

Lecture 9

Option Adjusted Spread Analysis (OAS)

- Different Types and Approaches to OAS?
- Use of Monte Carlo to determine the OAS for a callable bond
- Pitfalls in the use of OAS.



What is the Option Adjusted Spread?

- **Option adjusted spread (OAS)** analysis is an extension of the static spread that incorporates randomness in the cash flows from the security.
- The **static spread** is an extension of traditional yield spread analysis that accounts for the timing of the cash flows of the two securities being compared.
- **Traditional yield spread** analysis compares the value of bonds by observing the difference in the yields.
 - For corporate bonds, the yield spread being compared is the Treasury-corporate yield spread on other bonds of similar coupon and yield to maturity that are being assessed.

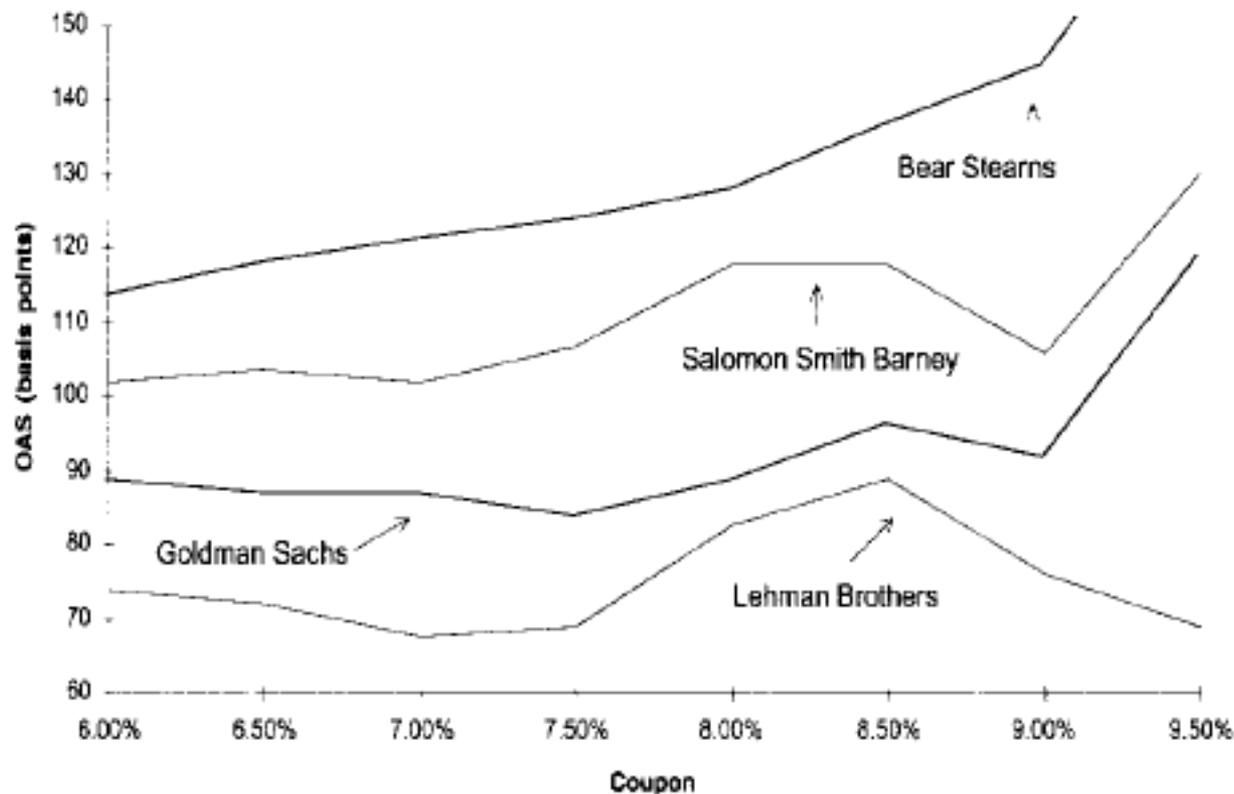


Example of an OAS chart provided by B of A/ML



Modelling the OAS for MBS is complicated and there can be wide differences in the OAS estimates depending on the proprietary model used for the contingencies and OAS calculation

