

Lecture 10

Basics of Fixed Income Derivatives

- Futures, Forwards, Swaps and Options
- Interest Rate Futures and Option Contracts
- Modelling the Interest Rate Process
- Prices of CME Bond Options
- Different Bond Option Pricing Formulas



Types of Pure Derivative Securities

Reading: RSD, Sec. 1.1

- What is are Derivative Securities?
 - Various possible designs:
 - Forward vs. Futures Contracts
 - Types of Options
 - Swaps and Other Variations



Basics of Futures, Forward and Options Contracts

- **Contract Features**
 - **Standardization vs. Customized**
- **Daily Marking to Market**
 - **Forwards vs. Futures**
 - **Hedgers vs. Speculators**
 - **Options vs. Futures**
- **How do Margins Work?**
 - **Types of Margin Requirements**
 - **Finding Margins on the CME site**



Figure 1.1 Comparison of Futures and Forward Contracts

	Forwards	Futures
Contract Amount	Depends on buyer and seller	Standardized
Price Movement Restrictions	No limit.	Varies; typically restricted by the exchange with provisions for increase or decrease
Position Limits	Market determined	Set by exchanges and regulators
Delivery Date	Depends on buyer and seller	Standardized
Market Location	Decentralized, often a telephone/computer network of dealers, brokers and other participants	Centralized exchange floor where trading is executed by open outcry between exchange members
Clearing	No direct, separate clearing mechanism	The exchange clearinghouse
Settlement	By delivery of goods as specified in contract	Marking-to-market daily using a margin system Some deliveries by specialized traders
Regulation	Self-regulation; contract law; general securities law	Exchange rules; Commodity Exchange Act; CFTC; State regulators; specific legislation



Chicago Mercantile Exchange Website: The Important Trading Link is Highlighted, next slide has the link content

The screenshot shows the CME Group website in a browser window. The address bar displays <https://www.cmegroup.com>. The navigation menu includes 'CME Group', 'Trading', 'Clearing', 'Regulation', 'Data', 'Technology', 'Education', and 'About'. The 'Trading' link is highlighted with a red rectangular box. The main banner features the text 'FX Options: Get Ready for the New 10 a.m. New York Cut' and a large 'FX OPTIONS' logo. Below the banner is an 'Economic Research' section with three article cards: '3% Yield for 10-Year Treasury Showcases Economic Anxiety', 'Trade War Costs to Consumers, Companies and Nations', and 'Did Quantitative Easing Help Spur Growth?'. A cookie consent banner is visible at the bottom of the page.



The Various Categories of Derivatives are identified

The screenshot displays the CME Group website's 'All Products Home' page. The navigation bar includes 'Trading', 'Clearing', 'Regulation', 'Data', 'Technology', 'Education', and 'About'. The main content area is organized into several sections:

- Featured Products:** A list of derivatives including Corn (ZC), Soybean (ZS), WTI Crude Oil (CL), Henry Hub Natural Gas (HG), E-mini S&P 500 (ES), E-mini NASDAQ 100 (NQ), Euro FX (GE), Eurodollar (GE), 10-Year T-Note (ZN), 5-Year T-Note (ZF), and Gold (GC).
- Product Groups:** A vertical list of categories such as Agricultural, Energy, Equity Index, FX, Interest Rates, Metals, Options, OTC, Real Estate, and Weather.
- Getting Started:** A section with links for 'Find a Broker', 'Holiday Calendar', 'Membership', and 'Tools & Resources'. It also features 'Active Trader' (Daily market data and insight for the Individual Active Trader) and 'Bitcoin Futures' (CME Bitcoin futures are now available for trading).
- New to Futures?:** A blue call-to-action box with the text 'Learn why traders use futures, how to trade futures, and what steps you should take to get started.' and a 'Start Here' button.

Below these sections are three news articles:

- 3% Yield for 10-Year Treasury Showcases Economic Anxiety:** By Bluford Putnam, August 16, 2018. The 3% yield level for the 10-Year Treasury.
- Trade War Costs to Consumers, Companies and Nations:** By Erik Norland, August 14, 2018. The trade war between the U.S. and China is.
- Did Quantitative Easing Help Spur Growth?:** By Erik Norland, August 09, 2018. As economies were struck down by the 2008.

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Clicking the Fixed Income link reveals the numerous types of Fixed Income Derivatives traded on the IMM division of the CME

Interest Rate futures and options

Explore the deepest centralized pool of liquidity, offering capital-efficient risk management solutions throughout the yield curve.

Trade across the yield curve

Use Interest Rate futures and options to manage exposure to government bonds and money market securities in a safe, capital-efficient way. Access a full range of benchmark products—Eurodollars, Fed Funds, SOFR, US Treasuries—across the USD yield curve, from one-week to 30-years.

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ZTZ2	101'310 14,814 +0'015 +0.05%	 OPT
2-Year T-Note Futures		
ZFZ2	106'185 31,056 +0'050 +0.15%	 OPT
5-Year T-Note Futures		
ZNZ2	110'205 51,602 +0'100 +0.28%	 OPT
10-Year T-Note Futures		
ZBZ2	120'14 9,434 +0'25 +0.65%	 OPT
U.S. Treasury Bond Futures		

Treasury CVOL Index >

Track forward-looking risk expectations on U.S. Treasuries with the CME Group Volatility Index (CVOL™), a robust measure of 30-day implied volatility derived from deeply liquid options on Treasury futures.

CODE: TVL

CVOL:

146.8129

CHANGE:

+0.7393

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10-YEAR T-NOTE FUTURES - SETTLEMENTS

TRADE DATE **FRIDAY 11 NOV 2022** ▾

FINAL DATA ⓘ

Last Updated 11 Nov 2022 06:00:00 PM CT

ESTIMATED VOLUME TOTALS **689,020**

PRIOR DAY OPEN INTEREST TOTALS **4,076,582**

MONTH	OPEN	HIGH	LOW	LAST	CHANGE	SETTLE	EST. VOLUME	PRIOR DAY OI
DEC 22	112'170	112'180	112'000	112'095	-'065	112'100	684,171	4,005,492
MAR 23	112'245	112'260	112'095	112'185	-'065	112'185	4,849	71,064
JUN 23	-	-	112'280A	112'280A	-'065	112'245	0	26

5-YEAR T-NOTE FUTURES - SETTLEMENTS

TRADE DATE **FRIDAY 11 NOV 2022** ▾

FINAL DATA ⓘ

Last Updated 11 Nov 2022 06:00:00 PM CT

ESTIMATED VOLUME TOTALS **454,982**

PRIOR DAY OPEN INTEREST TOTALS **4,286,604**

MONTH	OPEN	HIGH	LOW	LAST	CHANGE	SETTLE	EST. VOLUME	PRIOR DAY OI
DEC 22	107'292	107'310	107'197	107'267	-'032	107'260	452,933	4,237,150
MAR 23	108'042	108'080B	107'295A	108'037A	-'035	108'030	2,049	49,454
JUN 23	-	-	-	-	-'035	108'110	0	0

Pricing the 6% theoretical underlying for the 5 year Tnote future – there is a cheapest deliverable – the quoted price is for a theoretical bond, need to solve for the yield to maturity – see quoted price from previous slide

```
In[64]= f[y_] :=  
      (3 * ((1 / (y / 2)) - (1 / ((y / 2) * ((1 + (y / 2)) ^ (10)))))) + (100 / ((1 + (y / 2)) ^ (10)))  
      Solve[f[y] == (107 + (21 / 32)), y] // N
```

```
Out[65]= {{y → -3.98551}, {y → 0.0428268}, {y → -3.60635 - 1.167 i}, {y → -3.60635 + 1.167 i},  
          {y → -2.6137 - 1.88829 i}, {y → -2.6137 + 1.88829 i}, {y → -1.38671 - 1.88843 i},  
          {y → -1.38671 + 1.88843 i}, {y → -0.394032 - 1.16758 i}, {y → -0.394032 + 1.16758 i}}
```

```
In[60]= g[y_] := (6 * ((1 / (y)) - (1 / ((y) * ((1 + (y)) ^ (5)))))) + (100 / ((1 + (y)) ^ (5)))  
      Solve[g[y] == (107 + (21 / 32)), y] // N
```

```
Out[61]= {{y → 0.0426727}, {y → -1.79758 - 0.579315 i}, {y → -1.79758 + 0.579315 i},  
          {y → -0.695892 - 0.937712 i}, {y → -0.695892 + 0.937712 i}}
```

```
In[62]= (6 * ((1 / (.0427)) - (1 / ((.0427) * ((1 + (.0427)) ^ (5)))))) + (100 / ((1 + (.0427)) ^ (5)))
```

```
Out[62]= 107.644
```

```
In[66]= (3 * ((1 / (.0428 / 2)) - (1 / ((.0428 / 2) * ((1 + (.0428 / 2)) ^ (10)))))) +  
      (100 / ((1 + (.0428 / 2)) ^ (10)))
```

```
Out[66]= 107.669
```



SOFR and Eurodollars

SOFR >

The leading tools for hedging USD short-term interest rates, SOFR futures and options offer deep liquidity extending over five years out the term structure, alongside 80 options expiries for fine-tuning exposures from one week to four years.

