.34	0.43	0.34	0.44	0.34	0.43	0.44	0.43
Range(Ln Delta)		[0.5,0.9]		[0.65,1.05]		[0.9,1.3]		[0.35,075]		[0.53,0.93]		[0.06,0.46]	
Ln Delta		**0.68**		**0.87**		**1.01**		**0.61**		**0.86**		**0.21**	
Std Err	(1)	0.052	0.134	0.070	0.081	0.083	0.113	0.147	0.070	0.151	0.158	0.108	0.128
Std Err	(2)	0.100	0.155	0.113	0.099	0.130	0.130	0.151	0.086	0.147	0.133	0.143	0.158
Delta		1.97		2.39		2.75		1.84		2.36		1.23	
Eta		− 0.0059		− 0.0310		− 0.0380		0.0496		0.0672		− 0.0017	
Std Err	(1)	0.011	0.034	0.014	0.013	0.019	0.021	0.031	0.012	0.033	0.027	0.020	0.023
Std Err	(2)	0.019	0.027	0.021	0.014	0.026	0.026	0.031	0.012	0.034	0.023	0.020	0.026
Loss		1.49		2.31		1.43		3.86		1.63		2.05	
E(*Loss*)|*Null*	(1)	1.40	1.19	2.11	0.87	2.18	0.94	1.24	0.83	1.11	0.66	1.09	1.22
SE(*Loss*)|*Null*	(1)	1.12	1.58	1.71	0.82	1.78	1.03	1.17	0.84	1.20	0.91	1.06	1.23
P-value of Loss	(1)	37%	26%	35%	6%	58%	20%	3%	1%	21%	9%	13%	19%
P[*Loss* > *Loss*ı]	(1)	63%	74%	65%	94%	42%	80%	97%	99%	79%	91%	87%	81%
critical 10%	(1)	2.98	2.39	4.43	1.97	4.46	2.12	2.67	1.95	2.67	1.51	2.22	2.76
critical 5%	(1)	3.63	3.06	5.35	2.57	5.72	2.95	3.22	2.64	3.70	2.23	2.94	3.63
E(*Loss*)|*Null*	(2)	1.88	1.32	2.55	1.04	2.66	1.04	2.00	0.71	1.97	0.69	1.18	1.21
SE(*Loss*)|*Null*	(2)	1.61	1.19	2.25	0.96	2.42	0.91	1.96	0.66	1.90	0.68	1.17	1.25
P-value of Loss	(2)	47%	31%	40%	11%	63%	25%	14%	1%	43%	8%	17%	15%
P[*Loss* > *Loss*ı]	(2)	53%	69%	60%	89%	37%	75%	87%	100%	57%	92%	83%	85%
critical 10%	(2)	3.87	2.60	5.36	2.35	5.46	2.35	4.37	1.58	4.43	1.52	2.64	2.55
critical 5%	(2)	4.95	3.43	7.14	2.95	7.28	2.93	5.81	2.01	5.67	1.94	3.62	3.79
							Recreation and Restaurant Food						
Wavelength		0.45	0.33	0.45	0.26	0.45	0.64	0.33	0.26	0.33	0.64	0.26	0.64
Range(Ln Delta)		[0.5,0.9]		[0.65,1.05]		[0.9,1.3]		[0.35,075]		[0.53,0.93]		[0.06,0.46]	
Ln Delta		**0.80**		**0.66**		**0.90**		**0.75**		**0.86**		**0.25**	
Std Err	(1)	0.121	0.122	0.128	0.160	0.120	0.124	0.155	0.179	0.151	0.127	0.102	0.148
Std Err	(2)	0.137	0.122	0.142	0.142	0.144	0.142	0.130	0.110	0.122	0.096	0.143	0.151

