

# Lecture 5

## Review of Fixed Income Concepts

- Types of Fixed Income Securities
- Basic Fixed Income Calculations, How to Price Perpetuities, Mortgages and Bonds
- Review of Fixed Income Concepts: How to calculate spot and implied forward interest rates
- Introduction to Duration and Convexity
- Basics of Credit and Default Risk



# *Types of Securities and Markets*

- Reading: SAIS, sec. 4.1, 4.2 and 4.3
- Various classification schemes are available:
  - by types of issuer, e.g., corporate vs. government;
  - by country of issue, e.g., national vs. international;
  - by credit rating, e.g., investment grade vs. speculative;
  - classification by features, e.g., callable, puttable, convertible, and tax status;
  - by term to maturity, e.g., bond market vs. money market.



## Notes to the .zip files

- The BUS417 webpage has links to a number of .zip files with data on fixed income (Lecture 4). The size of many files makes it difficult to load into the slides
- **Bond Info:** Information about bonds that are available for trading at an on-line brokerage. CIBC-Fixed\_Income is the index file.
- **More Bond Info:** Has two publications on Canadian housing finance and Trust bonds; Trade info for IPO's and a few example pages from Moody's bond book.
- **Bond Market Info:** Has statistics for the US and Canadian bond markets.
- **Mortgage Info:** Has three publications (2 US and 1 Canada) on the housing market and an info sheet on mortgage insurance,



**Security: US Fixed Income Securities**

**Series: Outstanding**

**Units: \$ Billion**

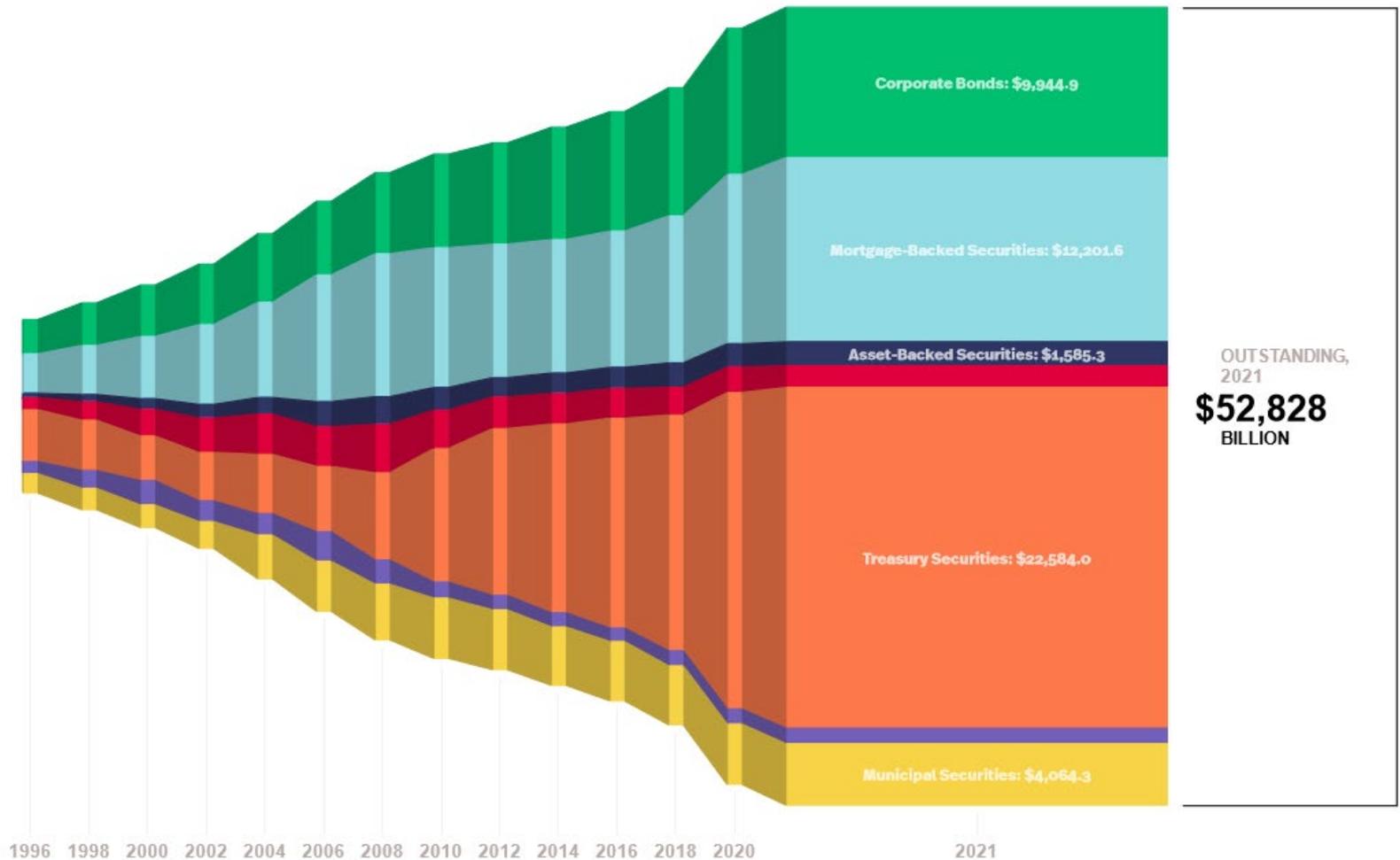
Source: Bloomberg, The Federal Reserve, US Agencies, US Treasury

Note: UST - US Treasury securities; interest bearing marketable coupon public debt. MBS - mortgage-backed securities; GNMA, FI

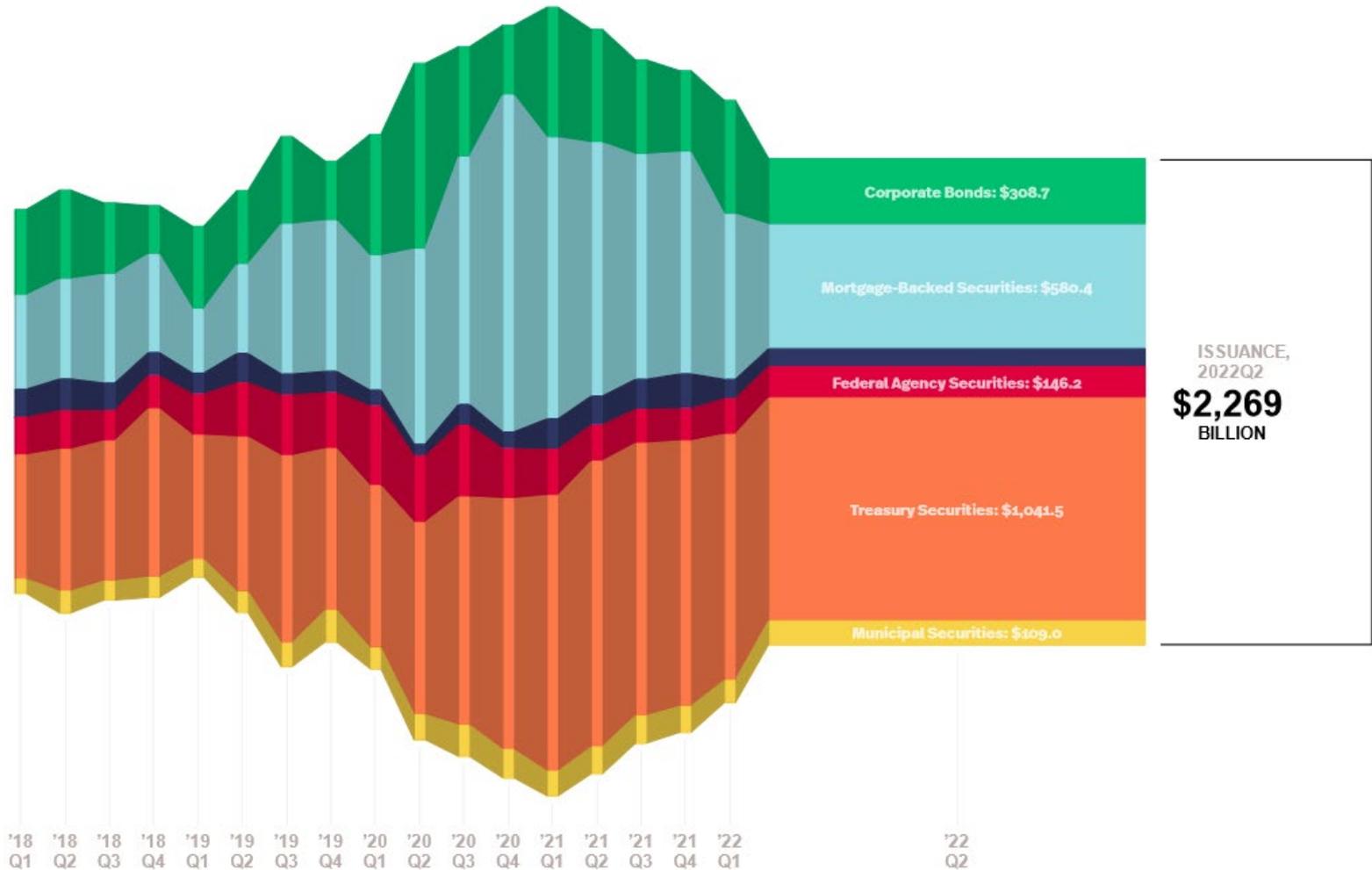
	US1	MBS	Corporates	Munis	Agency	ABS	MMs	Total
1980	623.2	111.4	466.0	399.4	164.3	n/a	163.8	1,928.2
1981	720.3	127.0	496.1	443.7	194.5	n/a	215.1	2,196.7
1982	881.5	177.1	541.8	508.0	208.8	n/a	226.8	2,544.1
1983	1,050.9	248.3	582.0	575.1	209.3	n/a	253.8	2,919.4
1984	1,247.4	302.9	658.7	650.6	240.4	n/a	305.7	3,405.7
1985	1,437.7	399.9	785.6	859.5	261.0	1.3	358.5	4,103.5
1986	1,619.0	614.7	971.4	920.4	276.6	11.8	384.9	4,798.8
1987	1,724.7	816.0	1,086.5	1,012.0	308.3	18.6	437.9	5,404.0
1988	1,821.3	973.6	1,202.2	1,080.0	370.7	26.2	513.4	5,987.3
1989	1,945.4	1,192.7	1,299.7	1,129.8	397.5	37.6	579.2	6,582.0
1990	2,180.8	1,340.1	1,354.9	1,178.6	421.5	67.2	609.9	7,153.0
1991	2,456.6	1,577.1	1,455.7	1,272.1	421.5	92.6	565.9	7,841.6
1992	2,739.1	1,774.3	1,554.4	1,295.4	462.4	117.3	579.0	8,522.0
1993	2,974.5	2,209.0	1,783.6	1,361.7	550.8	133.3	580.0	9,593.0
1994	3,111.0	2,352.9	1,937.6	1,325.8	727.7	162.8	623.5	10,241.3
1995	3,292.2	2,432.1	2,092.7	1,268.2	924.0	215.7	700.4	10,925.2
1996	3,444.7	2,606.4	2,264.6	1,261.6	925.8	298.2	803.0	11,604.3
1997	3,441.8	2,871.8	2,474.6	1,318.5	1,021.8	394.9	979.4	12,502.8
1998	3,340.5	3,243.4	2,814.3	1,402.7	1,302.1	478.0	1,172.6	13,753.5
1999	3,266.0	3,832.2	3,158.7	1,457.1	1,620.0	583.5	1,402.4	15,319.9
2000	2,951.9	4,119.3	3,401.2	1,480.7	1,853.7	701.9	1,614.0	16,122.7
2001	2,968.0	4,711.0	3,810.6	1,603.4	2,157.4	812.0	1,474.0	17,536.3
2002	3,205.3	5,289.4	3,996.5	1,762.8	2,377.7	904.8	1,374.9	18,911.4
2003	3,575.2	5,714.5	4,344.8	1,900.4	2,626.2	995.3	1,292.9	20,449.3
2004	3,945.8	6,301.7	4,540.5	2,888.1	2,700.6	1,100.2	1,399.1	22,874.1
2005	4,170.0	7,218.1	4,603.6	3,109.2	2,616.0	1,281.4	1,644.2	24,642.5
2006	4,328.0	8,389.9	4,869.5	3,295.3	2,631.7	1,656.9	1,958.4	27,129.6
2007	4,522.6	9,386.0	5,275.4	3,563.1	2,905.9	1,963.5	1,788.9	29,405.3
2008	5,783.6	9,467.4	5,366.5	3,677.4	3,208.5	1,829.5	1,599.8	30,932.7
2009	7,260.6	9,352.5	5,543.6	3,864.1	2,726.3	1,712.1	1,138.0	31,597.2
2010	8,853.0	9,258.4	6,192.6	3,981.6	2,537.5	1,507.8	1,057.6	33,388.5
2011	9,928.4	9,075.5	6,264.5	3,944.4	2,328.9	1,359.0	969.3	33,868.0
2012	11,046.1	8,838.1	6,706.8	3,953.2	2,095.8	1,280.3	952.3	34,872.7
2013	11,854.4	8,742.6	7,093.4	3,891.1	2,058.3	1,285.7	951.6	35,877.2
2014	12,504.8	8,842.0	7,429.4	3,854.4	2,028.7	1,349.4	930.4	36,939.2
2015	13,191.6	8,894.8	7,636.7	3,874.4	1,995.4	1,376.6	941.5	37,911.0
2016	13,908.2	9,023.2	7,910.5	3,923.0	1,971.7	1,391.8	894.9	39,013.2
2017	14,468.8	9,304.5	8,249.1	3,939.2	1,934.7	1,457.9	965.9	40,320.1
2018	15,608.0	9,732.3	8,481.0	3,994.0	1,841.6	1,615.6	996.0	42,168.5
2019	16,673.3	10,229.5	8,783.1	3,904.1	1,726.2	1,663.2	1,045.2	44,024.6
2020	20,973.1	11,214.0	9,684.6	3,988.2	1,688.6	1,535.5	986.9	50,071.0
2021	22,584.0	12,201.6	9,944.9	4,064.3	1,433.3	1,585.3	1,014.2	52,827.6
1Q20	17,154.0	10,296.0	9,162.6	3,910.7	2,049.1	1,555.1	1,088.8	45,216.2
2Q20	19,898.7	10,698.3	9,629.9	3,831.0	1,834.2	1,599.8	1,006.6	48,598.6
3Q20	20,367.7	10,906.8	9,631.3	3,965.8	1,747.9	1,572.4	956.9	49,148.8
4Q20	20,973.1	11,214.4	9,684.6	3,988.2	1,688.6	1,535.5	986.9	50,071.3
1Q21	21,380.8	11,452.1	9,772.5	4,014.7	1,603.9	1,509.1	1,105.2	50,838.3
2Q21	21,733.0	11,671.0	9,914.5	4,037.9	1,521.3	1,514.7	1,085.4	51,477.8
3Q21	21,872.6	11,905.5	9,934.6	4,055.0	1,446.8	1,475.6	1,071.4	51,761.4
4Q21	22,584.0	12,201.6	9,944.9	4,064.3	1,433.3	1,585.3	1,014.2	52,827.6
1Q22	23,280.1	n/a	10,020.4	4,041.3	1,463.5	n/a	1,102.0	n/a



