

**EXAM INSTRUCTIONS:** Please record all answers in the examination book provided. Be as complete as possible in recording the calculations made to arrive at specific answers. Calculators with enhanced capabilities such as the ability to attach external drives, whether such drives are attached or not, are prohibited. Other than an admissible calculator, no books or other materials are permitted to be used during the examination.

**Section I: Background and Definitions** (7 points each--35 total)

1. Derive the formula for (Macaulay) duration of an annuity assuming that coupons are paid semi-annually.
2. Derive the discounted dividend valuation formula for pricing a stock assuming that dividends stay constant at \$D until time T and then increase at a constant rate (g) from that time into perpetuity, i.e.,  $\{D_{T+1} = D(1+g)\}$ .
3. A ten year bond offers total annual coupon payments of \$8 which will be paid at **quarterly** intervals. The face value of \$100 will be repaid at the end of ten years. You observe that other similar bonds have yields to maturity of 5.48%. How much is this bond worth? (Assume the first payment will be paid one quarter from the valuation date.)
4. (Witt, 1613) A owes to B a debt to be paid semiannually over T years, in 2T equal installments of £Q paid every six months. However, A changes his mind and wants to make the semiannual payments over T/2 years, in T equal payments. The problem is to determine the formula for what each payment on the debt will be, with interest to be reckoned at r% interest, compounded effectively, e.g., an annuity of \$1 cash flow to be received every six months for two years is discounted as:

$$P = \frac{1}{(1 + \frac{r}{2})^{1/2}} + \frac{1}{(1 + \frac{r}{2})} + \frac{1}{(1 + \frac{r}{2})^{3/2}} + \frac{1}{(1 + \frac{r}{2})^2}$$

Simplify your answer as much as possible.

**Section II: Fixed Income Valuation** (15 points each--30 total)

1. You are in the market for a house. Your effective all-in market borrowing rate for a house mortgage from a bank is 7% (compounded annually). One of the houses you are considering purchasing has an assumable \$200,000, 6 year mortgage at 4%, with a 21 year amortization. In order to provide a further incentive to purchasing the house, the vendor is willing to defer the balance of the purchase price for one year, with no interest. Your bank has agreed to lock in the interest rate on the refinancing of the balance in one year's time (with a 20 year amortization) at the current 5 year interest rate of 6.75%. The asking price on the house is \$500,000. Assuming that your preferred mortgage maturity is five or six years and that mortgage rates are expected to be unchanged in six years time, what is the value of the concessionary financing for this house?
- 2.a) You are contemplating going to graduate school starting one year from now. You estimate that your tuition and living expenses will be \$50,000 each year for four years. Assuming current interest rates are 8%:  
a) what is the present value and duration of this obligation?

b) Assume a rich uncle is willing to give you enough money to cover your graduate education expenses and you have to decide how to invest this money to ensure that changes in interest rates do not affect your ability to meet your expenses. Assuming the yield curve is flat, what combination of 1 year, 2 year and 5 year zero coupon bond investments is needed to achieve immunization against interest rate changes?

### Section III: Term Structure Behaviour (15 points)

1. The following information about Government of Canada bonds is available (assume that all coupons are paid annually, yields are compounded annually and par values are 100):

Maturity (Years)	Coupon (%)	Price
1 Year tbill		94.47
2 Year bond	7.5	100.5
3 Year bond	8.0	101.5
4 Year bond	12.5	106.75
5 Year bond	10.0	99.5

- Calculate the term structure of (spot) interest rates up to five years.
- Assume that the liquidity premia (LP) hypothesis is correct, and that  $LP_{1,2} = LP_{2,3} = LP_{3,4} = LP_{4,5} = 1\%$ . Determine  $LP_{1,3}$ ,  $LP_{1,4}$ ,  $LP_{1,5}$ ,  $LP_{2,4}$ ,  $LP_{2,5}$ ,  $LP_{3,5}$ .
- Given that these LP are correct estimates, construct the market's expectation of the term structure of interest rates for years 2 to 5.

### Section IV: Stock Valuation Models (20 points)

1. Burnaby Air Cleaners has announced it intends to pay a \$12 common stock dividend next year. It is estimated that the dividend will increase at 4% per year for 10 years. (It is estimated that stocks of similar risk are currently priced to provide an expected return of 10%.) At that time the company will be split into two separate companies. In exchange for 1 share of the original stock, stockholders can receive 1 common share in both of the new companies. It is estimated that one of the new companies will pay a dividend in year 6 of \$8 which will increase at 5% per year thereafter. The other company will pay no dividends for five years after startup and will pay a fixed dividend of \$10 per year thereafter. (It is estimated that the expected return of the new companies will be the same as the old company.)

- What is the intrinsic value of Burnaby Air Cleaners common stock?
- What is the Macaulay duration of this security?

### Bonus Question: (5 pts.)

State the formula for the variance of the return on an equally weighted portfolio of k securities. Be sure to provide a precise statement of the value weights. Simplify your answer as much as possible.