Delta			2.23		1.93		2.46		2.12		2.36		1.28
Eta			− 0.0020		0.1038		0.0979		− 0.0092		0.0100		0.0060
Std Err	(1)	0.020	0.021	0.020	0.012	0.021	0.010	0.030	0.013	0.030	0.010	0.010	0.012
Std Err	(2)	0.023	0.018	0.021	0.013	0.024	0.011	0.026	0.011	0.026	0.009	0.014	0.012
Loss			0.74		5.01		7.50		17.60		15.71		0.75
E(*Loss*)|*Null*	(1)	1.11	1.55	1.98	2.23	2.10	0.35	1.62	1.87	1.30	0.25	2.60	0.30
SE(*Loss*)|*Null*	(1)	1.07	1.24	2.01	1.53	2.16	0.42	1.74	1.51	1.53	0.41	2.11	0.37
P-value of Loss	(1)	53%	72%	6%	6%	3%	0%	0%	0%	0%	0%	86%	17%
P[*Loss* > *Lossı*]	(1)	47%	28%	94%	94%	97%	100%	100%	100%	100%	100%	14%	83%
critical 10%	(1)	2.38	3.20	4.34	4.43	4.76	0.94	3.61	3.70	2.75	0.66	5.28	0.76
critical 5%	(1)	3.07	3.82	5.39	5.16	6.04	1.27	5.37	4.87	4.14	1.04	6.63	1.07
E(*Loss*)|*Null*	(2)	1.24	1.63	2.99	2.51	2.74	0.54	2.79	1.55	2.07	0.36	2.46	0.47
SE(*Loss*)|*Null*	(2)	1.19	1.29	2.80	1.81	2.71	0.64	2.60	1.13	1.87	0.44	1.86	0.53
P-value of Loss	(2)	57%	18%	9%	6%	0%	0%	0%	0%	0%	0%	89%	19%
P[*Loss* > *Lossı*]	(2)	43%	26%	82%	91%	94%	100%	100%	100%	100%	100%	11%	81%
critical 10%	(2)	2.71	3.27	6.46	4.86	6.26	1.27	6.05	2.99	4.44	0.89	5.00	1.09
critical 5%	(2)	3.65	4.06	8.10	6.25	8.23	1.83	8.19	3.81	5.87	1.27	6.09	1.53
							Clothing						
Wavelength		0.37	0.51	0.37	0.28	0.37	0.56	0.51	0.28	0.51	0.56	0.28	0.56
Range(Ln Delta)			[0.5,0.9]		[0.65,1.05]		[0.9,1.3]		[0.35,075]		[0.53,0.93]		[0.06,0.46]
Ln Delta			**0.63**		**1.04**		**1.22**		**0.45**		**0.63**		**0.29**
Std Err	(1)	0.109	0.137	0.162	0.129	0.144	0.126	0.131	0.116	0.123	0.099	0.132	0.127
Std Err	(2)	0.142	0.152	0.161	0.113	0.157	0.108	0.156	0.151	0.158	0.159	0.155	0.155
Delta			1.88		2.83		3.39		1.57		1.88		1.34
Eta			− 0.0180		− 0.0174		− 0.0181		− 0.0007		− 0.0022		− 0.0075
Std Err	(1)	0.008	0.009	0.042	0.009	0.079	0.010	0.026	0.008	0.009	0.007	0.009	0.008
Std Err	(2)	0.009	0.054	0.054	0.008	0.011	0.009	0.010	0.010	0.011	0.009	0.010	0.019
Loss			0.15		0.61		0.04		0.41		0.02		0.33
E(*Loss*)|*Null*	(1)	0.56	0.27	0.91	1.15	0.71	0.32	0.21	1.16	0.17	0.30	1.19	0.40
SE(*Loss*)|*Null*	(1)	0.60	0.28	0.80	0.89	0.84	0.35	0.35	0.95	0.25	0.39	1.05	0.43
P-value of Loss	(1)	80%	58%	54%	69%	98%	90%	14%	81%	88%	94%	86%	41%
P[*Loss* > *Lossı*]	(1)	20%	42%	46%	31%	2%	10%	86%	19%	12%	6%	14%	59%
critical 10%	(1)	1.23	0.57	1.86	2.27	1.51	0.81	0.50	2.42	0.40	0.70	2.46	0.94
critical 5%	(1)	1.67	0.81	2.44	2.88	1.99	1.15	0.68	3.30	0.57	1.00	3.37	1.23
E(*Loss*)|*Null*	(2)	0.68	0.19	1.25	1.46	0.88	0.45	0.32	0.98	0.31	0.26	1.25	0.39
SE(*Loss*)|*Null*	(2)	0.63	1.94	1.41	1.05	0.83	0.49	0.36	0.82	0.34	0.26	0.96	0.44
P-value of Loss	(2)	90%	66%	65%	80%	100%	98%	24%	78%	98%	98%	91%	43%
P[*Loss* > *Lossı*]	(2)	10%	34%	35%	20%	0%	2%	76%	22%	2%	2%	10%	57%
10% critical	(2)	1.40	0.70	2.51	2.76	1.93	0.96	0.72	1.96	0.70	0.57	2.51	0.88
critical 5%	(2)	1.79	0.94	3.25	3.51	2.49	1.37	0.97	2.51	0.96	0.78	3.25	1.24
							All Shares (system estimation)						
Range(Ln Delta)			[0.5,0.9]		[0.65,1.05]		[0.9,1.3]		[0.35,075]		[0.53,0.93]		[0.06,0.46]
Ln Delta			**0.69**		**0.77**		**0.96**		**0.75**		**0.93**		**0.23**