# US Fixed Income Outstanding, 2021-Q4, 2 Year Intervals (SIFMA)



# US Fixed Income Issuance, up to 2022-Q2



## Global Fixed Income Markets Outstanding – Value

\$ Billions

	Australia	Canada	China	EU	HK	Japan	Singapore	UK	US	DM	EM	Total
2007	1,251.4	1,542.9	1,704.5	19,303.5	108.2	9,852.1	166.3	4,706.8	26,154.9	601.1	808.9	66,200.7
2008	1,145.3	1,369.7	2,228.0	20,246.4	105.6	12,329.9	167.6	4,098.4	28,038.7	602.5	749.0	71,081.1
2009	1,529.5	1,671.6	2,577.9	24,098.0	154.2	12,268.5	185.0	5,280.3	28,684.8	775.9	840.7	78,066.5
2010	1,749.0	1,866.6	3,065.3	23,312.7	186.7	14,604.1	219.0	5,209.8	29,765.3	833.3	991.9	81,803.7
2011	1,879.8	1,963.4	3,528.4	23,453.7	214.0	15,809.1	240.9	5,587.2	30,506.7	837.2	1,001.7	85,022.1
2012	2,045.6	2,173.5	4,294.1	24,301.4	253.4	14,253.3	294.8	5,928.8	31,695.6	951.2	1,140.0	87,331.7
2013	1,885.9	2,241.9	4,960.8	24,993.7	306.8	11,906.2	331.4	5,851.0	32,682.1	960.9	1,100.9	87,221.6
2014	1,923.8	2,206.1	5,835.4	22,097.1	345.2	10,551.7	349.3	6,169.5	33,705.6	890.5	1,150.4	85,224.5
2015	1,827.2	2,767.5	7,752.5	19,884.9	374.1	10,633.4	322.3	5,891.4	34,712.7	858.0	1,199.2	86,223.2
2016	1,825.7	2,943.5	9,408.8	19,381.9	405.8	11,239.5	326.4	5,439.2	36,061.9	936.0	1,272.1	89,240.6
2017	1,997.8	3,267.4	11,757.1	22,133.1	474.0	11,912.1	412.2	6,078.4	37,268.9	1,034.1	1,497.8	97,832.9
2018	1,892.9	3,151.0	12,906.9	21,440.5	513.5	12,461.7	441.1	5,787.1	38,816.6	1,002.3	1,501.9	99,915.5
2019	1,943.0	3,371.7	14,725.9	21,790.4	534.5	12,816.5	492.9	6,331.0	40,534.2	1,110.9	1,599.6	105,250.5
2020	2,490.5	3,913.5	18,555.6	25,286.2	559.1	14,669.9	542.8	7,252.8	46,444.2	1,323.9	1,758.9	122,797.3
2021	2,408.1	4,041.9	21,816.4	24,657.9	582.4	13,371.1	613.4	7,189.7	49,106.3	1,337.8	1,740.5	126,865.5
Average	1,853.0	2,566.1	8,341.2	22,425.4	341.2	12,578.6	340.4	5,786.8	34,945.2	937.0	1,223.6	91,338.5
Y/Y % Change	-3.3%	3.3%	17.6%	-2.5%	4.2%	-8.9%	13.0%	-0.9%	5.7%	1.0%	-1.1%	3.3%
5-Year CAGR	3.8%	4.3%	13.2%	2.2%	4.2%	2.3%	8.3%	3.4%	5.7%	5.3%	3.0%	5.3%
10-Year CAGR	1.6%	6.4%	17.6%	0.1%	8.7%	-0.6%	7.6%	1.9%	4.5%	3.5%	4.3%	3.8%



Bonds outstanding: Government of Canada, provincial, municipal, corporate, and other bonds  
 Encours des obligations : Gouvernement canadien, provinces, municipalités, sociétés et autres emprunteurs

Millions of dollars En millions de dollars

Gross amount as at 31 December Montants bruts au 31 décembre

2006 2007 2008 2009 2010 2011 2012 2013 2014 2015 2016  
 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015 2016

Government of Canada direct													Gouvernement canadien (obligations émises)
Canadian dollars only	270,024	259,905	271,764	355,790	404,348	443,227	458,929	476,995	478,901	491,921	526,608	Dollars canadiens seulement	
Other currencies	8,822	7,577	2,627	3,374	5,700	5,743	8,646	9,528	11,222	18,598	18,447	Monnaies étrangères	
Provincial direct and guaranteed													Provinces (obligations émises ou garanties)
Canadian dollars only	296,558	316,667	324,355	349,693	381,554	419,357	456,662	506,236	532,883	577,549	617,582	Dollars canadiens seulement	
Other currencies	108,846	91,140	107,612	111,824	120,900	130,629	130,396	134,725	142,761	151,084	138,047	Monnaies étrangères	
Municipal direct and guaranteed													Municipalités (obligations émises ou garanties)
Canadian dollars only	36,597	38,824	40,089	43,292	48,517	53,203	57,572	60,823	60,842	64,179	64,103 k	Dollars canadiens seulement	
Other currencies	2,110	1,800	2,037	1,741	1,522	1,440	1,432	1,393	1,194	978	790	Monnaies étrangères	
Corporate													Sociétés
Financial													Financières
Canadian dollars only	143,057	159,377k	168,563k	164,835k	181,454k	184,790k	204,478k	229,774k	247,386k	257,314k	266,340 k	Dollars canadiens seulement	
Other currencies	99,035	105,640	139,398	121,662	112,489	137,535	158,973	227,496	285,982	383,955	416,609	Monnaies étrangères	
Non-financial													Non financières
Canadian dollars only	122,048	124,582	127,103	145,345	163,548	183,054	202,131	228,264k	246,370k	260,720	273,131 k	Dollars canadiens seulement	
Other currencies	118,472	107,990	146,849	145,467	148,740	156,045	170,196	188,817	238,684	304,960	304,145	Monnaies étrangères	
Institutions													Institutions
Canadian dollars only	3,088	3,248	3,108	3,356	4,486	5,099	5,167	5,015	4,987	5,073	5,603	Dollars canadiens seulement	
Other currencies	-	-	-	-	-	-	-	-	-	-	-	Monnaies étrangères	
Foreign debtors													Emprunteurs étrangers
Canadian dollars only	39,700	69,495	67,535	59,466	59,241	58,068	55,422	56,952	55,711k	55,356k	53,166 k	Dollars canadiens seulement	
Term securitizations													Titrisation à terme
Canadian dollars only	177,687	226,207	308,668	373,926	398,104	436,128	454,469	468,762	485,702	492,557	512,797	Dollars canadiens seulement	
Total													Total
Canadian dollars only	1,088,760	1,198,304 k	1,311,184 k	1,495,703 k	1,641,251 k	1,782,927 k	1,894,831 k	2,032,821 k	2,112,782 k	2,204,668 k	2,319,331 k	Dollars canadiens seulement	
Other currencies	337,285	314,148	398,524	384,068	389,350	431,393	469,644	561,959	679,843	859,573	878,036	Monnaies étrangères	



# Types and Amounts of Fixed Income

- Outstanding Level of US Public and Private Debt
  - A breakdown by type of issuer reveals the relative importance of mortgage related issues. This category and the corporate debt category are the two largest and are the two categories most characterized by issues with embedded options.
  - See 'bond market info' file on webpage
    - Data is from SIFMA (link on Fixed Income on links page)
  - Canadian data from Bank of Canada in bfs.pdf in this .zip file
    - Table K8 is only published occasionally
- Summary of Treasury Securities, Maturity Distribution and Average Length ...
  - See 'SIFMA factbook' file on webpage (from SIFMA)
    - Data also available on Bureau of Public Debt (see links page)
  - The STRIPS program and pricing of zeroes



## Trading and Researching Bonds

- Bond prices and information on specific issues is not as readily accessible as common stocks
  - There are specialized fee-service firms that provide detailed information
  - Liquidity in the bond market concentrates on primary issues (unlike the stock market that is mostly concerned with secondary issues)
    - Easiest to get quotes and trade for investment grade issues
    - Possible to gather information about the bond issues of specific Canadian companies – using SEDAR prospectus info + Annual Reports



# Types of Special Contingency Features

- *Call feature*
- *Sinking fund provision*
- *Put feature*
- *Extendible feature*
- *Convertibility*
- *Warrant bond structure*
- *Floating rate provision*
- *Cap, floor and collar features*



# Canadian versus US Mortgage (1)

- Basic Mortgage Characteristics
  - Term to Maturity and Amortization Period
    - **Amortization Period** refers to the time required to fully pay the mortgage
      - In US 30 year amortization conventional
      - In Canada 25 year amortization (this changes depending on Dept. of Finance, OSFI and CMHC)
    - The **Term to Maturity** (Term) determines the time for which the mortgage contract applies
      - In US possible to have a 30 year mortgage term = amortization period – mortgage payment fixed over the of the mortgage contract
      - In Canada, term < amortization period, mortgage payment resets at end of term



# Basic Overview of US vs. Canadian Mortgages

Table 1. Comparison of US and Canadian Mortgage Contracts.