- [View SOFR options](#)
- [View SOFR swaps](#)
- [View Term SOFR Rates](#)

SR1J3

One-Month SOFR Futures

95.06
13
+0.025
+0.03%



OPT

SR3M3

Three-Month SOFR Futures

94.985
4,699
+0.02
+0.02%



OPT

Eurodollars >

A cost-effective way to manage risk for short-term interest rates, Eurodollar futures and options are the preferred tools for traders who want to express a view on future interest rate moves.

GEU3

Eurodollar Futures

94.86
1,336
+0.02
+0.02%



OPT



EURODOLLAR FUTURES - SETTLEMENTS

TRADE DATE WEDNESDAY 09 NOV 2022 -

FINAL DATA ⓘ

Last Updated 09 Nov 2022 06:00:00 PM CT

ESTIMATED VOLUME TOTALS **532,815**

PRIOR DAY OPEN INTEREST TOTALS **7,973,351**

MONTH	OPEN	HIGH	LOW	LAST	CHANGE	SETTLE	EST. VOLUME	PRIOR DAY OI
NOV 22	95.3200	95.3275	95.3125	95.3250	+ .0025	95.3250	11,731	142,569
DEC 22	94.8900	94.9200	94.8750	94.9100	+ .0150	94.9000	110,492	1,552,703
JAN 23	94.8850	94.9150	94.8550	94.9100A	UNCH	94.8900	9,041	95,386
FEB 23	94.7450	94.7900B	94.7350A	94.7950B	+ .0050	94.7650	857	12,915
MAR 23	94.6550	94.7000	94.6100	94.6950	+ .0100	94.6600	74,181	902,017
APR 23	94.6550	94.6550	94.6550	94.6650B	+ .0100	94.6300	150	1,013
JUN 23	94.6300	94.6900	94.5850	94.6800	+ .0200	94.6450	55,730	712,724
SEP 23	94.8050	94.8900	94.7600	94.8700	+ .0450	94.8400	31,760	713,261
DEC 23	95.0050	95.1100	94.9700	95.0900	+ .0700	95.0600	51,496	925,122
MAR 24	95.2550	95.3850	95.2300	95.3600	+ .0850	95.3250	25,940	397,003
JUN 24	95.4000	95.6150	95.4550	95.5900	+ .0900	95.5550	24,762	399,468

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- ✓ Quarterly/Serial Options
- 1 Year Mid-Curve Options
- 1 Year Weekly Mid-Curve Options
- 2 Year Mid-Curve Options
- 2 Year Weekly Mid-Curve Options
- 3 Month Mid-Curve Options
- 3 Year Mid-Curve Options
- 3 Year Weekly Mid-Curve Options
- 4 Year Mid-Curve Options
- 5 Year Mid-Curve Options
- 6 Month Mid-Curve Options
- 9 Month Mid-Curve Options
- Calendar Spread Option

EURODOLLAR OPTIONS - SETTLEMENTS

EXPIRATION **DEC 2022** TRADE DATE **WEDNESDAY 09 NOV 2022**

FINAL DATA

Last Updated 09 Nov 2022 06:00:00 PM CT

ESTIMATED VOLUME TOTALS **432,178**

PRIOR DAY OPEN INTEREST TOTALS

CALLS								STRIKE PRICE	PUTS							
EST. VOL	PRIOR DAY OI	HIGH	LOW	OPEN	LAST	SETTLE	CHANGE		CHANGE	SETTLE	LAST	OPEN	LOW	HIGH	PRIOR DAY OI	EST. VOL
0	0	-	-	-	-	5.6500	+0150	8925	UNCH	CAB	-	-	-	-	0	0
0	0	5.3900B	-	-	-	5.4000	+0150	8950	UNCH	CAB	-	-	-	-	0	0
0	0	-	-	-	-	5.1500	+0150	8975	UNCH	CAB	-	-	-	-	0	0
0	0	4.9050B	-	-	-	4.9000	+0150	9000	UNCH	CAB	-	-	-	-	0	0
0	0	-	-	-	-	4.6500	+0150	9025	UNCH	CAB	-	-	-	-	0	0
0	0	4.4000B	-	-	-	4.4000	+0150	9050	UNCH	CAB	-	-	-	-	0	0
0	0	4.1550B	-	-	-	4.1500	+0150	9075	UNCH	CAB	-	-	-	-	0	0
0	0	3.8950B	-	-	-	3.9000	+0150	9100	UNCH	CAB	-	-	-	-	1	0
0	0	3.6550B	-	-	-	3.6500	+0150	9125	UNCH	CAB	-	-	-	-	0	0



Clicking the FX link reveals the numerous types of FX Derivatives traded on the IMM division of the CME (FX is fixed income!)



Efficient products for a responsive market

Manage FX exposure in our highly liquid marketplace using our cleared and listed futures and options, and award-winning FX Link. Benefit from open and transparent pricing to identify opportunities and find efficient alternatives to forwards, swaps, and options.

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[FX Product Guide](#)

FX NEWS AND EVENTS

Get the latest news on futures and options in the FX markets with product news and information, macro trends, and more.

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New: The October 2022 FX Report

Webinar: CME FX Options Products for Evolving...

Will new capital rules be a SA-CCR punch for FX markets?

Case study: Reducing uncleared gross notional to reduce your...

New: The Jul

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G10 >

Manage risk and find opportunities in futures and options on G10 currencies, with the most liquid and capital-efficient FX futures market.

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6EZ2

Euro FX Futures

1.0364
188,986
-0.0026
-0.25%



OPT

6JZ2

Japanese Yen Futures

0.007149
154,580
-0.000102
-1.41%



OPT

6BZ2

British Pound Futures

1.1743
83,846
-0.0122
-1.03%



OPT

6AZ2

Australian Dollar Futures

0.67065
89,633
-0.00155
-0.23%



OPT

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6MZ2

Mexican Peso Futures

0.051230
42,941
+0.000260
+0.51%



OPT

6LZ2

Brazilian Real Futures

0.1868
9,178
+0.0002
+0.11%



OPT

6RZ2

Russian Ruble Futures

-
0
-
-



OPT

6ZZ2

South African Rand Futures

0.057550
1,033
-0.000250
-0.43%



OPT

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BRITISH POUND FUTURES - SETTLEMENTS

TRADE DATE MONDAY 14 NOV 2022 ▾

FINAL DATA ⓘ

Last Updated 14 Nov 2022 06:00:00 PM CT

ESTIMATED VOLUME TOTALS 104,597

PRIOR DAY OPEN INTEREST TOTALS 232,776

MONTH	OPEN	HIGH	LOW	LAST	CHANGE	SETTLE	EST. VOLUME	PRIOR DAY OI
NOV 22	1.1854	1.1854	1.1725A	1.1725A	-.0106	1.1748	44	859
DEC 22	1.1795	1.1840	1.1720	1.1769	-.0074	1.1791	104,441	227,418
JAN 23	1.1781	1.1852B	1.1743A	1.1780A	-.0075	1.1808	15	1,301
FEB 23	-	-	1.1755A	1.1755A	-.0074	1.1816	0	0
MAR 23	1.1831	1.1870B	1.1755A	1.1796A	-.0074	1.1823	97	2,414
JUN 23	-	-	1.1784A	1.1784A	-.0074	1.1845	0	178
SEP 23	-	-	1.1804A	1.1804A	-.0073	1.1862	0	406

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British Pound

Futures and Options

GLOBEX CODE

6BZ2

LAST

1.1763

CHANGE

-0.0028 (-0.24%)