Table 8 Continued

		Singles	Couples 0	Singles	Couples 1	Singles	Couples 2	Couples 0	Couples 1	Couples 0	Couples 2	Couples	Couples 2
Std Err	(1)	0.049	0.117	0.096	0.049	0.109	0.141	0.168	0.088	0.170	0.171	0.065	0.148
Std Err	(2)	0.121	0.156	0.146	0.122	0.156	− 0.176	0.164	0.024	0.167	0.138	0.129	0.158
Delta			1.99		2.16		2.61		2.12		2.53		1.26
Eta (Food)			− 0.003		0.046				− 0.074	0	− 0.08		− 0.001
Std Err	(1)	0.010	0.022	0.018	0.009	0.022	0.026	0.039	0.013	0.042	0.031	0.013	0.028
Std Err	(2)	0.023	0.027	0.025	0.019	0.030	0.030	0.035	0.007	0.038	0.083	0.021	0.027
Eta (Recreation)			− 0.008		− 0.092		− 0.090		0.010		0.001		− 0.006
Std Err	(1)	0.012	0.019	0.020	0.007	0.022	0.012	0.034	0.009	0.036	0.012	0.009	0.013
Std Err	(2)	0.017	0.022	0.022	0.012	0.025	0.012	0.026	0.007	0.027	0.088	0.014	0.012
Eta (Clothing)			0.024		− 0.000		0.003		0.019		0.019		0.004
Std Err	(1)	0.006	0.008	0.008	0.005	0.010	0.010	0.012	0.007	0.012	0.011	0.006	0.010
Std Err	(2)	0.008	0.009	0.010	0.007	0.011	0.008	0.011	0.007	0.012	0.065	0.008	0.008
Loss			2.69		11.61		9.69		20.80		14.39		2.95
E(*Loss*)|*Null*	(1)	3.78	3.73	9.03	4.15	8.11	2.49	2.52	6.10	1.77	1.76	6.69	2.87
SE(*Loss*)|*Null*	(1)	2.40	2.06	4.88	2.30	5.04	1.52	1.78	3.03	1.27	1.18	3.65	1.88
P-value of Loss	(1)	58%	62%	23%	1%	29%	0%	0%	0%	0%	0%	89%	36%
P[$Loss > Loss\imath$]	(1)	42%	38%	77%	99%	71%	100%	100%	100%	100%	100%	11%	64%
critical 10%	(1)	6.91	6.57	15.85	7.04	14.74	4.42	4.91	10.13	3.36	3.231	1.31	5.26
critical 5%	(1)	8.35	7.61	18.89	8.21	17.78	5.34	5.99	11.80	4.22	4.03	13.14	6.50
E(*Loss*)|*Null*	(2)	5.16	3.87	10.54	3.89	9.79	2.30	4.56	4.43	3.80	1.68	6.80	2.57
SE(*Loss*)|*Null*	(2)	2.95	2.19	5.73	2.00	5.40	1.52	3.03	2.20	2.30	0.97	3.29	1.60
P-value of Loss	(2)	81%	66%	34%	0%	41%	0%	0%	0%	0%	0%	92%	29%
P[$Loss > Loss\imath$]	(2)	19%	34%	66%	100%	60%	100%	100%	100%	100%	100%	8%	71%
critical 10%	(2)	9.09	6.80	17.37	6.44	16.60	4.30	8.31	7.31	6.83	2.87	11.05	4.85
critical 5%	(2)	10.78	8.09	20.43	7.93	19.32	5.09	10.40	8.92	8.14	3.611	3.515.	62

[a]Wavelengths were found by cross-validation; *Range (Ln Delta)* describes the search space for the log equivalence scale; (1) and (2) refer to simulation techniques.Technique (1) samples from the data pairs,́ share, log expenditurė, and Technique (2) Samples fromthe distribution of shares conditional on log expenditure. Simulated standard errors and loss function distributions are reported using both simulation techniques, and using both household types as the reference type.

References

Banks, J., Blundell, R., Lewbel, A., 1994. Quadratic Engel curves, indirect tax reform and welfare measurement. Discussion Paper, University College of London, 94–04.

Banks, B., Lewbel, A., 1997. Quadratic Engel curves and consumer demand. Review of Economics and Statistics 79 (4), 527–539.

Blackorby, C., Donaldson, D., 1989. Adult equivalence scales, interpersonal comparisons of well-being and applied welfare economics. University of British Columbia, Department of Economics Discussion Paper 89–24.