	<b>Prepayment Penalty</b>	<b>Default Recourse</b>	<b>Main Funding Source</b>
US Contract	No Penalty: Not Assumable	Depends on State Usually No Recourse	Government Sponsored Enterprises (GNMA, FNMA, Freddie Mac)
Canadian Contract	Loss of Interest: Assumable	Full Recourse	Depository Institutions
	<b>Benchmark Term</b>	<b>Benchmark Amortization</b>	<b>Floating Rate Benchmark</b>
US Contract	30 Year	30 Year	ARM: Fixed Reset Date
Canadian Contract	5 Year	25 Year	Variable Rate: Prime Rate Reset



Fixed Rate Mortgages	Term	Rate	Special Offers <sup>2</sup>
<p><b>CIBC Fixed Rate Closed Mortgage</b></p> <p>The security of a fixed interest rate, so you always know exactly what your payments will be.</p>	1 year	2.84%	<a href="#">Ask for details</a>
	2 years	2.79%	<a href="#">Ask for details</a>
	3 years	3.39%	2.44% <sup>i</sup>
	4 years	4.09%	<a href="#">Ask for details</a>
	5 years	4.79%	<a href="#">Ask for details</a>
	7 years	5.85%	3.54% <sup>ii</sup>
	10 years	6.09%	<a href="#">Ask for details</a>
<p><b>CIBC Fixed Rate Open Mortgage</b></p> <p>The security of a fixed interest rate and the flexibility to pay off as much of your mortgage as you want, when you want.</p>	6 months	6.95%	
	1 year	6.35%	
<p><b>CIBC Convertible Mortgage</b></p> <p>A short-term closed mortgage with a fixed interest rate and the flexibility to convert to a long-term closed mortgage at any time.</p>	6 months	3.14%	
Variable Rate Mortgages	Term	Rate	Special Offers
<p><b>CIBC Variable Flex Mortgage<sup>®</sup></b></p>	3 years	2.70%	<a href="#">Ask for details</a>



<b>Product</b>	<b><u>Interest Rate</u></b>	<b><u>APR</u></b>
<b><u>Conforming</u></b> and <b><u>Government Loans</u></b>		
<b>30-Year Fixed Rate</b>	3.625%	3.649%
<b>30-Year Fixed-Rate FHA</b>	3.500%	4.532%
<b>30-Year Fixed-Rate VA</b>	3.250%	3.525%
<b>15-Year Fixed Rate</b>	2.875%	2.944%
<b>7/1 ARM</b>	3.250%	3.432%
<b>5/1 ARM</b>	3.000%	3.383%
<b>5/1 ARM VA</b>	2.875%	2.746%
<b><u>Jumbo Loans</u></b> - Amounts that exceed conforming loan limits		
<b>30-Year Fixed-Rate Jumbo</b>	3.375%	3.381%
<b>15-Year Fixed-Rate Jumbo</b>	3.250%	3.261%
<b>7/1 ARM Jumbo</b>	2.750%	3.161%



# Canadian versus US Mortgage (2)

## □ Prepayment penalties

- US Mortgages have no mortgage prepayment penalties (still legal/origination fees for re-issuing mortgage)
  - Prepayment penalty is often barred by state law
  - If interest rates fall this allows mortgage holders to 'refi' (refinance)
- In Canada prepayment penalty equal to 3 months payment or loss of interest (whichever is greater)
  - Most mortgages permits an annual prepayment of up to 10% of initial principal value without penalty
  - At end of mortgage term, mortgage balance can be paid without penalty by refinancing balance at another financial institution or mortgage can be renewed with the same financial institution
  - Possible to change term and amortization period when the mortgage is reissued (will be legal/origination fees for reissuing that can be negotiated)



# Canadian versus US Mortgage (3)

- Mortgage Funder (Mortgagee) Recourse for **Default** by Borrower (Mortgagor)
  - High Loan-to-Value (LTV) > 80% require mortgage insurance
    - Insurance premium can be charged as a fixed payment at origination or as 'points' which increase the monthly payment; premium increases with LTV; 95% maximum in Canada
    - Both public (FHA and the GSE's in US; CMHC in Canada) and private insurance (e.g., Genworth, Canada Guarantee in Canada)
- **Lender Recourse** depends on mortgage contract terms
  - In Canada, typically full recourse
  - In US depends on state, usually no recourse
    - Mortgage insurance does not guarantee that value of (foreclosed) property will be sufficient to avoid loss, i.e.,  $V < LV$
    - Average Default Loss more likely on non-insured (LTV < 80%) mortgages



# Valuation Formula and Examples

- Perpetuities, SAIS, p.207-8, e.g., British consol.
  - Application of geometric series
  - Fixed and variable coupon
- Semi-annual and Quarterly coupons
  - SAIS, p.209
  - Effective Yield
- Mortgage Valuation, SAIS, p.243.
  - See midterm mortgage question and solution on class webpage
  - See Mortgage Info on class webpage for background



## Perpetuity with Annual Payment (\$A)

$$\begin{aligned} P_{perp} &= \frac{\$A}{1+r} + \frac{\$A}{(1+r)^2} + \frac{\$A}{(1+r)^3} + \dots = \frac{\$A}{1+r} \left( 1 + \frac{1}{1+r} + \frac{1}{(1+r)^2} + \dots \right) \\ &= \frac{\$A}{1+r} \left( \frac{1}{1 - \frac{1}{1+r}} \right) = \frac{\$A}{r} \end{aligned}$$

## Term Annuity: Mortgage with Annual Payment (\$A)

$$P_{mort} = \frac{\$A}{r} - \left( \frac{\$A}{r} \div \frac{1}{(1+r)^T} \right) = \$A \left( \frac{1}{r} - \frac{1}{r(1+r)^T} \right)$$

## Term Annuity: Mortgage with Monthly Payment (\$A / 12)

$$P_{mort} = \frac{\$A}{12} \left( \frac{1}{\frac{r}{12}} - \frac{1}{\frac{r}{12} \left(1 + \frac{r}{12}\right)^{12T}} \right)$$

```
In[1]:= 100 * ((1 / .06) - (1 / (.06 ((1.06) ^ (25)))))
```

```
Out[1]= 1278.34
```

```
In[3]:= (100 / 12) * ((1 / (.06 / 12)) - (1 / ((.06 / 12) ((1 + (.06 / 12)) ^ (25 * 12)))))
```

```
Out[3]= 1293.39
```

```
In[4]:= (100 / 2) * ((1 / (.06 / 2)) - (1 / ((.06 / 2) ((1 + (.06 / 2)) ^ (25 * 2)))))
```

```
Out[4]= 1286.49
```



# How to Price Bonds

- Simplifications required to use the basic formulas to determine the bond price/yield:
  - Bond is valued on the issue date or coupon payment date
    - No need to take account of accrued interest.
    - It is possible to specify more complicated, exact formulas for price between payment dates.
  - The bond has no embedded options.
  - ‘Straight bonds’ with a bullet maturity are used.



# Bond Price and Yield to Maturity

- **Yield to Maturity**
  - Key method of comparing values, see Table 4.5
  - Basis of traditional yield spread analysis
- For an **annual coupon bond** valued on the issue date, the yield to maturity is determined by solving:

$$P_B = \left\{ \sum_{t=1}^T \frac{C}{(1+y)^t} \right\} + \frac{M}{(1+y)^T}$$



# Interpretations of Yield to Maturity

- Because the calculation assumes that future coupon cash flows can be reinvested at the stated  $y$ , it follows that  $y$  is only a ***promised*** yield to maturity.
- Reinvestment of coupons at the promised yield is required in order for the bond to actually earn the stated yield if the bond is held to maturity. As such, the yield to maturity is an ***ex ante forecast of the ex post realized yield***.
- When  $C = 0$ , the bond is referred to as a pure discount or ***zero coupon bond***. If held to maturity, a default and option free zero coupon bond will have the promised yield to maturity equal to the realized yield.



# Spot Interest Rate (Implied Zero Coupon Interest Rate)

- The **spot interest rate** address a conceptual problem that arises with the yield to maturity
  - applying the same interest rate to discount cash flows occurring at different points in time
- The spot interest rate  $z_t$  used to discount cash flows at time  $t$  is specified by:

$$P_B = \left\{ \sum_{t=1}^T \frac{C}{(1+y)^t} \right\} + \frac{M}{(1+y)^T} = \left\{ \sum_{t=1}^T \frac{C}{(1+z_t)^t} \right\} + \frac{M}{(1+z_T)^T}$$



# Use of Implied Zeroes

- Calculation of implied forward rates
  - Used to empirically model term structure behavior
  - Used in Monte Carlo estimation procedures for OAS (see sec. 6.3)
- To calculate the value of bonds with contingencies
- To provide riskfree discount rates for cash flows occurring at a particular time  $t$  or over a horizon  $[0, T]$



# Calculation of Implied Zeroes

- ▣ Implied zeroes are calculated from the US Treasury yield curve (Government of Canada curve for Canada)
  - ▣ Introduction of contingencies and default risk modeled as an add-on to the implied zero
- ▣ Calculated implied zeroes typically differ from rates in the STRIPS market (SAIS, p.201-2)



### Example: Solving the Canadian Spot Interest Rates

Assume annual coupon payments

From Figures 4.6 and 4.10 pick the following bills/bonds

1 year tbill  $z_1 = .0720$

6.5% 1 Aug 1996  $P_2 = 97.505$  ( $y_2 = .07887$ )

7.5% 1 Jul 1997  $P_3 = 98.225$  ( $y_3 = .08197$ )

6.5% 1 Sep 1998  $P_4 = 93.350$  ( $y_4 = .08474$ )

7.75 1 Sep 1999  $P_5 = 96.800$  ( $y_5 = .08542$ )

Two Possible Bootstrap Solution Techniques: the direct approach and the par bond approach.

The direct approach involves using the observed price and coupon to solve for the spot interest rate.

Solving for  $z_2$ :  $97.505 = 6.5/(1 + z_1) + 106.5/(1 + z_2)^2$

Using this method,  $z_2 = .0792$

For the Par Bond approach, use the result that when the stated yield to maturity equals the C/M then the bond sells at par:

$100 = 7.887/(1 + z_1) + 107.887/(1 + z_2)^2$

Using this method  $z_2 = .0791428$

Why the difference, when the method would appear to be exactly the same?

Yields used are semi-annual, not annual, compounded while the prices/coupons are exact (except that annual coupon payments have been assumed in making the calculations).

The differences involved are generally small:

Par bond:  $z_3 = .082423$   $z_4 = .0854697$   $z_5 = .0861374$

Price/Coupon:  $z_3 = .08232$   $z_4 = .08595$   $z_5 = .08630$

Selected quotations, with changes since the previous day, on  
minion Securities. Yields are calculated to full maturity, unless  
midpoint between final bid and ask quotations Aug. 8, 1994.