VOLUME

952

Add to portfolio

as of November 14 2022, 06:45pm CT

OVERVIEW QUOTES **SETTLEMENTS** VOLUME & OI TIME & SALES SPECS MARGINS CALENDAR

FUTURES **OPTIONS**

GBP/USD MONTHLY OPTIONS - SETTLEMENTS

EXPIRATION DEC 2022

TRADE DATE MONDAY 14 NOV 2022

OVERVIEW QUOTES **SETTLEMENTS** VOLUME & OI TIME & SALES SPECS MARGINS CALENDAR

FUTURES **OPTIONS**

CALLS								STRIKE PRICE	PUTS							
EST. VOL	PRIOR DAY OI	HIGH	LOW	OPEN	LAST	SETTLE	CHANGE		CHANGE	SETTLE	LAST	OPEN	LOW	HIGH	PRIOR DAY OI	EST. VOL
0	1,041	-	2.04A	-	2.04A	2.45	-.51	1165	+22	1.04	1.26B	-	-	1.27B	756	0
0	283	-	1.89A	-	1.89A	2.28	-.50	1167	+24	1.13	1.36B	-	-	1.37B	10	2
12	613	1.96	1.76A	1.96	1.76A	2.13	-.48	1170	+26	1.22	1.47B	-	-	1.48B	157	5
0	271	-	1.62A	-	1.62A	1.97	-.47	1172	+28	1.32	1.58B	-	-	1.59B	3	2
2	929	2.05B	1.50A	2.00	1.50A	1.83	-.44	1175	+29	1.42	1.70B	-	-	1.71B	619	2
0	0	-	1.38A	-	1.38A	1.69	-.42	1177	+31	1.53	1.83B	-	-	1.83B	68	12
21	588	1.61	1.26A	1.61	1.26A	1.55	-.41	1180	+33	1.64	1.97B	-	-	1.97B	61	0

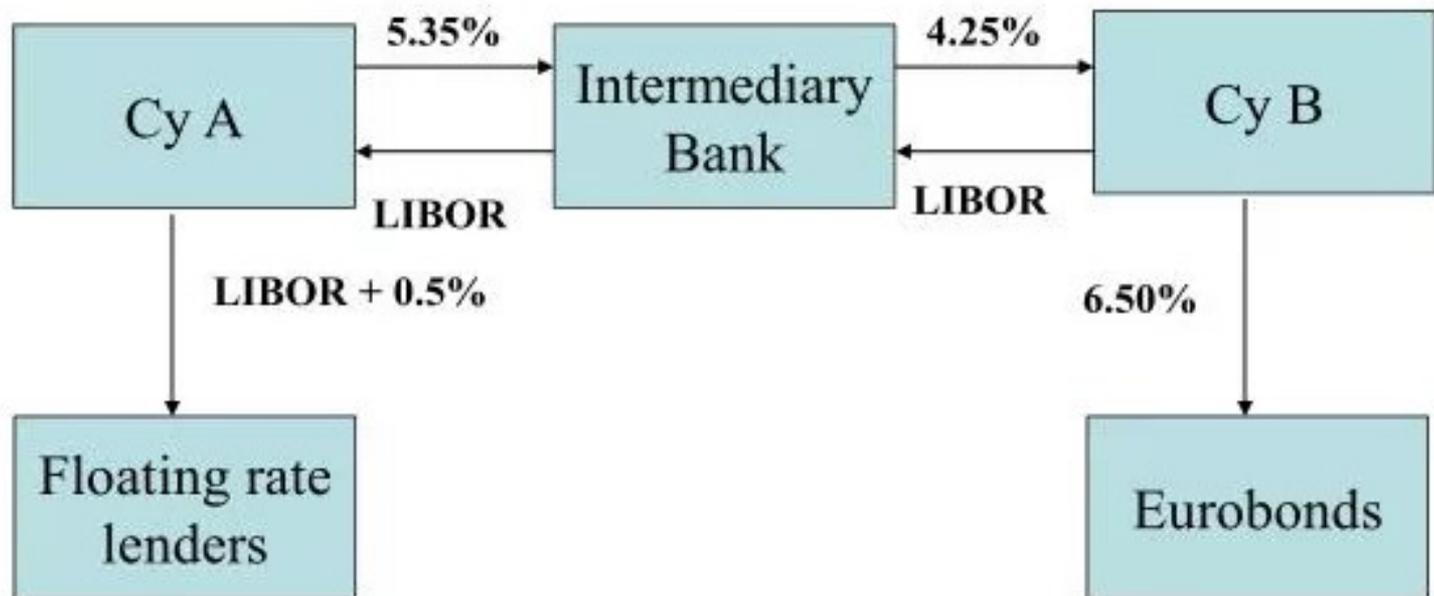
Basis Relationships

Generally, basis is the difference between two prices

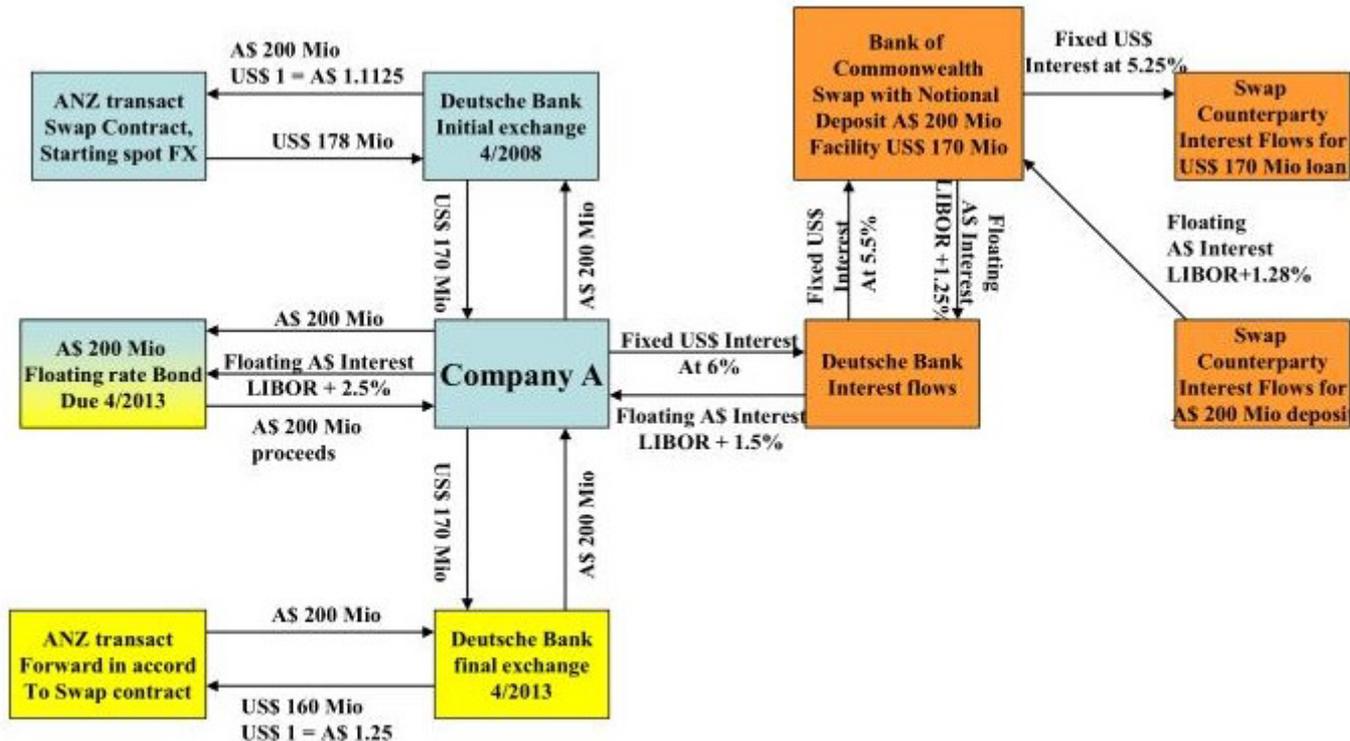
- Study of basis relationships was important in analysis of derivative securities prior to the financial futures 'revolution' (see Lecture 1)
- Hieronymous, *The Economics of Futures Trading* (1977) is an excellent example of the traditional approach



INTEREST RATE SWAP: Floating rate borrower (often a lower quality credit) exchanges cash flows with a fixed rate borrower (usually a higher quality credit). The role of the intermediary is to absorb credit risk and reduce transactions costs over a party-to-party trade and capture part of the cash flows as compensation.



Cross Currency Interest Rate Swap: Transactions in the cross currency interest rate swap market are facilitated by the contract standardization provided by the International Swap Dealers Association <https://www.isda.org>



Basis Definitions

- **The Basis:** $F(t, T) - S(t)$
- **The Futures Basis:** $F(t, T) - F(t, N)$

Notice that as t changes, then the carrying charges embedded in the basis will decline due to the reduction in $(T-t)$ while the carrying charges in the futures basis will not decline because $(T-N)$ does not change



More on the Futures Basis

- ▣ **Definitions Applicable to the Futures Basis**

- ▣ Contango: $F(t, T) - F(t, N) > 0$

Example: Gold

- ▣ Backwardation: $F(t, T) - F(t, N) < 0$

Example: Tbonds

Do not confuse this with the Normal
Backwardation Hypothesis for the **Yield** curve



Absence of Arbitrage

- What is an arbitrage?

An *arbitrage* opportunity is defined here as: ***a riskless trading strategy that generates a positive profit with no net investment of funds.***

Key words: riskless and trading strategy.