Blackorby, C., Donaldson, D., 1991. Adult-equivalence scales, interpersonal comparisons of well-being and applied welfare economics. In: Elster, J., Roemer, J. (Eds.), Interpersonal Comparisons of Well-Being. Cambridge University Press, Cambridge, pp. 164–199.

Blackorby, C., Donaldson, D., 1993. Adult equivalence scales and the economic implementation of interpersonal comparisons of well-being social choice and welfare 10, 335–361.

Blackorby, C., Donaldson, D., 1994. Measuring the costs of children: A theoretical framework. In: Blundell, R., Preston, I., Walker, I. (Eds.), The Economics of Household Behaviour. Cambridge University Press, Cambridge, pp. 51–69.

Blundell, R., Lewbel, A., 1991. Information content of equivalence scales. Journal of Econometrics 50, 49–68.

Blundell, R., Pashardes, P., Guglielmo, W., 1993. What do we learn about consumer demand patterns from micro data? American Economic Review 83 (3) 570–597

Browning, M., 1989. The Effects of Children on Demand Behaviour and Household Welfare. Unpublished mimeo, McMaster University.

Browning, M., 1992. Children and household behaviour. Journal of Economic Literature 30, 1434–1475.

Dickens, R., Fry, V., Pashardes, P., 1993. Nonlinearities, aggregation and equivalence scales. Economic Journal 103, 359–368.

Gozalo, P., 1997. Nonparametric Bootstrap analysis with applications to demographic effects in demand functions. Journal of Econometrics 81, 357–393.

Härdle, W., 1993. Applied Nonparametric Regression. Cambridge University Press, New York.

Härdle, W., Kelly, G., 1987. Nonparametric kernel regression estimation—optimal choice of bandwidth. Statistics 18, 21–35.

Härdle, W., Marron, J.S., 1990. Semiparametric comparison of regression curves. Annals of Statistics 18 (1), 63–89.

Härdle, W., Marron, J.S., 1993. Comparing nonparametric versus parametric regression Fits. Annals of Statistics 21 (4), 1926–1947.

Jorgenson, D., Lau, L., Stoker, T., 1960. The Trancendental Logarithmic Model of Aggregate Consumer Behaviour. In: Bassman, R., Rhodes, G. (Eds.), Advances in Econometrics: A Research Manual, JAI Press, Greenwich, CT, 1982.

Jorgenson, D., Slesnick, D., 1987. Aggregate consumer behaviour and household equivalence scales. Journal of Business and Economic Statistics 5 (2), 219–232.

Lewbel, A., 1989a. Identification and estimation of equivalence scales under weak separability. Review of Economic Studies 52 (6), 311–316.

Lewbel, A., 1989b. Household equivalence scales and welfare comparisons. Journal of Public Economics 39 (3), 377–391.

Lewbel, A., 1991. Cost of characteristics indices and household equivalence scales. European Economic Review 35 (6), 1277–1293.

Nelson, J., 1991. Independent of a Base Equivalence Scale Estimation, unpublished mimeo, University of California, Davis.

Nicol, Christopher J., 1994. Identifiability of household equivalence scales through exact aggregation. Canadian Journal of Economics 27 (2), 307–328.

Pashardes, P., 1991. Contemporaneous and intertemporal child costs. Economic Journal.

Pashardes, P., 1995. Equivalence scales in a rank-3 demand system. Journal of Public Economics 58, 143–158.

Pendakur, K., 1994. A Household Equivalence Scale for Canada, unpublished mimeo, Simon Fraser University.

Phipps, S., 1996. Distributional Effects of Different Models of Intra-household Allocation. Canadian Public Policy forthcoming.

Phipps, S., 1990, Price Sensitive Equivalence Scale for Canada, unpublished, Dalhousie University.

Pinkse, C.A.P., Robinson, P.M., 1995. Pooling nonparametric estimates of regression functions with a similar shape. In: Maddala, G., Phillips, P., Srinivisan, T.N. (Eds.), Advances in Econometrics and Quantitative Economics pp. 172–195.

Pollak, R., Wales, T., 1979. Welfare comparisons and equivalence scales. American Economic Review, 69 (2) May 1979, pp. 216–221

Pollak, R., 1991. Welfare comparisons and situation comparisons. Journal of Econometrics 50, 31–48.

U.S. House of Representatives, Committee on Ways and Means, 1994. Where Your Money Goes...The 1994–1995 Green Book, Washington D.C, U.S. Government Printers.

Working, H., 1943. Statistical Laws of Family Expenditure. Journal of the American Statistical Association 38, 43–56.