Issuer	Coupon	Maturity	Price	Yield	\$ Chg	Is:
<b>GOVERNMENT OF CANADA</b>						
CANADA	4.75	15 MAR 96	95.625	7.708	+0.300	NE
CANADA	6.50	1 AUG 96	97.505	7.867	+0.380	NE
CANADA	7.75	15 SEP 96	99.796	7.852	+0.420	ON
CANADA	7.50	1 JUL 97	98.225	8.197	+0.425	ON
CANADA	6.25	1 FEB 98	93.600	8.416	+0.450	ON
CANADA	6.50	1 SEP 98	93.350	8.474	+0.500	ON
CANADA	5.75	1 MAR 99	89.850	8.486	+0.600	ON
CANADA	7.75	1 SEP 99	96.800	8.542	+0.600	ON
CANADA	9.25	1 DEC 99	102.550	8.634	+0.600	ON
CANADA	9.75	1 JUN 01	104.650	8.821	+0.700	ON
CANADA	9.50	1 OCT 01	103.425	8.638	+0.700	PE
CANADA	9.75	1 DEC 01	104.750	8.648	+0.700	PE
CANADA	8.50	1 APR 02	97.950	8.572	-0.700	OU
CANADA	7.25	1 JUN 03	89.650	8.973	-0.700	OU
CANADA	7.50	1 DEC 03	90.800	8.977	+0.760	OU
CANADA	10.25	1 FEB 04	107.750	9.014	-0.850	OU
CANADA	6.50	1 JUN 04	84.000	8.938	-0.750	SA
CANADA	9.00	1 DEC 04	99.863	9.017	+0.863	SA
CANADA	10.00	1 JUN 08	105.650	9.139	-0.900	SA
CANADA	9.50	1 JUN 10	102.850	9.152	-0.950	TOP
CANADA	9.00	1 MAR 11	98.650	9.158	+0.950	TOP
CANADA	10.25	15 MAR 14	109.400	9.204	+1.000	ALT
CANADA	9.75	1 JUN 21	105.650	9.177	+1.150	BEL
CANADA	8.00	1 JUN 23	88.150	9.174	+1.050	BEL

**This spot rate calculation example makes the simplifying assumption that the quoted yields are annual, which the actual yields are semi-annual. This will result in a small deviation from the 'true spot rate'**



# Bootstrapping Implied Zeros (An 'Exact' Semi-annual Coupon Bond Example from Stigum – 1980's interest rates)

- Assume:
  - Bonds are sold at par and pay coupons semi-annually.
  - The observed six month Tbill yield is 8.87%, where the six month Treasury bill discount rate is converted to a true yield.
  - The observed one year Tbond yield is 9.04%

$$100 = \frac{4.52}{1 + \frac{.0887}{2}} + \frac{104.52}{\left(1 + \frac{z_1}{2}\right)^2}$$



# Next Step in the Bootstrap

- Solving for  $z_1$  -- the implied one year zero coupon rate -- gives 0.090438.
- This solution can now be used to solve the next step in the bootstrap where the observed 1.5 year Treasury yield is 9.155:

$$100 = \frac{4.5775}{1 + \frac{0.0887}{2}} + \frac{4.5775}{\left(1 + \frac{z_1}{2}\right)^2} + \frac{104.5775}{\left(1 + \frac{z_{1.5}}{2}\right)^3}$$



# Finishing the Bootstrap

- Solving for  $z_{1.5}$  -- the implied one and a half year zero coupon rate -- gives 0.091629.
- This solution can now be used to solve the next step in the bootstrap where the observed 2 year Treasury yield is 9.2:
- solve for  $z_2$  and ETC ETC ETC to  $T$ .

$$100 = \frac{4.6}{1 + \frac{z_5}{2}} + \frac{4.6}{\left(1 + \frac{z_1}{2}\right)^2} + \frac{4.6}{\left(1 + \frac{z_{1.5}}{2}\right)^3} + \frac{104.6}{\left(1 + \frac{z_2}{2}\right)^4}$$



# The yield curve and the term structure

- Basic definitions:
  - The **yield curve** is the relationship between term to maturity and yield to maturity
    - See Bloomberg and other sites for a plot of the yield curve
  - The **term structure of interest rates** is the relationship between spot interest rates and term to maturity
    - Demonstrate that if the yield curve slopes up then the term structure of interest rates will lie above the yield curve
- Note that the 'continuous' yield curve is a 'best fit' to a discrete sample of available bonds, there is a considerable literature on the appropriate methodology to do this.



Supplied by RBC Dominion Securities Inc. (3pm close, bid)

## FEDERAL

	Coupon	Mat. date	Bid \$	Yld%
Canada	12.000	Mar 01/05	110.38	2.44
Canada	3.500	Jun 01/05	101.39	2.45
Canada	6.000	Sep 01/05	105.33	2.59
Canada	12.250	Sep 01/05	115.11	2.59
Canada	8.750	Dec 01/05	110.89	2.70
Canada	12.500	Mar 01/06	119.28	2.99
Canada	5.750	Sep 01/06	106.81	3.01
Canada	14.000	Oct 01/06	128.06	3.05
Canada	7.000	Dec 01/06	110.56	3.11
Canada	7.250	Jun 01/07	112.43	3.30
Canada	4.500	Sep 01/07	103.71	3.40
Canada	13.000	Oct 01/07	133.03	3.39
Canada	10.000	Jun 01/08	125.53	3.61
Canada	6.000	Jun 01/08	109.42	3.64
Canada	4.250	Sep 01/08	102.30	3.70
Canada	11.000	Jun 01/09	134.23	3.86
Canada	5.500	Jun 01/09	107.64	3.90
Canada	4.250	Sep 01/09	101.41	3.97
Canada	10.750	Oct 01/09	134.43	3.93
Canada	9.750	Mar 01/10	130.40	4.07
Canada	9.500	Jun 01/10	130.04	4.08
Canada	5.500	Jun 01/10	107.62	4.12
Canada	9.000	Mar 01/11	128.91	4.24
Canada	6.000	Jun 01/11	110.50	4.32
Canada	8.500	Jun 01/11	126.30	4.29
Canada	5.250	Jun 01/12	105.55	4.45
Canada	5.250	Jun 01/13	105.38	4.54
Canada	10.250	Mar 15/14	145.82	4.56
Canada	5.000	Jun 01/14	103.15	4.61
Canada	11.250	Jun 01/15	158.81	4.56
Canada	9.750	Jun 01/21	155.36	4.96
Canada	9.250	Jun 01/22	150.97	4.98
Canada	8.000	Jun 01/23	135.78	5.07
Canada	9.000	Jun 01/25	149.65	5.14
Canada	8.000	Jun 01/27	138.32	5.16
Canada	5.750	Jun 01/29	108.30	5.16
Canada	5.750	Jun 01/33	109.35	5.13

Yield curve for Government of Canada bonds observed in Jan. 2004

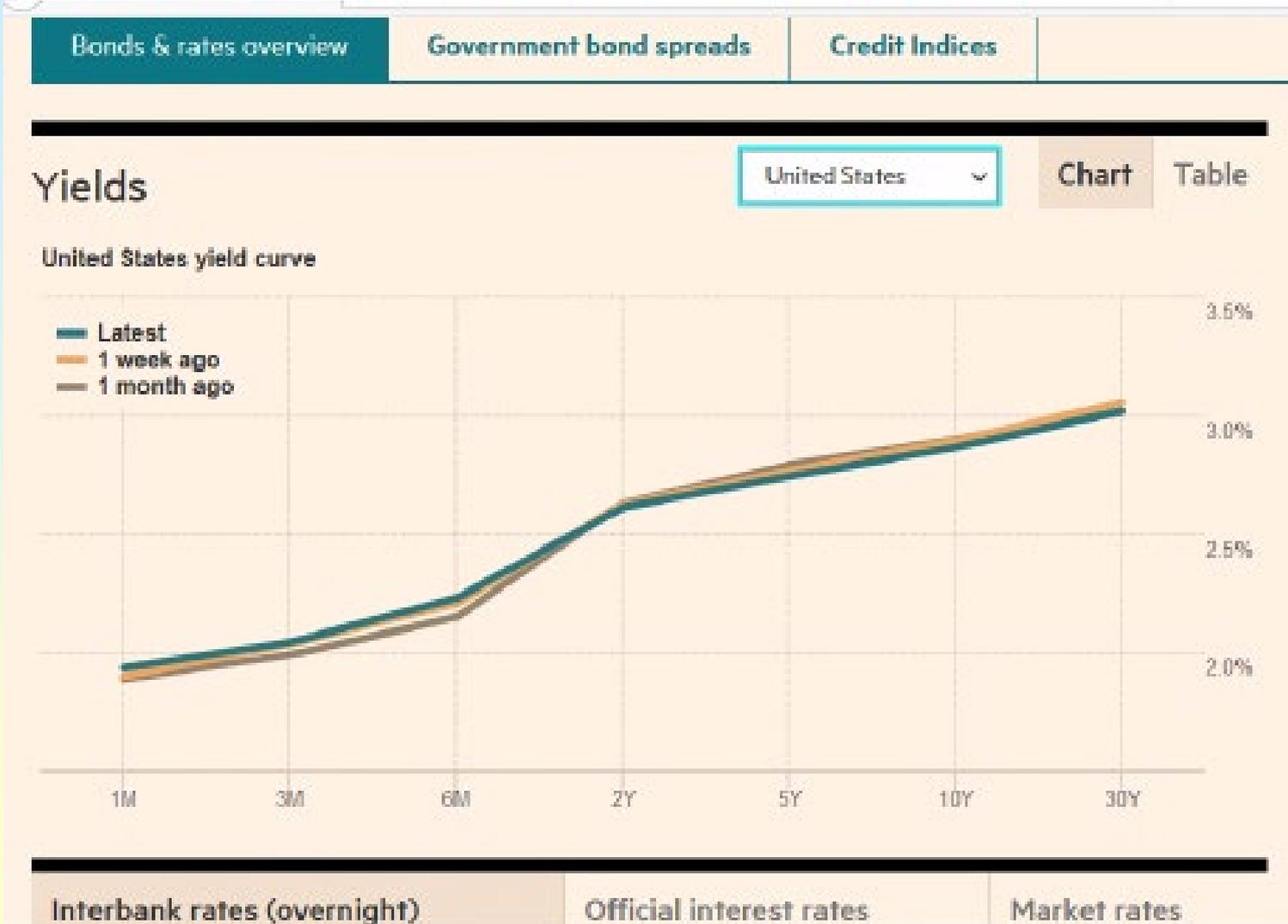
Notice that the yield curve specifies the relationship between the maturity date column and the yield column

Observe that two bonds with the same maturity date, e.g., June 01/09, don't have the same yield due to differences in the coupon → the bond with the higher coupon is actually a 'shorter' bond as more of the cash flow will be received sooner. Put differently, the return of principal at maturity is a larger fraction of the total cash flow for a lower coupon bond.

When the yield curve is plotted, the sequence of individual points are connected with a 'best fit' line



# US Yield Curve (from Financial Times, UK) 17-8-18



# Specifying implied forward interest rates

- An important application of implied zero coupon interest rates is to calculate **implied forward interest rates**
- A useful interpretation of the implied forward interest rate is as a **breakeven interest rate** in comparing a 2 period *rollover portfolio* with a 2 period buy and hold portfolio.
- These two types of portfolios are defined as:

**Portfolio A:** Buy and hold a 2 year zero coupon bond. If the initial investment is \$1 the return at the end of 2 years is  $(1 + z_2)^2$

**Portfolio B:** Buy and mature a 1 year zero coupon and use the proceeds to purchase another 1 year zero coupon bond, 1 year in the future. If the initial investment in this portfolio is \$1 then the expected return at end of year two is:  $(1 + z_1)(1 + E[z_{1,2}])$

Portfolio A is referred to as the *buy and hold* portfolio and Portfolio B as the *rollover* portfolio.