No Arbitrage Condition

- A fundamental *theoretical* requirement of pricing in financial markets is that there is

NO ARBITRAGE OPPORTUNITIES

In terms of the profit function, this condition is expressed as $B_{arb} < 0$ (can also be a weak inequality)



Some Possible Confusions

- Do not confuse the **No Arbitrage Condition** where $B_{arb} < 0$ with the profit function for a speculative trade where $B(1) > 0$ is the desired result.
- The arbitrage profit function is time dated as $B_{arb}(0)$ to reflect all variables in the profit function being known at $t=0$ (needed for riskless trading)



Types of Arbitrage

- The type of arbitrage that is most important in the analysis of derivative securities is the *cash and carry arbitrage*.
- Other types of arbitrage include:
 - Triangular Arbitrage* (RSD, p.237)
 - Geographical Arbitrage* – price of goods in different locations differ only by the cost of purchasing and transporting, e.g., cement.



History of Arbitrage

- On the history of the concept and definition of arbitrage see Poitras, *Early History of Financial Economics 1478-1776*, p.243-7.
- The word arbitrage has a Latin root, e.g., 'arbitrio' in Italian and initially referred to trading strategies aimed at profiting from differences in exchange rates.
- Arbitrage was active in 16th C. Antwerp and has roots in antiquity.



Cash and Carry Arbitrage

- Reading: RSD, p.213-220.
- The Cash and Carry Arbitrage is the fundamental arbitrage applicable to derivative securities traded on **storable commodities**.
- Key Point: there are two arbitrages for any derivative contract – the **short arbitrage** and the **long arbitrage**.



What are Perfect Capital Markets?

Various presentations of perfect capital markets are available, with different versions emphasizing elements that are of importance to the argument at hand. One particularly complete set is provided in Haley and Schall (1979).

A.1 Costless capital markets: No capital market transactions costs (including commissions and bid/offer spreads), no government restrictions which interfere with capital market transactions, and the costless ability to make financial assets infinitely divisible.

A.2 Neutral Taxes: There are no personal or corporate taxes.

A.3 Competitive Markets: There are many perfect substitutes for all securities of a firm at any point in time and there is no discrimination in the pricing of these securities such that any security can be acquired at the same market price by all investors. In addition, firms and investors are price takers in investing, borrowing and lending activities.

A.4 Equal Access: Investors and firms can borrow, lend and issue claims on the same terms. This assumption requires that borrowing and lending rates be equal.

A.5 Homogeneous Expectations: All capital market participants have the same expectations about relevant random variables.

A.6 No Information Costs: Firms and individuals have the same available information and this information is acquired at zero cost.

A.7 No Costs of Financial Distress: Firms and individuals incur no costs of financial distress or bankruptcy such as legal costs and disruption of operations. This assumption does not rule out the possibility of bankruptcy.



The Case of Gold

- The short and long arbitrages are referenced to the position in the spot commodity
- The long gold arbitrage (RSD, p.216) involves borrowing money to buy gold and simultaneously selling the gold for forward delivery
- Solving the profit function for the long arbitrage gives:
$$F(0, T) < S(0) \{1 + r(0, T)\}.$$



Figure 4.1: Profit Function for a Long Gold Cash-and-Carry Arbitrage

<i>DATE</i>	<i>Cash Position</i>	<i>Futures Position</i>
$t=0$	Borrow $\$[Q_G S(0)]$ at interest rate $r(0,T)$ and buy Q_G ounces of gold at $S(0)$ for storage until $t=T$	Short Q_G units at $F(0,T)$

-- The cash gold position provide no pecuniary return between $t=0$ and $t=T$

$t=T$	Deliver the Q_G units against the maturing futures contract and use the proceeds to repay the maturity value of the loan, $\$[Q_G S(0)]\{1 + r(0,T)\}$	
-------	--	--

In this case, the profit function can be specified:

$$\pi(0) \leq \{F(0,T) - S(0)(1 + r(0,T))\} Q_G$$



More on the Gold Arbitrage

- The long arbitrage provides an *upper bound* on gold forward prices while the short arbitrage provides a *lower bound*
- Solving the profit function for the short arbitrage:
$$F(0, T) > S(0) \{1 + i(0, T)\}$$

Notice that $i(0, T)$ is an investing rate and $r(0, T)$ is a borrowing rate



Figure 4.2: Profit Function for a Short Gold Cash-and-Carry Arbitrage

<i>DATE</i>	<i>Cash Position</i>	<i>Futures Position</i>
$t=0$	Borrow Q_G ounces and sell at $S(0)$. Invest the funds received at interest rate $i(0, T)$	Long Q_G ounces at $F(0, T)$
$t=T$	Take delivery of the Q_G units against the maturing futures contract, pay with the proceeds of the investment, $\$[Q_G S(0)] \{1 + i(0, T)\}$, returning the Q_G units to settle the short position	

In this case, the profit function can be specified:

$$\pi(0) = \{S(0)(1 + i(0, T)) - F(0, T)\} Q_G$$



Cash and Carry Arbitrage Condition for Gold

- Under perfect markets assumptions, lending and borrowing rates will be equal ($i = r$) and the short and long arbitrage restrictions reduce to the cash and carry arbitrage condition for gold:

$$F(0, T) = S(0) \{1 + r(0, T)\}$$

Gold forward/ futures prices will be in contango.
Examine the NYMEX/COMEX prices on CME website (also RSD, Fig. 4.1, p.217)



Solving for the Gold Futures-Futures Condition

- In the case of gold, if

$$F(t, T) = S(t) \{1 + r(t, T)\} \text{ and } F(t, N) = S(t) \{1 + r(t, N)\}$$

Then dividing $F(t, T)$ by $F(t, N)$ gives:

$$F(t, T) = F(t, N) \{1 + r(t, T-N)\}$$

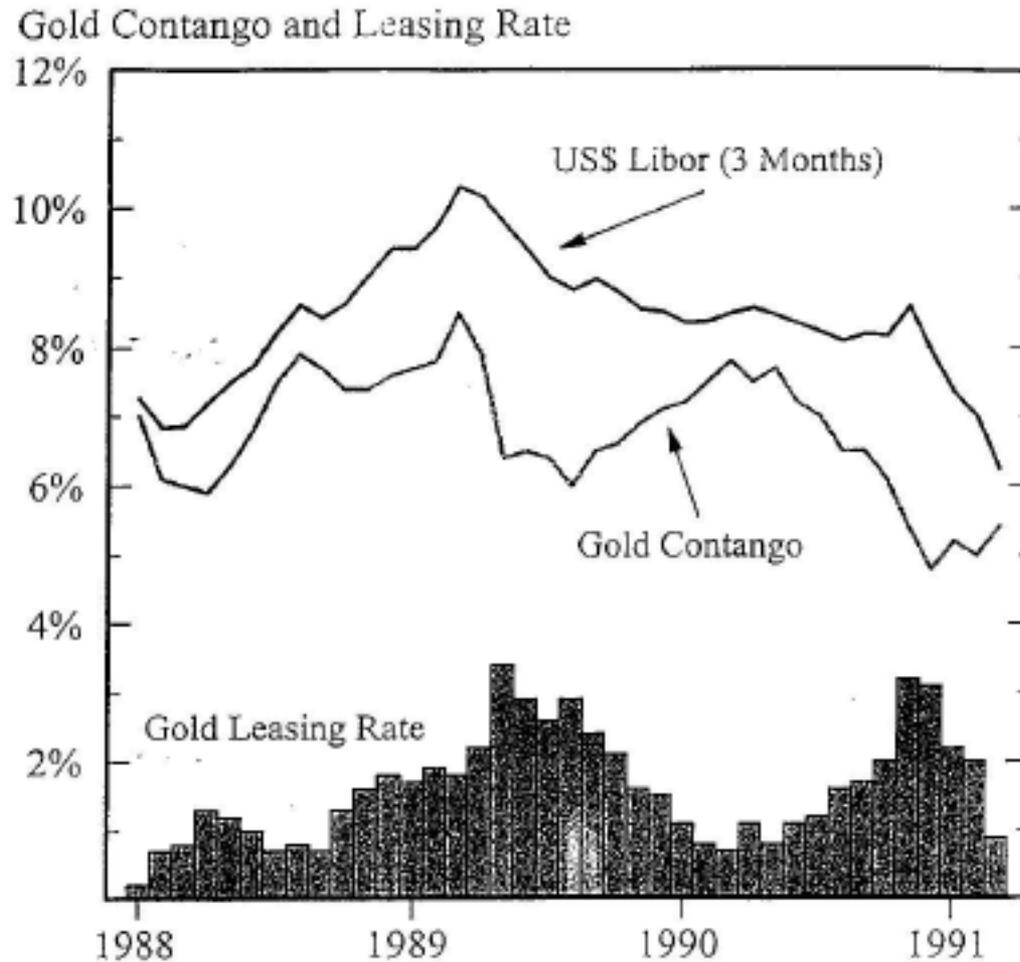
Where:

$$1 + r(t, T-N) = \{1 + r(t, T)\} / \{1 + r(t, N)\}$$

For example if 0 to T is 6 months and 0 to N is 3 months, then $1 + r(t, T-N)$ is a 3 month interest rate that starts at N and matures at T



Illustration of a Golden turtle trade



Source: Consolidated Gold Fields, Gold 1991



The Basic Long Hedger Profit Function

Figure 2.2 Profit Function for a Grain Elevator Hedge using Futures Contracts

<i>DATE</i>	<i>Cash Position</i>	<i>Futures Position</i>
$t=0$	Buy Q_A units of grain at $S(0)$ for storage in grain elevator	Short Q_H units at $F(0,T)$
$t=1$	Q_A units are sold at $S(1)$ and loaded for shipment	Close out position with Long Q_H units at $F(1,T)$

If costs associated with carrying the commodity are ignored, the profit function for this type of hedge can be specified:

$$\pi(1,T) = \{S(1) - S(0)\} Q_A + \{F(0,T) - F(1,T)\} Q_H$$



Extending the Basic Hedger Profit Function to a 1-1 Calendar Spread

Figure 3.3 Profit Function for an One-to-One Intra-commodity Futures Spread Position

<i>DATE</i>	<i>Nearby Position</i>	<i>Deferred Position</i>
$t=0$	Short Q units at $F(0,N)$	Long Q units at $F(0,T)$
$t=1$	Close out position with Long Q units at $F(1,N)$	Close out position with Short Q units at $F(1,T)$

Taking Q to be always positive, the profit function (π) can be specified by observing that the profit for each leg of the spread is equal to the contract selling (short) price minus the purchase (long) price:

$$\begin{aligned}\pi/Q &= \{F(0,N) - F(1,N)\} + \{F(1,T) - F(0,T)\} \\ &= \{F(1,T) - F(1,N)\} - \{F(0,T) - F(0,N)\}\end{aligned}\tag{3.1}$$



Extending to the 1-1 Calendar Spread to the Case where the Position sizes are not equal

Figure 3.4 Profit Function for a General Intra-commodity Futures Spread Position

<i>DATE</i>	<i>Nearby Position</i>	<i>Deferred Position</i>
<i>t=0</i>	Short Q_N units at $F(0,N)$	Long Q_T units at $F(0,T)$
<i>t=1</i>	Close out position with Long Q_N units at $F(1,N)$	Close out position with Short Q_T units at $F(1,T)$

In this case, the profit function can be specified:

$$\pi(1,T) = \{F(0,N) - F(1,N)\} Q_N + \{F(1,T) - F(0,T)\} Q_T \quad (3.4)$$



Solving the Profit Function for the Calendar Spread

($\pi > 0$ for Short N, Long T; $\pi < 0$ for Short T, Long N)

$$\frac{\pi}{Q} = (F(1,T) - F(1,N)) - (F(0,T) - F(0,N))$$

$$\text{where } F(t,T) = F(t,N) (1 + ic(t,T-N))$$

$$\rightarrow ((F(1,N)(1 + ic(1)) - F(1,N)) - ((F(0,N)(1 + ic(0)) - F(0,N))) = F(1,N) ic(1) - F(0,N) ic(0)$$

$$\Delta X = X(1) - X(0) \rightarrow X(1) = X(0) + \Delta X$$

$$\rightarrow F(1,N) ic(1) - F(0,N) ic(0) = (F(0,N) + \Delta F) (ic(0) + \Delta ic) - F(0,N) ic(0)$$

$$(F(0,N) + \Delta F) (ic(0) + \Delta ic) - F(0,N) ic(0)$$

$$= F(0,N) ic(0) + \Delta F ic(0) + F(1,N) \Delta ic - F(0,N) ic(0) = ic(0) \Delta F + F(1,N) \Delta ic$$



Tailed Spread Profit Function

- The tailed spread profit function is:

$$\pi(1) = \{F(0, N) - F(1, N)\} Q_N + \{F(1, T) - F(0, T)\} Q_T$$

- In order to be dollar equivalent on the two legs of the spread the following condition has to be satisfied:
- $Q_N F(0, N) = Q_T F(0, T)$
- To solve this let $Q_T = 1$ to get the result that
- $Q_N = \{F(0, T) / F(0, N)\} = \{1 + ic(0)\}$



Solving for the General Futures-Futures Condition with Implied carry ($ic(t, T)$)

- Gold is unusual in having **no carry return**, either pecuniary or a non-pecuniary convenience yield
- More generally, the relationship between spot and futures/forward prices has both **carry cost** ($cc(t, T)$) and a **carry return** ($cr(t, T)$) $\rightarrow ic(t, T) = cc(t, T) - cr(t, T)$

- ***This produces the general results:***

$$F(t, T) = S(t) \{1 + ic(t, T)\}$$

$$F(t, T) = F(t, N) \{1 + ic(t, T-N)\}$$



Solving the Profit Function

- Substituting the restriction on Q_N into the profit function and using the cash and carry arbitrage condition gives the final form of the profit function:

$$B(1) = F(1, N) \Delta ic$$

Compare to the one-to-one case and observe that the impact of ΔF has been eliminated.

- Note: Some tails are not dollar equivalent (e.g., RSD, p.268)



Application to Tbond futures spreads

Figure 3.3 Profit Function for a Tailed Tbond Spread

DATE	Nearby (N) Position	Deferred Position (T)
$t=0$	Short $[F(0,T)/F(0,N)] Q$ Tbonds at $F(0,N)$	Long Q Tbonds at $F(0,T)$
$t=1$	Long $[F(0,T)/F(0,N)] Q$ at $F(1,N)$	Short Q at $F(1,T)$

From (3.5), the profit function for the short-the-nearby, long-the-deferred tailed Tbond spread takes the form:

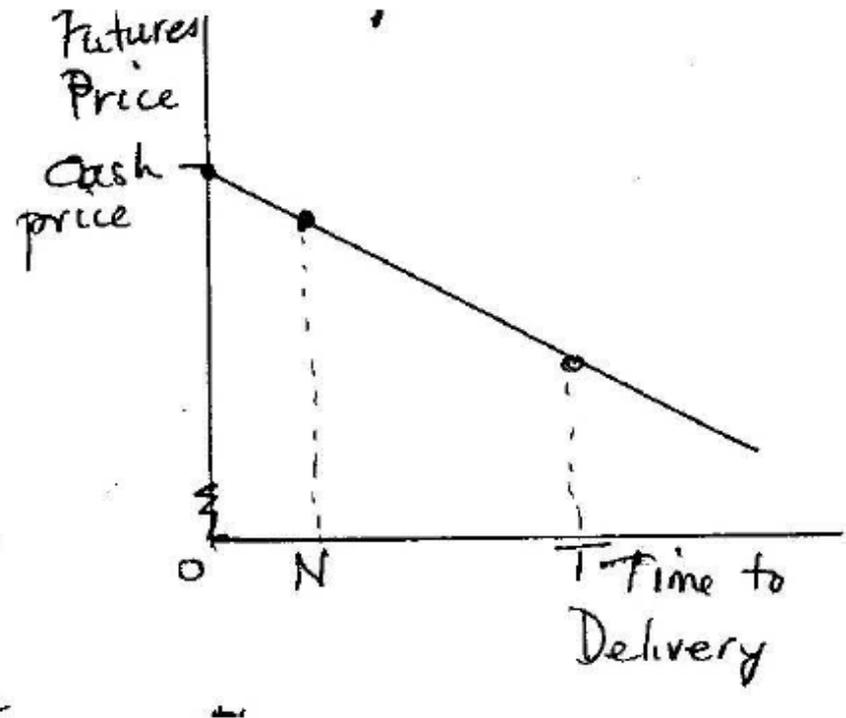
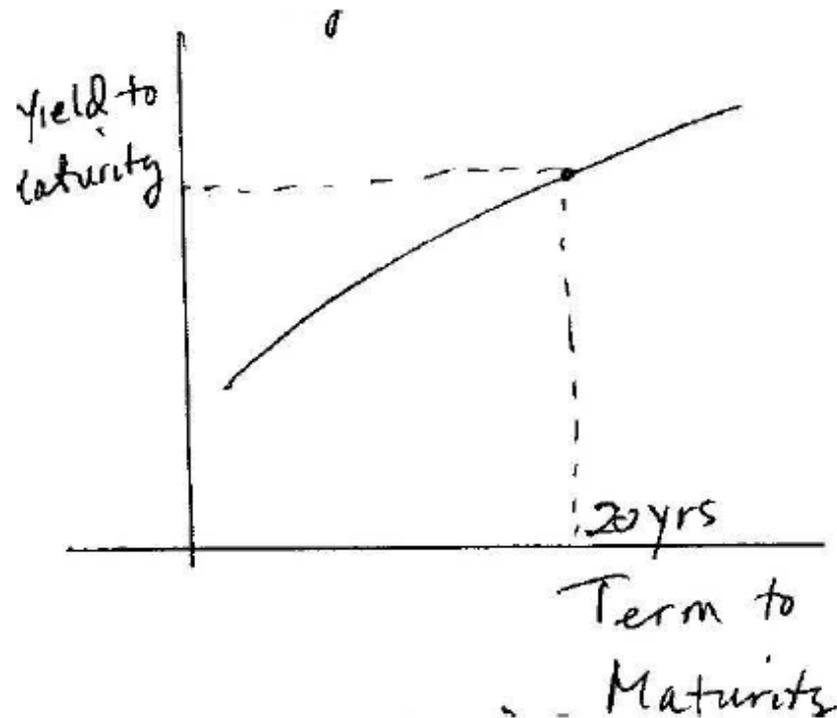
$$\pi(1) = F(1,N) \Delta ic = F(1,N) \{ \Delta irr(N,T) - \Delta R(N,T) \}$$