# Calculation of one period implied forward interest rates

To derive the *breakeven interest rate*, assume that the expected returns on the two portfolios are equal, then:

$$(1 + z_2)^2 = (1 + z_1)(1 + E[z_{1,2}])$$

It follows that the breakeven expected interest rate can be calculated as:

$$(1 + E[z_{1,2}]) = \frac{(1 + z_2)^2}{(1 + z_1)}$$

Using this result, the correspondence between the breakeven interest rate and the definition of the implied forward rate  $f_{1,2}$  follows appropriately:  $f_{1,2} = E[z_{1,2}]$ . (The notation for the implied forward rate differs across the various textbooks on the subject, i.e., there is no generally accepted notational convention.)



# A variety of implied forward rates:

The following can be used to specify:

- i) a one year implied forward rate starting at time  $t$
- ii) a five year implied period forward rate starting at time 5 maturing at time 10
- iii) a three year implied forward rate starting at time 2 and maturing at time 5
- iv) a  $k$  period implied forward rate starting at time  $t$  and maturing at time  $t + k$

$$(1 + f_{t,t+1}) = \frac{(1 + z_{t+1})^{t+1}}{(1 + z_t)^t}$$

$$(1 + f_{5,10}) = \sqrt[5]{\frac{(1 + z_{10})^{10}}{(1 + z_5)^5}}$$

$$(1 + f_{2,5}) = \sqrt[3]{\frac{(1 + z_5)^5}{(1 + z_2)^2}}$$

$$(1 + f_{t,t+k}) = \sqrt[k]{\frac{(1 + z_{t+k})^{t+k}}{(1 + z_t)^t}}$$



# Interpreting yield curve shape with implied forward rates

- Three basic interpretations of the term structure
  - The unbiased expectations hypothesis
    - This theory predicts that implied forward rates are unbiased expectations of future interest rates
      - An upward sloping yield curve implies that the market expects higher future interest rate levels
  - The liquidity premium hypothesis
    - Interprets the implied forward rate as a biased predictor by the amount of a liquidity premium
      - Liquidity premia are monotonically increasing with the term to maturity
  - Market segmentation/Preferred Habitat hypothesis
    - Implied forward rates are biased and it is not possible to state that liquidity premia are monotonically increasing



# Introduction to Duration

- Duration is a fundamental concept in the analysis of fixed income securities, both for individual securities and for portfolios.
- Duration has at least three distinct interpretations:
  - Adjusted (weighted average) term to maturity
  - Elasticity of bond price with respect to interest rate changes
  - A tool for bond portfolio management



# 20<sup>th</sup> Century Contributions

- **Frederick Macaulay (1882-1970)**
  - Early member of NBER, Canadian, son of Sun Life pioneers, founder of Macaulay-Bernstein
  - *The Movements of Interest Rates. Bond Yields and Stock Prices in the United States since 1856*, (1938) seminal contribution of Macaulay Duration
- **John Hicks (1904-1989)**
  - *Value and Capital* (1939) contains duration as an elasticity
  - Nobel Prize in Economics winner, IS-LM model
- **Frank Redington (1906-1984)**
  - Duration and Convexity as tools in bond portfolio management
  - Seminal development of classical immunization theory



# Duration as an Elasticity Measure

- **Macaulay duration** is the elasticity of the bond's price with respect to a change in interest rates.
  - Elasticity is usually dimensionless (dollars divided by dollars) but, due to the presence of compound interest it is acceptable to express duration in years
  - For those unfamiliar with 'point elasticity' see the link 'Point Elasticity Note' on class webpage

$$\frac{dP_B}{d(1+y)} = - \left\{ \sum_{t=1}^T \frac{tC}{(1+y)^{t+1}} + \frac{TM}{(1+y)^{T+1}} \right\}$$

$$\rightarrow -\frac{1+y}{P_B} \frac{dP_B}{d(1+y)} = \frac{\sum_{t=1}^T \frac{tC}{(1+y)^t} + \frac{TM}{(1+y)^T}}{P_B} = D^* = \text{MacaulayDuration}$$



# Macaulay Duration as Adjusted Maturity

- The Macaulay duration ( $D$ ) of a zero coupon bond equals the term to maturity ( $T$ ). To see this, set  $C = 0$  in the duration formula and use the price of zero coupon bond for  $P_B$
- For  $C > 0$ ,  $D < T$ .
  - Macaulay duration adjusts the term to maturity of a fixed income security to be equal to the duration of a zero coupon bond with term to maturity equal to  $D$ .
  - Macaulay duration is sometimes referred to as the weighted average term to maturity.



## Duration for Semi-annual Coupon Bonds

- As an elasticity measure, the “length” of duration is equal to the frequency of the cash flows, i.e., when coupons are paid semi-annually, the duration is measured in  $\frac{1}{2}$  years (need to divide by 2 to get duration as an annualized number):

$$\frac{dP_B}{d\left(1 + \frac{y}{2}\right)} = - \left\{ \sum_{t=1}^{2T} \frac{t (C/2)}{\left(1 + \frac{y}{2}\right)^{t+1}} + \frac{2T M}{\left(1 + \frac{y}{2}\right)^{2T+1}} \right\}$$

$$\rightarrow - \frac{1 + \frac{y}{2}}{P_B} \frac{dP_B}{d\left(1 + \frac{y}{2}\right)} = \frac{\sum_{t=1}^{2T} \frac{t (C/2)}{\left(1 + \frac{y}{2}\right)^t} + \frac{2T M}{\left(1 + \frac{y}{2}\right)^{2T}}}{P_B} = D^{*SA} = \text{Macaulay Duration} \\ \text{( in half years )}$$



Some special Macaulay durations: Term Annuity and Par Bond (see Macaulay Duration and Term Annuity file on class webpage) and the Convexity of a Par Bond (in Par-Bond-Convexity file contained in 16-1 midterm answer .zip file) → notice that the ANNUAL COUPON Par Bond assumption allows the use of simple results to program formulas

### Term Annuity

$$\frac{1 + y}{y} - \frac{T}{(1 + y)^T - 1}$$

### Par Bond Duration

$$\frac{1 + y}{y} \left[ 1 - \frac{1}{(1 + y)^T} \right]$$

### CONVEXITY OF A PAR BOND

$$\frac{2}{y} \left[ \frac{1}{y} - \frac{1}{y(1 + y)^T} \right] - \frac{2T}{y(1 + y)^{T+1}}$$



# Macaulay Duration versus Modified Duration

- Recall the three different uses of Duration: An Elasticity Measure; Weighted Average Term to Maturity; and, Tool in Fixed Income Portfolio Management → these three applications apply to Macaulay Duration
- In theoretical applications in fixed income P.M., it is useful to use the simplified **Modified Duration** ( $D$ ) where (annualized form):

$$D = (D^* / (1 + y))$$

Note: Modified duration is not an exact measure of either the elasticity or weighted average term to maturity but eliminates the theoretically unnecessary  $(1 + y)$



# Basics of Credit Risk and Default Risk

**Note:** The following section will only be covered if time permits

- ***The Legal Aspects of Default***
  - ABA Model Indenture
  - Absolute Priority Rule
- **Credit risk** can be decomposed into three parts:
  - **default risk** (bond default)
  - credit spread risk (no change in rating)
  - downgrade risk (change down in rating)



# Assessment of Credit and Default Risk

- Ratings Services
  - Standard and Poor's Corporation,
  - Moody's Investors Services Inc.
  - Fitch Ratings (created by the merger of Fitch IBCA with Duff and Phelps Credit Rating Co.)
    - See next slide
- There are also firms that specialize in ratings for specific industries such as Demotech, Inc. that assesses insurance companies and HMO's
  - See links in Fixed Income on links page.



**Table 7.1**  
**Summary of Corporate Bonds Rating Systems and Symbols**

Fitch	Moody's	S&P	Summary Description
<b>Investment Grade – High Credit Worthiness</b>			
AAA	Aaa	AAA	Gilt edge, prime, maximum safety
AA+	Aa1	AA+	High grade, high-credit quality
AA	Aa2	AA	
AA-	Aa3	AA-	
A+	A1	A+	Upper-medium grade
A	A2	A	
A-	A3	A-	
BBB+	Baa1	BBB+	Lower-medium grade
BBB	Baa2	BBB	
BBB-	Baa3	BBB-	
<b>Speculative – Lower Credit Worthiness</b>			
BB+	Ba1	BB+	Low-grade, speculative
BB	Ba2	BB	
BB-	Ba3	BB-	
B+	B1	B	Highly speculative
B	B2		
B-	B3		
<b>Predominately Speculative, Substantial Risk, or in Default</b>			
CCC+	Caa	CCC+	Substantial risk, in poor standing
CCC		CCC	
CC	Ca	CC	May be in default, very speculative
C	C	C	Extremely speculative
		C1	Income bonds—no interest being paid
DDD			Default
DD			
D		D	

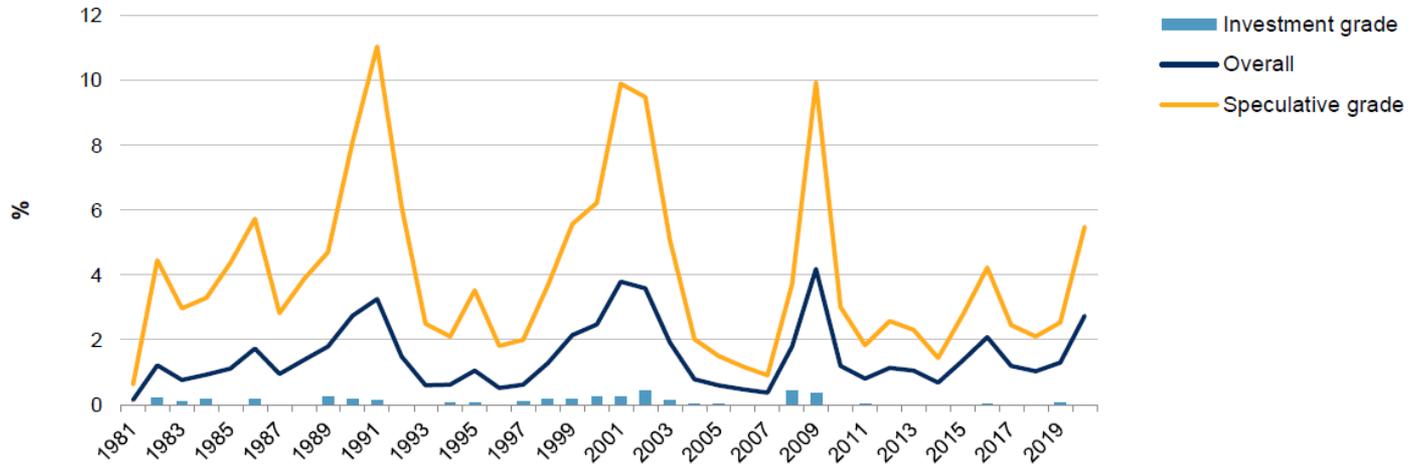


# Evidence on Defaults

- How to measure default rates?
  - Figures + Tables – altman.pdf
  - S&P Global Default Study (on class webpage)
- Loss depends on probability of default and the recovery rate
  - Tables --altman.pdf
- Ratings Drift
  - S&P Global Default Study