where irr is the implied repo rate (irr), the repurchase agreement financing rate implied in Tbond futures prices, and R is the return earned on the cash Tbond position during the period between the two delivery dates, N and T . With suitable modification, this type of profit function also applies to all other debt futures contracts.



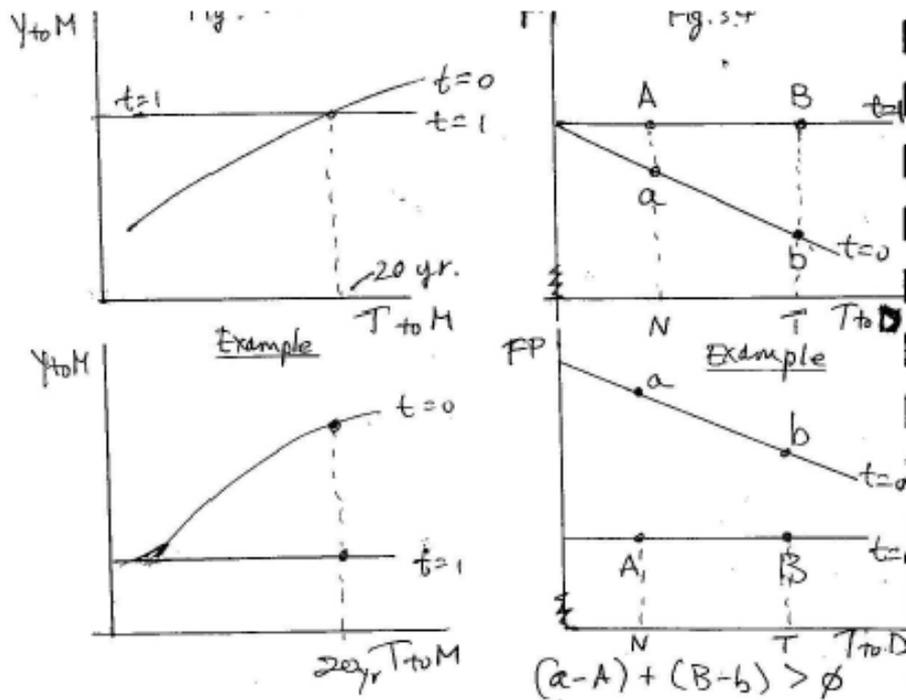
Relation between Tbond term structure of futures prices and the yield curve

Graphs 3.1 and 3.2 The Relationship between the Cash Yield Curve and the Futures Term Structure

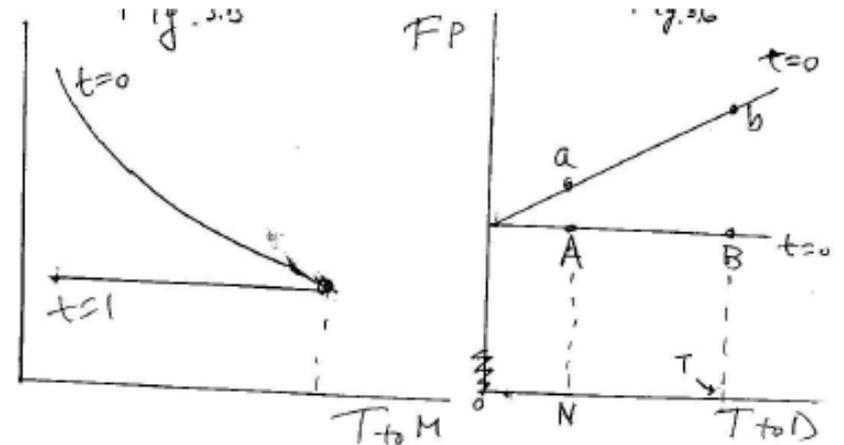


For Tailed Tbond spread the level of the yield curve does not matter only the slope and shape (see RSD chap. 3)

Graphs 3.3 and 3.4 (with examples)



Graphs 3.5 and 3.6



$$(a-A) + (B-b) < 0$$

∴ Long the nearby / Short the Deferred is Preferred trade

Covered Interest Arbitrage

- The cash and carry arbitrage for currencies is given a special name: ***covered interest arbitrage***.
- Covered interest arbitrage is based on the notion that, ***in markets where arbitrage is active and unrestricted, securities that differ only by currency of denomination should exhibit fully hedged returns that are approximately equal***



History and Background

- Historically, forward trading of currencies was usually bundled together with a loan to form a **bill of exchange** – money is borrowed (lent) in one location, in a given currency, and repaid (redeemed) in another location using a different currency. Reading: Poitras (2000, ch.7)
- Reading on covered interest arbitrage:
History, RSD, p.224-6; Arbitrage Trades, **RSD, p.226-40.**
(those unfamiliar with CIP go over example on p.227-8).



A Stylized Example of Covered Interest Arbitrage

Patrick Yamada, a trader in the foreign exchange department of Sanwa Bank, Singapore office, specializes in arbitraging U.S. dollars against Deutschemarks. He observes the following rates at 9:10 am Singapore time:

Spot rate: $\text{DM}1.8200 = \$1.0000$

Three Month Forward Rate: $\text{DM}1.8000 = \$1.0000$

Yamada can borrow or invest U.S. dollars for three months at 9% per annum or Deutschemarks for three months at 5% per annum. He has a borrowing limit of \$5,000,000 or the equivalent in DM.

- Ignoring transactions costs, how can Yamada make a riskless arbitrage profit? Assume that Yamada desires to take any profits in dollars.
- If the dollar three month interest rate on US dollars were 10%, instead of 9%, all other factors remaining the same, would Yamada still make a profit using the strategy outlined in a)? If not, is there another set of transactions which would provide an arbitrage profit?
- If the transactions costs in a) or b) were above \$7000 and were to be paid out of final proceeds, would this change the strategies described in a) or b)?



Solution to Yamada's Stylized Arbitrages

a) Yamada can make an arbitrage profit by doing a *long (DM)* covered interest arbitrage. The arbitrage is short because it involves borrowing in US and investing in DM. This arbitrage involves the following sequence of transactions which will all be executed at 9:10 am Singapore time:

Borrow \$5,000,000 for three months. In three months time, the amount owing on this borrowing will be: $(\$5 \text{ mil})(1 + (.09/4)) = \$5,112,500$

Exchange the \$5 mil. at the spot exchange rate to get $(\$5 \text{ mil})(1.82) = 9.1 \text{ mil DM}$.

Invest the 9.1 mil. DM for three months. In three months time, the investment will mature to a value: $(9.1 \text{ mil})(1 + (.05/4)) = 9,213,750 \text{ DM}$

Sell the maturing value of the DM investment for US dollars using a three month forward exchange contract. At the quoted forward exchange rate of 1.8, the DM investment will produce $(9,213,750/1.8) = \$5,118,750$

In three months time, the DM investment will mature and the proceeds delivered on the forward exchange contract. The proceeds of the forward contract will be used to settle the maturing three month loan producing an arbitrage profit of $\$5,118,750 - \$5,112,500 = \$6250$.

b) If the US interest rate is 10%, instead of 9%, then the cost of the US\$ borrowing would be $(5 \text{ mil})(1 + (.1/4)) = \$5,125,000$. Because this exceeds the covered return which could be received on the DM investment, the short arbitrage would not be profitable. However, in the absence of transactions costs, it would now be possible to do the *long* arbitrage, which would involve borrowing in DM and investing in the US. In this case the profit would be $\$5,125,000 - \$5,118,750 = \$6250$.

c) The presence of a \$7000 transaction cost would prevent either the long or the short arbitrage from being executed. This illustrates the point that covered interest arbitrage only provides upper and lower boundaries on the available combinations of interest rates and exchange rates that are consistent with absence of arbitrage at a specific point in time.

NOTE: In actual practice, the presence of transaction costs dictates that the spot and forward transactions will combined into one transaction, a foreign exchange swap.



Figure 4.4 Selected Foreign Exchange Rates

	CROSS RATES								
	Canadian dollar	U.S. dollar	British pound	German mark	Japanese yen	Swiss franc	French franc	Dutch guilder	Italian lira
Canada dollar	—	1.3797	2.1289	0.8735	0.013610	1.0362	0.2550	0.7777	0.000875
U.S. dollar	0.7248	—	1.5430	0.6331	0.009864	0.7510	0.1648	0.5637	0.000634
British pound	0.4697	0.6481	—	0.4103	0.006393	0.4867	0.1198	0.3653	0.000411
German mark	1.1448	1.5795	2.4372	—	0.015581	1.1853	0.2919	0.8903	0.001002
Japanese yen	73.48	101.37	156.42	64.18	—	76.14	18.74	57.14	0.064291
Swiss franc	0.9651	1.3315	2.0545	0.8430	0.013135	—	0.2461	0.7505	0.000844
French franc	3.9216	5.4106	8.3486	3.4255	0.053373	4.0635	—	3.0498	0.003431
Dutch guilder	1.2858	1.7741	2.7374	1.1232	0.017500	1.3324	0.3279	—	0.001125
Italian lira	1142.86	1576.80	2433.03	998.29	15.554286	1184.23	291.43	888.80	—

Mid-market rates in Toronto at noon, Aug. 8, 1994. Prepared by the Bank of Montreal Treasury Group.



		\$1 U.S. in Cdn.\$ =	\$1 Cdn. in U.S.\$ =
U.S./Canada spot		1.3797	0.7248
1 month forward		1.3808	0.7242
2 months forward		1.3818	0.7237
3 months forward		1.3827	0.7232
6 months forward		1.3862	0.7214
12 months forward		1.3973	0.7157
3 years forward		1.4457	0.6917
5 years forward		1.4917	0.6704
7 years forward		1.5622	0.6401
10 years forward		1.6547	0.6043
Canadian dollar	High	1.3083	0.7644
in 1994:	Low	1.3990	0.7148
	Average	1.3712	0.7293

Country	Currency	Cdn. \$ per unit	U.S. \$ per unit
Britain	Pound	2.1289	1.5430
1 month forward		2.1294	1.5421
2 months forward		2.1297	1.5412
3 months forward		2.1297	1.5402
6 months forward		2.1320	1.5380
12 months forward		2.1385	1.5304
Germany	Mark	0.8735	0.6331
1 month forward		0.8739	0.6329
3 months forward		0.8751	0.6329
6 months forward		0.8787	0.6339
12 months forward		0.8891	0.6363
Japan	Yen	0.013510	0.009864
1 month forward		0.013550	0.009836
3 months forward		0.013728	0.009928
6 months forward		0.013873	0.010008
12 months forward		0.014211	0.010170
Algeria	Dinar	0.0435	0.0316
Antigua, Grenada and St. Lucia	E.C.Dollar	0.5119	0.3711
Argentina	Peso	1.38205	1.00170
Australia	Dollar	1.0213	0.7402
Austria	Schilling	0.12403	0.08990
Bahamas	Dollar	1.3797	1.0000
Barbados	Dollar	0.6933	0.5025
Belgium	Franc	0.04245	0.03077
Bermuda	Dollar	1.3797	1.0000
Brazil	Real	1.509519	1.094092
Bulgaria	Lev	0.0258	0.0187
Chile	Peso	0.003281	0.002378
China	Renminbi	0.1604	0.1162
Cyprus	Pound	2.8744	2.0833
Czech Rep	Koruna	0.0490	0.0355
Denmark	Krona	0.2220	0.1609
Egypt	Pound	0.4076	0.2954

Country	Currency	Cdn. \$ per unit	U.S. \$ per unit
Fiji	Dollar	0.9548	0.6920
Finland	Markka	0.2652	0.1929
France	Franc	0.2550	0.1848
Greece	Drachma	0.00578	0.00419
Hong Kong	Dollar	0.1786	0.1294
Hungary	Forint	0.01258	0.00912
Iceland	Krona	0.01972	0.01429
India	Rupee	0.04397	0.03187
Indonesia	Rupiah	0.000636	0.000461
Ireland	Punt	2.1068	1.5270
Israel	N Shekel	0.4531	0.3284
Italy	Lira	0.000875	0.000634
Jamaica	Dollar	0.04415	0.03200
Jordan	Dinar	1.9852	1.4388
Lebanon	Pound	0.000824	0.000597
Luxembourg	Franc	0.04245	0.03077
Malaysia	Ringgit	0.5345	0.3874
Mexico	N Peso	0.4074	0.2953
Netherlands	Guilder	0.7777	0.5637
New Zealand	Dollar	0.8340	0.6045
Norway	Krone	0.1999	0.1449
Pakistan	Rupee	0.04519	0.03275
Philippines	Peso	0.05276	0.03824
Poland	Zloty	0.0000603	0.0000437
Portugal	Escudo	0.00859	0.00623
Romania	Leu	0.0008	0.0006
Russia	Ruble	0.000661	0.000479
Saudi Arabia	Riyal	0.3679	0.2667
Singapore	Dollar	0.9164	0.6642
Slovakia	Koruna	0.0437	0.0317
South Africa	Rand	0.3821	0.2770
South Korea	Won	0.001719	0.001246
Spain	Peseta	0.01062	0.00770
Sudan	Dinar	0.0445	0.0322
Sweden	Krona	0.1787	0.1295
Switzerland	Franc	1.0362	0.7510
Taiwan	Dollar	0.0524	0.0380
Thailand	Baht	0.0553	0.0401
Trinidad, Tobago	Dollar	0.2475	0.1794
Turkey	Lira	0.0000441	0.0000320
Venezuela	Bollivar	0.00812	0.00589
Zambia	Kwacha	0.002090	0.001515
European Currency Unit		1.5701	1.2105
Special Drawing Right		1.9950	1.4460

The U.S. dollar closed at \$1.3772 in terms of Canadian funds, down \$0.0095 from Friday. The pound sterling closed at \$2.1201, down \$0.0182.

In New York, the Canadian dollar closed up \$0.0050 at \$0.7261 in terms of U.S. funds. The pound sterling was down \$0.0026 to \$1.5394.

see:

Mail, Monday, August 8, 1994.

[o first page](#)



Exchange Rates and the Domestic Country

- To implement the **covered interest parity condition** it is necessary to identify which country is the domestic and which is the foreign. (See RSD, Fig. 4.2)
- Observe that FX rates can be quoted in ratio form either as $\$US/\$C = .7497$ or $\$C/US\$ = 1.3339$ – the currency on top in the FX ratio is the domestic currency
- Note: different methods are used to quote the FX rate (e.g., in East Asia the convention is reversed).



Figure 4.9 Money Market Interest Rates

MONEY RATES

ADMINISTERED RATES	
Bank of Canada	5.70%
Canadian prime	7.25%
MONEY MARKET RATES	
(for transactions of \$1-million or more)	
3-mo. T-bill(when-issued)	5.58%
1-month treasury bills	5.21%
2-month treasury bills	5.40%
3-month treasury bills	5.50%
6-month treasury bills	6.10%
1-year treasury bills	7.20%
10-year Canada bonds	9.03%
30-year Canada bonds	9.18%
1-month banker's accept.	5.46%
2-month banker's accept.	5.56%
3-month banker's accept.	5.61%
Commercial Paper (R-1 Low)	
1-month	5.60%
2-month	5.68%
3-month	5.73%
Call money	5.25%

Supplied by Dow Jones
Telerate Canada

UNITED STATES
NEW YORK (AP) — Money rates for Monday as reported by Telerate Systems Inc:
Telerate interest rate index: 4.820
Prime Rate: 7.25
Discount Rate: 3.50
Broker call loan rate: 6.00
Federal funds market rate: High 4.375, low 4.3125, last 4.3125
Dealers commercial paper: 30-180 days: 4.48-5.15
Commercial paper by finance company: 30-270 days: 4.43-4.71
Bankers acceptances dealer indications: 30 days, 4.45; 60 days, 4.64; 90 days, 4.77; 120 days, 4.86; 150 days, 5.05; 180 days, 5.12
Certificates of Deposit Primary: 30 days, 3.40; 90 days, 3.85; 180 days, 4.23

Certificates of Deposit by dealer: 30 days, 4.47; 60 days, 4.67; 90 days, 4.80; 120 days, 4.91; 150 days, 5.12; 180 days, 5.21

Eurodollar rates: Overnight, 4.25-4.375; 1 month, 4.50-4.5625; 3 months, 4.8125-4.875; 6 months, 5.25-5.3125; 1 year, 5.75-5.8125

London Interbank Offered Rate: 3 months, 4.75; 6 months, 5.1875; 1 year, 5.5625

Treasury Bill auction results: average discount rate: 3-month as of Aug. 8: 4.43; 6-month as of Aug. 8: 4.93

Treasury Bill, annualized rate on weekly average basis, yield adjusted for constant maturity, 1-year, as of Aug. 1: 5.51

Treasury Bill market rate, 1-year: 5.29-5.27

Treasury Bond market rate, 30-year: 7.53



The Short Arbitrage Condition

- The **short** condition refers to borrowing in the domestic currency and lending offshore, fully covering the currency exposure.