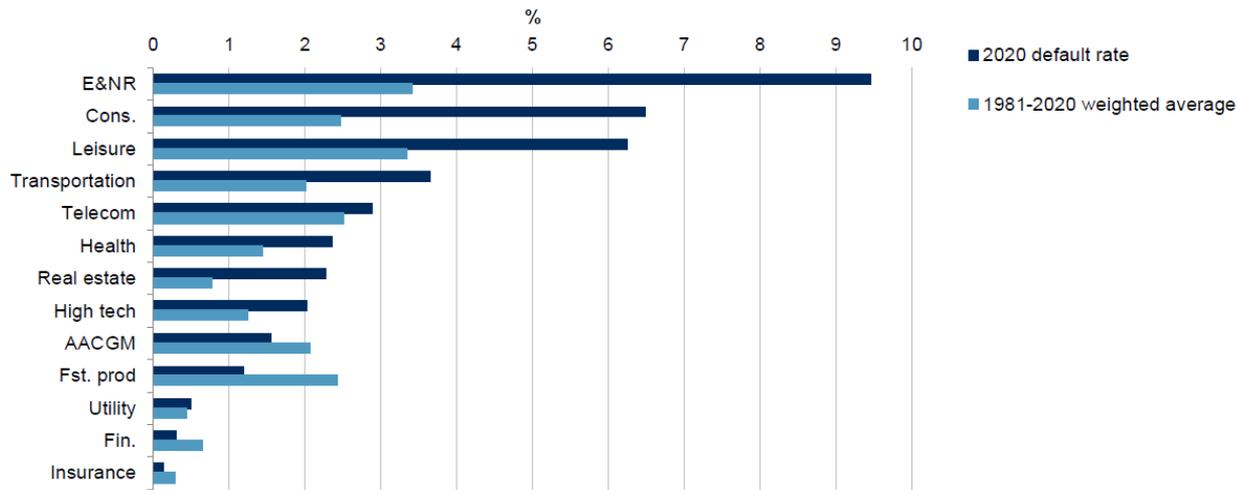


## Global Default Rates: Investment Grade Versus Speculative Grade



Sources: S&P Global Ratings Research and S&P Global Market Intelligence's CreditPro®.

## Global Corporate Default Rates By Industry: 2020 Versus Long-Term Average



Leisure--Leisure time/media. High tech--High technology/computers/office equipment. Health--Health care/chemicals. Fst. prod--Forest and building products/homebuilders. Fin.--Financial institutions. E&NR--Energy and natural resources. Cons.--Consumer/service sector.

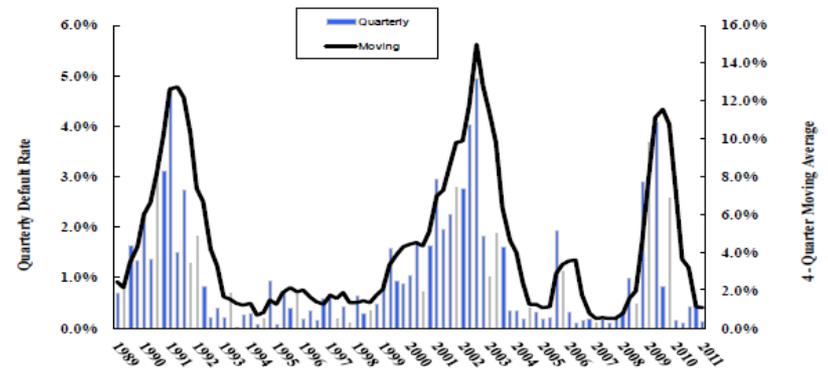
AACGM--Aerospace/automotive/capital goods/metal. Sources: S&P Global Ratings Research and S&P



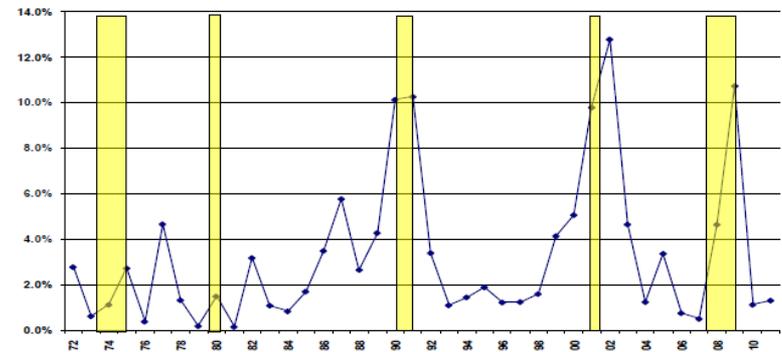
**Figure 1. Historical Default Rates – Straight Bonds Only, Not Including Defaulted Issues From Par Value Outstanding, 1971-2011 (Dollars in Millions)**

Year	Par Value Outstanding <sup>a</sup>		Default Rates (%)
	Outstanding (\$)	Defaults (\$)	
2011	1,354,649	17,813	1.315
2010	1,221,569	13,809	1.130
2009	1,152,952	123,878	10.744
2008	1,091,000	50,763	4.653
2007	1,075,400	5,473	0.509
2006	993,600	7,559	0.761
2005	1,073,000	36,209	3.375
2004	933,100	11,657	1.249
2003	825,000	38,451	4.661
2002	757,000	96,858	12.795
2001	649,000	63,609	9.801
2000	597,200	30,295	5.073
1999	567,400	23,532	4.147
1998	465,500	7,464	1.603
1997	335,400	4,200	1.252
1996	271,000	3,336	1.231
1995	240,000	4,551	1.896
1994	235,000	3,418	1.454
1993	206,907	2,287	1.105
1992	163,000	5,545	3.402
1991	183,600	18,862	10.273
1990	181,000	18,354	10.140
1989	189,258	8,110	4.285
1988	148,187	3,944	2.662
1987	129,557	7,486	5.778
1986	90,243	3,156	3.497
1985	58,088	992	1.708
1984	40,939	344	0.840
1983	27,492	301	1.095
1982	18,109	577	3.186
1981	17,115	27	0.158
1980	14,935	224	1.500
1979	10,356	20	0.193
1978	8,946	119	1.330
1977	8,157	381	4.671
1976	7,735	30	0.388
1975	7,471	204	2.731
1974	10,894	123	1.129
1973	7,824	49	0.626
1972	6,928	193	2.786

**Figure 2. Quarterly and the Four-Quarter Moving Average Default Rate 1989-2011**



**Figure 4. Historical Default Rates and Recession Periods in the US High-Yield Bond Market, 1972-2011**

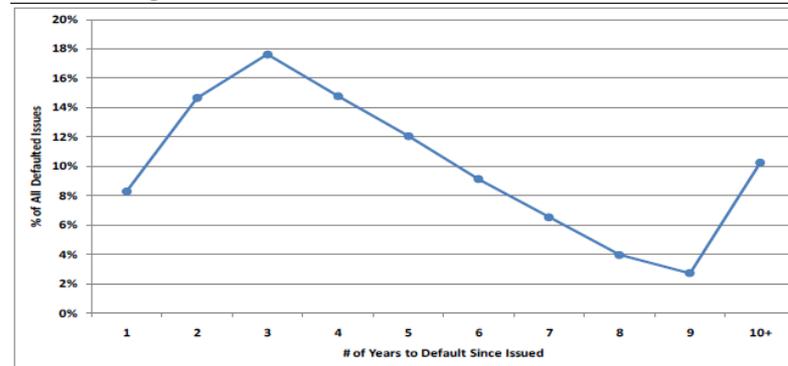


**Figure 8. Corporate Bond Defaults by Industry (Number of Companies)**

Industry	1970																					Total			
	-89	90	91	92	93	94	95	96	97	98	99	2000	01	02	03	04	05	06	07	08	09		10	11	
Auto/Motor Carrier	6	3					1				1		1				4	3		8	9		1	37	
Conglomerates	5	1	3	3								1			1	1									15
Energy	35	4	2	3			1			1	13	1		7	6		1		1	4	8		5	92	
Financial Services	21	7	14	3	2	1	2	1	2	6	1	6	4	5	6	2	3	2		5	7	7	2	109	
Leisure/Entertainment	9	8	2	4	3	4	3	1	5	5	8	9	6	5	6			3		10	12	3	1	107	
General Manufacturing	26	5	8	8	7	3	8	6	7	6	16	23	43	22	13	17	12	6	7	10	36	4	1	294	
Health Care	4	2	1	1	1		2			2	8	6	3	4	3		2		1	1		2	1	44	
Miscellaneous Industries	16	4	4	3	1	1	1			3	3	16	34	38	25	16	6	1	4	3	4	4	3	2	192
RealEstate/Construction	14	7	5	1			2	1	2	1	4	6	4	3			2	1			6	11	5	2	77
REIT	12			1											1										14
Retailing	10	6	15	6	4	5	6	3	6	6	12	7	12	5	5	3	2	2	3	5	5	3	8	139	
Comm. & Media	17	3	4	1	1	3	2	2	1	6	11	8	39	26	21	6	3	2	4	10	20	6	3	199	
Transportation (Non Auto)	9	1	2			2				2	1	8	5	7	7	6	2	5	1			7		68	
Utilities	2			1				1	1			1		1	3							1	2	13	
<b>Total</b>	<b>186</b>	<b>47</b>	<b>62</b>	<b>34</b>	<b>22</b>	<b>19</b>	<b>28</b>	<b>15</b>	<b>29</b>	<b>37</b>	<b>98</b>	<b>107</b>	<b>156</b>	<b>112</b>	<b>86</b>	<b>39</b>	<b>34</b>	<b>23</b>	<b>19</b>	<b>63</b>	<b>119</b>	<b>34</b>	<b>31</b>	<b>1,400</b>	

Source: NYU Salomon Center.

**Figure 11. Distribution of Years to Default From Original Issuance**  
Date: Summary Chart, 1991-2011



Source: NYU Salomon Center.



**Figure 12. Fallen Angels Versus Original(S&P) Issue and All High Yield Default Rates<sup>a</sup> (In Percent), 1985-2011**

Year	Fallen Angel Average 12-Mo. Default Rate	Original Issue Speculative Grade Default Rates	All Speculative Grade Bond Default Rates	Altman Dollar Weighted Annual Default Rates
2011	3.36 <sup>a</sup>	1.96 <sup>a</sup>	2.26 <sup>a</sup>	1.31
2010	1.76	3.43	3.27	1.13
2009	8.07	11.89	10.93	10.77
2008	3.07	4.34	4.02	4.65
2007	0.86	1.00	0.97	0.51
2006	1.40	1.23	1.26	0.76
2005	2.74	3.70	2.48	3.37
2004	0.83	2.65	2.23	1.25
2003	5.88	5.46	5.53	4.66
2002	6.59	8.55	8.32	12.79
2001	8.46	10.14	10.99	9.81
2000	7.01	7.10	7.03	5.07
1999	4.01	5.10	4.62	4.15
1998	3.31	2.75	2.23	1.60
1997	2.04	2.10	1.71	1.25
1996	1.38	2.00	1.71	1.23
1995	0.25	3.90	3.07	1.90
1994	0.00	2.31	1.70	1.45
1993	1.72	1.99	1.79	1.10
1992	4.50	5.48	5.45	3.40
1991	7.53	10.86	11.66	10.27
1990	5.77	8.30	8.20	10.14
1989	3.74	4.93	5.33	4.29
1988	4.25	3.39	3.95	2.66
1987	4.36	2.92	2.41	5.78
1986	2.46	6.29	4.78	3.50
1985	6.77	4.06	3.24	1.71
Arithmetic Average	3.78	4.73	4.49	4.09
Standard Deviation	2.45	2.94	3.07	3.51

<sup>a</sup> Issue based. <sup>b</sup> All S&P issuer based except for Altman rates and 2011.

Sources: NYU Salomon Center and S&P.



# More Evidence of Credit Risk

- Default and Original Rating
  - Figures 28-9 – altman.pdf
    - See also Table 4.10 (webpage Copies of Tables)
- Years to Default
  - Table 4.11 (webpage Copies of Tables)
- Credit Spreads
  - Figure 4,1 and Table 4.12 (see next slides or Copies of Tables)



Figure 16. Weighted Average (by Issue) Recovery Rates on Defaulted Debt by Seniority per \$100 Face Amount, 1978-2011

Default Year	Senior Secured			Senior Unsecured			Senior Subordinated			Subordinated			Discount and Zero Coupon			All Seniorities	
	No.	%	\$	No.	%	\$	No.	%	\$	No.	%	\$	No.	%	\$	No.	\$
2011	15	23	59.02	45	69	64.01	3	5	42.76	2	3	19.98	0	0	0.00	65	60.28
2010	6	24	39.46	12	48	57.86	5	20	30.64	2	8	12.67	0	0	0.00	25	46.62
2009	28	9	43.35	226	76	37.22	31	10	24.06	4	1	12.57	7	2	16.84	296	36.08
2008	18	14	30.52	79	63	49.56	23	18	30.25	4	3	21.09	1	1	2.71	125	42.52
2007	10	36	87.24	10	36	47.70	6	21	63.98	2	7	46.53	0	0	0.00	28	66.65
2006	9	18	90.60	26	52	60.90	8	16	50.24	1	2	60.33	6	12	78.31	50	65.32
2005	67	54	76.50	44	36	45.88	7	6	32.67	0	0	0.00	5	4	74.21	123	61.10
2004	27	39	63.67	33	48	56.77	2	3	37.44	0	0	0.00	7	10	43.06	69	57.72
2003	57	28	53.51	108	53	45.40	29	14	35.98	1	0	38.00	8	4	32.27	203	45.58
2002	37	11	52.81	254	75	21.82	21	6	32.79	0	0	0.00	28	8	26.47	340	25.30
2001	9	3	40.95	187	67	28.84	48	17	18.37	0	0	0.00	37	13	15.05	281	25.62
2000	13	8	39.58	47	29	25.40	61	37	25.96	26	16	26.62	17	10	23.61	164	26.74
1999	14	11	26.90	60	47	42.54	40	31	23.56	2	2	13.88	11	9	17.30	127	27.90
1998	6	18	70.38	21	62	39.57	6	18	17.54	0	0	0.00	1	3	17.00	34	40.46
1997	4	16	74.90	12	48	70.94	6	24	31.89	1	4	60.00	2	8	19.00	25	57.61
1996	4	17	59.08	4	17	50.11	9	38	48.99	4	17	44.23	3	13	11.99	24	45.44
1995	5	15	44.64	9	27	50.50	17	52	39.01	1	3	20.00	1	3	17.50	33	41.77
1994	5	23	48.66	8	36	51.14	5	23	19.81	3	14	37.04	1	5	5.00	22	39.44
1993	2	6	55.75	7	22	33.38	10	31	51.50	9	28	28.38	4	13	31.75	32	38.83
1992	15	22	59.85	8	12	35.61	17	25	58.20	22	33	49.13	5	7	19.82	67	50.03
1991	4	3	44.12	69	44	55.84	37	24	31.91	38	24	24.30	9	6	27.89	157	40.67
1990	12	10	32.18	31	27	29.02	38	33	25.01	24	21	18.83	11	9	15.63	116	24.66
1989	9	12	82.69	16	21	53.70	21	28	19.60	30	39	23.95				76	35.97
1988	13	21	67.96	19	31	41.99	10	16	30.70	20	32	35.27				62	43.45
1987	4	13	90.68	17	55	72.02	6	19	56.24	4	13	35.25				31	66.63
1986	8	14	48.32	11	20	37.72	7	13	35.20	30	54	33.39				56	36.60
1985	2	7	74.25	3	11	34.81	7	26	36.18	15	56	41.45				27	41.78
1984	4	29	53.42	1	7	50.50	2	14	65.88	7	50	44.68				14	50.62
1983	1	13	71.00	3	38	67.72				4	50	41.79				8	55.17
1982				16	80	39.31				4	20	32.91				20	38.03
1981	1	100	72.00													1	72.00
1980				2	50	26.71				2	50	16.63				4	21.67
1979										1	100	31.00				1	31.00
1978				1	100	60.00										1	60.00
<b>Total/Avg</b>	<b>409</b>	<b>15</b>	<b>57.60</b>	<b>1,348</b>	<b>51</b>	<b>38.77</b>	<b>482</b>	<b>18</b>	<b>30.65</b>	<b>263</b>	<b>10</b>	<b>30.63</b>	<b>164</b>	<b>6</b>	<b>25.45</b>	<b>2,707</b>	<b>38.24</b>
				9													
Median			57.39			46.79			32.73			27.50			17.40		42.15
Standard			17.94			13.48			13.73			17.15			20.70		13.75



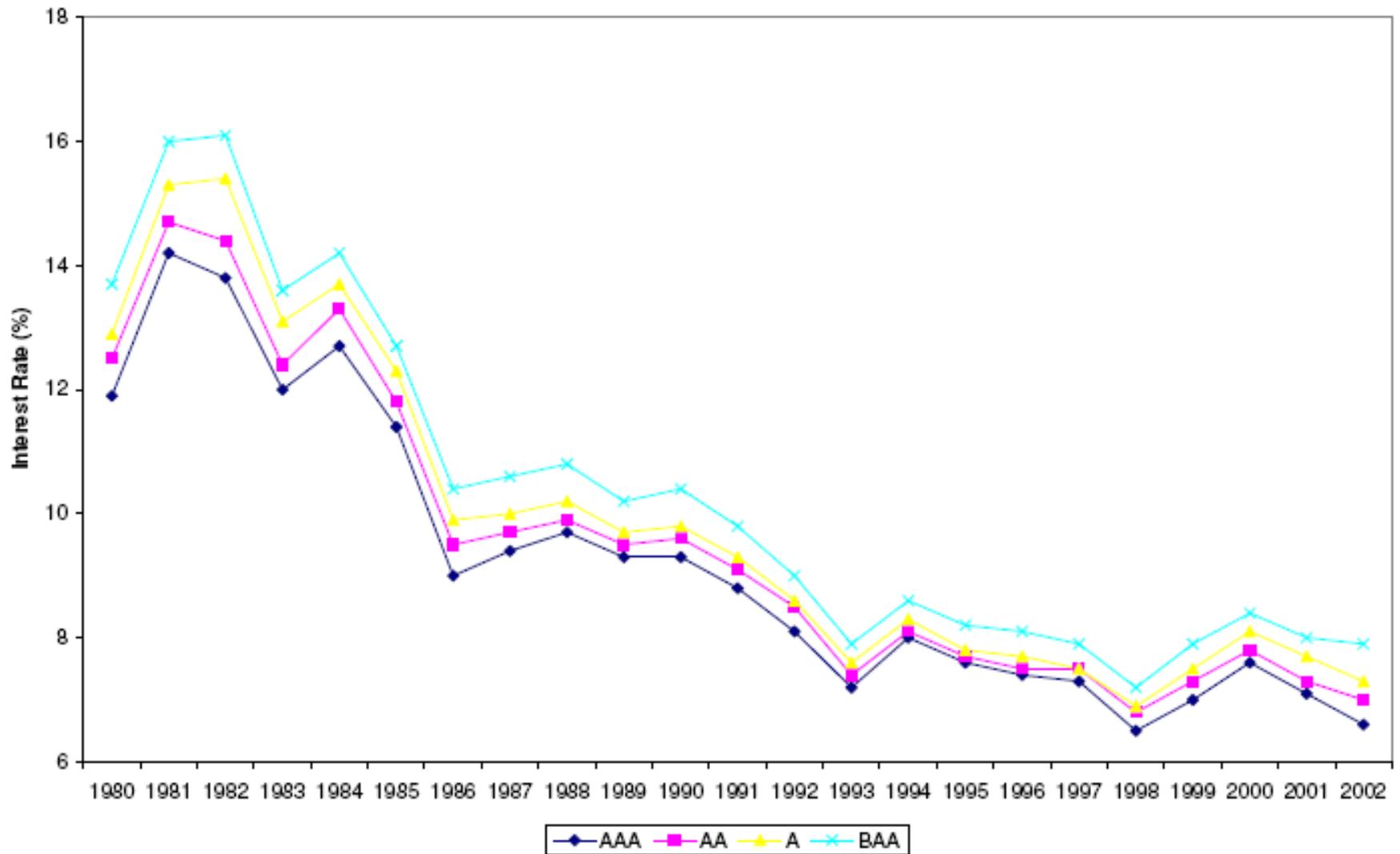
**Chapter 70**  
**Reuters Corporate Spreads for Banks**  
**for February 10, 2003**  
**(in basis points,)**

Ratings	1 yr	2yr	3 yr	5 yr	7 yr	10 yr	30 yr
Aaa/AAA	27	36	47	61	74	88	107
Aa1/AA+	33	49	53	61	84	99	118
Aa2/AA	35	54	56	75	87	101	121
Aa3/AA-	37	57	58	80	91	105	130
A1/A+	61	73	77	95	107	123	145
A2/A	64	76	79	97	108	125	148
A3/A-	68	79	82	101	115	128	150
Baa1/BBB+	81	98	104	124	153	180	202
Baa2/BBB	84	106	112	130	158	183	207
Baa3/BBB-	91	111	117	135	163	190	212
Ba1/BB+	605	615	625	635	655	675	695
Ba2/BB	615	625	635	645	665	685	705
Ba3/BB-	625	635	645	655	675	695	715
B1/B+	775	785	795	825	865	905	955
B2/B	785	795	805	835	875	915	965
B3/B-	795	805	815	845	885	925	975
Caa/CCC	1195	1205	1215	1240	1270	1330	1280

Source: [www.bondsonline.com](http://www.bondsonline.com) (Bond Market Association website)