The arbitrage profit function for the short arbitrage:

$$B_s(0) = F(0, 1)\{Q/S(0)\}(1+r^*) - Q(1+r) < 0$$

Here, r is a borrowing rate and r^* is a lending rate.



Figure 4.5: Short Covered Interest Arbitrage Trade

At $t=0$

US asset	Exchange Market	Foreign (Canadian) asset
Borrow $\$Q$ for 1 year at $r(0,1)$	Buy $\$Q/S(0)$ Canadian dollars, spot	Invest $\$Q/S(0)$ for 1 year at $r^*(0,1)$
	Sell $(\$Q/S(0))(1+r^*(0,1))$ Canadian dollars forward at $F(0,1)$	

At $t=1$ Use the funds from the maturing foreign asset to settle the forward exchange position by paying the foreign currency and receiving US dollars. Use these dollars to settle the US dollar loan.

where: $F(0,1)$ = the 1 year forward exchange rate in US direct terms; $S(0)$ = the spot exchange rate in US direct terms; $r(0,1)$ = the domestic (US) interest rate on a 1 year zero coupon security (quoted on a 365 day basis); $r^*(0,1)$ = the foreign (Canadian) one year interest rate (quoted on a 365 day basis).



The Long Arbitrage Condition

- The long arbitrage involves borrowing offshore and investing domestic, fully covering the currency exposure.

- The long arbitrage condition is:

$$B_L(0) = Q (1+y) - F(0,1)\{Q/S(0)\}(1+y^*) < 0$$

Here y is an investing rate and y^* is a borrowing rate.



Foreign Exchange Swaps

- The discussion of covered interest arbitrage assumes that the trader will do a spot exchange transaction and (simultaneously) a forward exchange transaction to cover the future currency exposure
- As discussed in Lecture 2 (see RSD, Fig. 1.4) in practice, to reduce transactions costs, these two trades are done as one trade known as a **foreign exchange swap**.



Figure 4.7: Short Forward-Forward Arbitrage

At t=0

US (Domestic) Market

Borrow $\$Q(1+r^*(0,N))$
at $r(0,T)$

Invest $Q\$(1+r^*(0,N))$
at $r(0,N)$

Exchange Market

Sell US\$ forward
 $Q\$(1+r^*(0,N))(1+r(0,N))$
at $F(0,N)$

Buy US\$ forward
 $(\$Q/F(0,N))(1+r(0,N))(1+r^*(0,T))$
at $F(0,T)$

Foreign Market

Borrow $(\$Q/F(0,N))(1+r(0,N))$
at $r^*(0,N)$

Invest $(\$Q/F(0,N))(1+r(0,N))$
at $r^*(0,T)$

At t=N

The US investment will mature to give $\$Q(1+r^*(0,N))(1+r(0,N))$ that is used to deliver on the forward position that matures at $t=N$. The amount of foreign currency received will be $(\$Q/F(0,N))(1+r^*(0,N))(1+r(0,N))$ which is the amount owing on the foreign borrowing maturing at $t=N$. *The cash flows at $t=N$ all cancel.*

At t=T

The T period foreign investment will mature to $(\$Q/F(0,N))(1+r(0,N))(1+r^*(0,T))$. This amount is delivered against the forward contract to obtain US\$ which can be used to settle the loan. The resulting US\$ cash flow will have to be less than or equal to the maturing value of the US\$ T period loan in order to ensure absence of arbitrage opportunities.



Covered Interest Parity

- Observing that the long (short) arbitrage condition bounds the forward rate above (below), imposing perfect market assumptions produces the **covered interest parity condition**:

$$F(0, T) = \frac{1 + r(0, T)}{1 + r^*(0, T)} S(0)$$

- Exercise: Solve this equation for $r(0, T)$ on the lhs to provide a more revealing connection to the terminology 'covered interest parity'.



Solving for the Covered Parity Condition

The solution demonstrates that the domestic interest rate ($r(0, T)$) is equal to the fully hedged foreign rate interest rate ($r^*(0, T)$) where the second term on rhs is the cost of hedge

$$F(0, T) = \frac{1 + r(0, T)}{1 + r^*(0, T)} S(0) \quad \rightarrow \quad \frac{F(0, T)}{S(0)} = \frac{1 + r(0, T)}{1 + r^*(0, T)}$$

$$\frac{F(0, T)}{S(0)} - 1 = \frac{1 + r(0, T)}{1 + r^*(0, T)} - 1 \quad \rightarrow \quad \frac{F(0, T)}{S(0)} - 1 = \frac{1 + r(0, T)}{1 + r^*(0, T)} - \frac{1 + r^*(0, T)}{1 + r^*(0, T)}$$

$$\frac{F(0, T)}{S(0)} - 1 = \frac{r(0, T) - r^*(0, T)}{1 + r^*(0, T)} = \frac{F(0, T) - S(0)}{S(0)}$$

$$\rightarrow \quad r(0, T) = r^*(0, T) + (1 + r^*(0, T)) \frac{F(0, T) - S(0)}{S(0)}$$



What is the $ic(0, T-N)$ for CIP?

$$\begin{aligned} F(0, T) &= \frac{1 + r(0, T-N)}{1 + r^*(0, T-N)} F(0, N) = \left(1 + \frac{1 + r(0, T-N)}{1 + r^*(0, T-N)} - 1 \right) F(0, N) \\ &= \left(1 + \frac{1 + r(0, T-N)}{1 + r^*(0, T-N)} - \frac{1 + r^*(0, T-N)}{1 + r^*(0, T-N)} \right) F(0, N) \\ &= \left(1 + \frac{r - r^*}{1 + r^*(0, T-N)} \right) F(0, N) = (1 + \theta) F(0, N) \end{aligned}$$



Contango versus Backwardation in Currency Future and Forwards

$$\text{For } F(0, T) = (1 + \beta) F(0, T)$$

If $r(0, T-N) > r^*(0, T-N)$ then $\beta > 0$ and futures price term structure is in contango for CME futures prices (where US is the domestic)

Example: US rates are above Yen rates

If $r(0, T-N) < r^*(0, T-N)$ then $\beta < 0$ and futures price term structure is in backwardation

Example: US rates are below Aussie rates



Australian Dollar Futures Settlements

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Trade Date: Wednesday, 31 May 2017 (Final)

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Month	Open	High	Low	Last	Change	Settle	Estimated Volume	Prior Day Open Interest
JUN 17	.7458	.7473	.7422	.7430	-.0031	.7432	88,031	125,431
JUL 17	.7440	.7469B	.7420A	.7425A	-.0031	.7429	56	49,000
AUG 17	.7445	.7466B	.7418A	.7422A	-.0031	.7426	14	31,000
SEP 17	.7450	.7463B	.7413	.7419B	-.0030	.7423	953	2,950
OCT 17	-	-	-	-	-.0030	.7421	0	0
DEC 17	.7413	.7449B	.7409A	.7413	-.0030	.7415	78	15,000
MAR 18	-	-	.7412A	.7412A	-.0030	.7408	0	0
JUN 18	-	-	-	-	-.0030	.7402	0	1,000
SEP 18	-	-	-	-	-.0030	.7396	0	0
DEC 18	-	-	-	-	-.0029	.7391	0	0
MAR 19	-	-	-	-	-.0029	.7385	0	0
JUN 19	-	-	-	-	-.0028	.7379	0	0
SEP 19	-	-	-	-	-.0028	.7370	0	0
DEC 19	-	-	-	-	-.0028	.7360	0	0



Sample Final Question

- On March 1, 1990 the spot and 3 month forward rates for the Canadian dollar (per US dollar) were \$1.1922 and \$1.2072 respectively. What "risk-free" discount rate on U.S. dollar instruments would be consistent with the interest-rate-parity theorem if the 3 month (annualized) risk-free rate on Canadian dollar instruments was 13.10%?
- Use the CIP equation to solve. Remember that US discount rates have to be converted from the true yield (this is BUS 315).



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