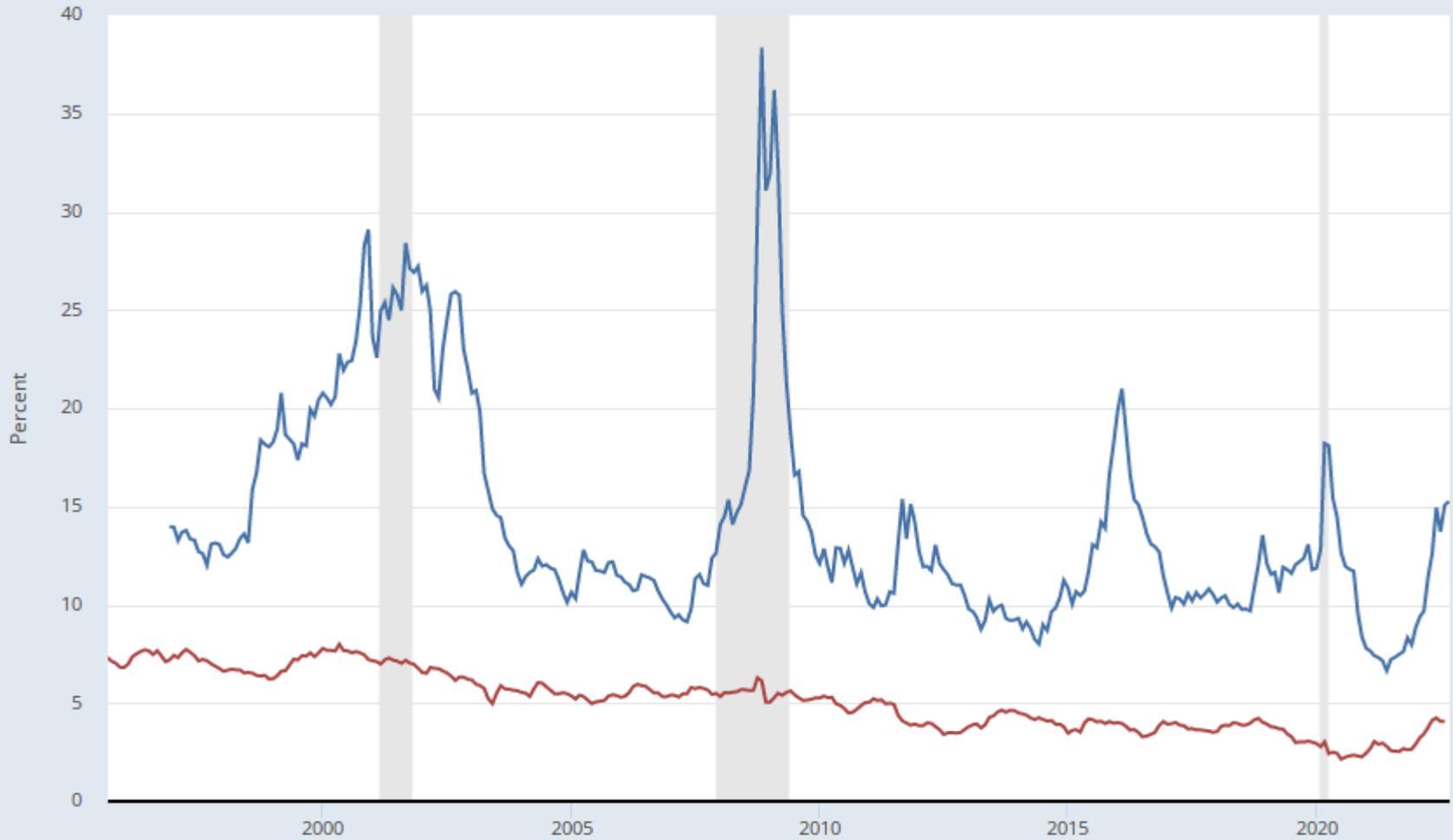


Graph 4-a Interest Spreads for Investment Grade Bonds, 1980-2002





— ICE BofA CCC & Lower US High Yield Index Effective Yield  
— Moody's Seasoned Aaa Corporate Bond Yield



Sources: Ice Data Indices, LLC; Moody's

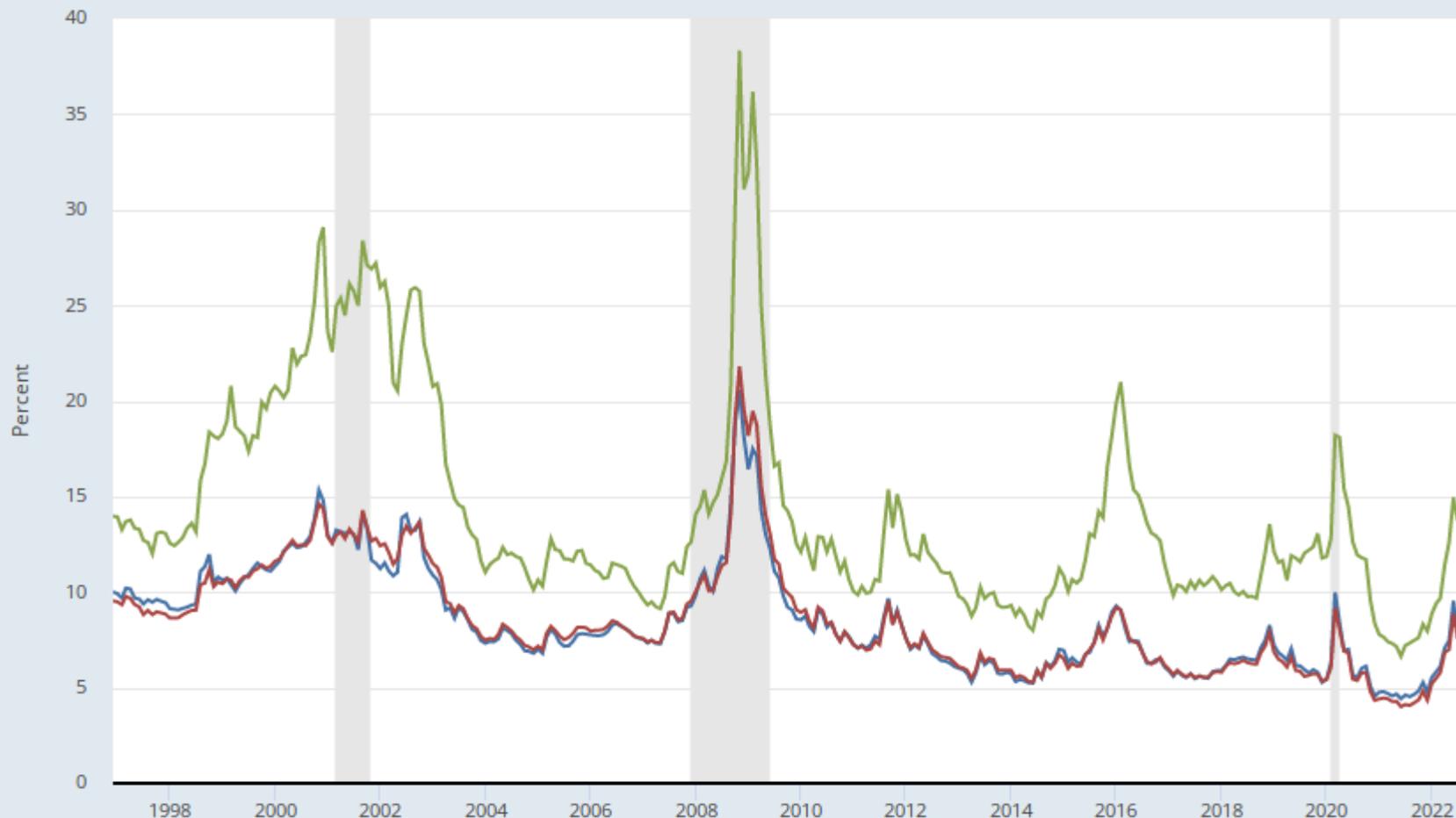
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**FRED**

- ICE BofA Single-B US High Yield Index Effective Yield
- ICE BofA US High Yield Index Effective Yield
- ICE BofA CCC & Lower US High Yield Index Effective Yield



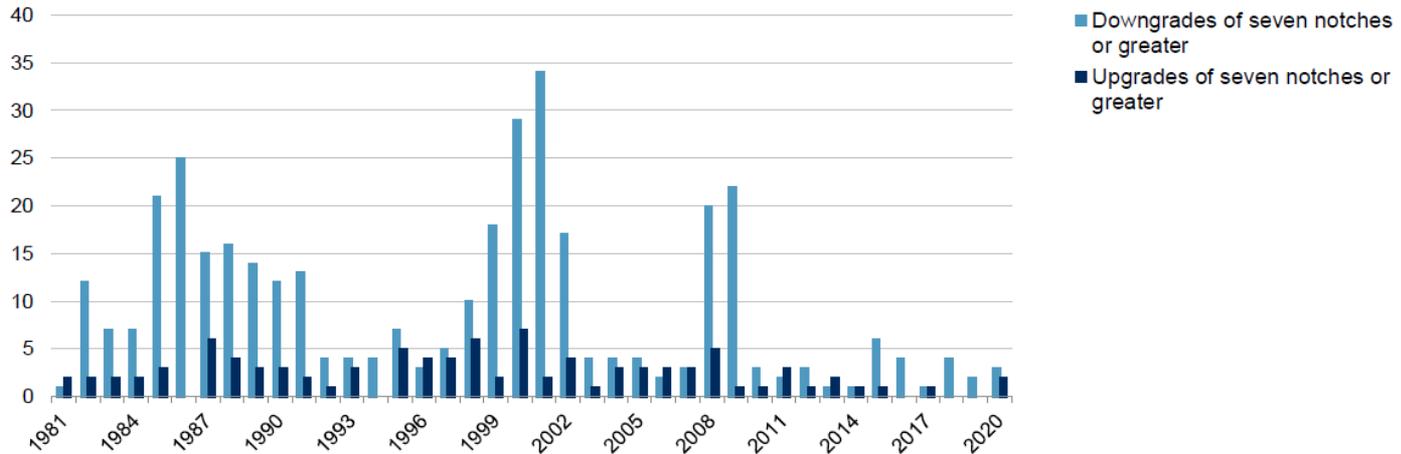
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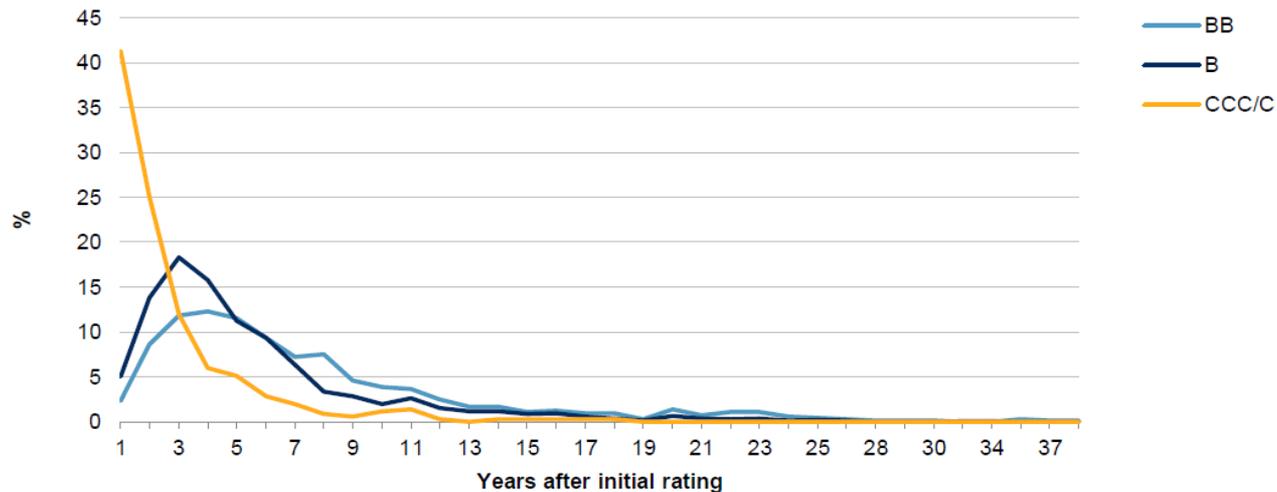


## Count Of Global Corporate Issuers With "Big" Rating Moves



Note: "Big" rating moves defined as upgrades or downgrades of seven notches or greater. Downgrades include movements to default. Sources: S&P Global Ratings Research and S&P Global Market Intelligence's CreditPro®.

## Tracking Global Corporate Default Rates From Initial Rating Category



Note: Data based on universe of defaulters from 1981-2020. Sources: S&P Global Ratings Research and S&P



## Time To Default From Original Rating For Global Corporate Defaulters (1981-2020)

Original rating	Defaults	Average years from original rating*	Median years from original rating	Standard deviation of years from original rating	Range
AAA	8	18.0	18.5	11.4	23.0
AA	32	17.4	19.6	10.6	37.8
A	101	14.1	10.9	9.1	37.7
BBB	221	9.2	7.3	6.7	36.1
BB	650	7.1	5.4	5.9	37.8
B	1,735	5.1	3.7	4.3	33.1
CCC/C	351	2.2	1.3	2.7	17.4
Total	3,098	5.9	4.1	5.7	39.4

\*Or Dec. 31, 1980, whichever is later. Sources: S&P Global Ratings Research and S&P Global Market Intelligence's CreditPro®.