8/16/99 Dealer OAS



Illustrations of OAS

- Table 6.10, p.339 (Also in 'Copies of Tables for Lecture 6' on class webpage)
 - The OAS depends on the assumed level of volatility
 - For bonds with seller's options, the OAS declines as volatility increases (Why?)
 - OAS for straight bond is constant across volatilities (Why?)
 - CMO only mitigates part of randomness of the pass-through



**Option-Adjusted Spreads for Various types
of Bonds with Embedded Options***

Callable Bonds (9/88)

Issuer	S&P Rating	Maturity	Next	Next	Curr. Price	Coupon (%)	Yield	Treasury.	OAS at Volatility of		
			Call Date	Call Price				Yield Spread	10%	15%	20%
ITT Fin.	A	07/01/92	07/01/89	100.00	101.58	10.800	10.27	168	57	30	-4
Marriot	A-	02/01/96	02/01/93	100.00	99.59	9.625	9.70	85	65	50	37
GMAC	AA-	07/15/07	now	104.00	84.19	8.000	9.86	82	71	54	41

Mortgage Backed Pass Through (PT) vs. Fixed Coupon Agency (6/88)

Security	Price	Term to Maturity	Avg. Life	Yield	Treasury	OAS at Volatility		
					Spread	10%	15%	20%
FNMA 9% PT	95.3125	28.05	9.80	9.98	115	104	84	64
FNMA 9.35%	101.1875	7.7	7.70	9.13	38	38	38	38

GNMA 9 and Collateralized Mortgage Obligations (5/88)

Class	Par Amt	Price*	Coupon.	Avg. Life	Yield	Treasury	OAS at Volatility of		
						Spread	10%	15%	20%
GNMA 9	100	92.656	9.0	11	10.48	123	104	88	71
A	35.1	98.378	8.0	2	9.10	100	67	48	21
B	19.0	95.508	8.5	5	9.85	115	83	59	25
C	18.0	94.748	9.0	7	10.27	125	103	80	52
Z	27.9	80.999	9.0	17	10.87	150	102	79	52

* Callable bond values are based on closing prices and Treasury rates on September 20, 1988. The MBS values are based closing prices on June 29, 1988. All prices shown in decimal form. The GNMA 9 is assumed to have a



Determining the Static Spread

- The static spread adds a fixed number of basis points to each implied zero interest rate to determine the price of the (semi-annual coupon) corporate bond (P_C)
 - ss is used instead of taken the traditional yield spread difference between the riskless government bond yield (used to derive the z 's) and the corporate yield

$$P_C = \sum_{t=1}^{2T} \frac{C/2}{\left(1 + \frac{z_t}{2} + \frac{ss}{2}\right)^2} + \frac{M}{\left(1 + \frac{z_T}{2} + \frac{ss}{2}\right)^{2T}} = \sum_{t=1}^{2T} \frac{C/2}{\left(1 + \frac{y_C}{2}\right)^2} + \frac{M}{\left(1 + \frac{y_C}{2}\right)^{2T}}$$



Calculating the OAS using a Monte Carlo

- A number of methodologies can be used to determine the OAS – e.g., Kupiec and Kah (1999) (see Readings Link) report sizable differences in dealer valuations of MBS – unlike yield calculations, **OAS methodology is not standardized.**
- There are a number of methods used to calculate OAS, the following is only a simple illustration of one possible method
- **SAIS, p.333**, details a simplified nine-step Monte Carlo process that can be used to estimate the OAS (see copy on next slide)



Description of the OAS Monte Carlo methodology

1. Calculate the implied zero coupon interest rates from the observed Treasury yield curve. Use the implied zeros to calculate a full set of one period ahead implied forward rates ($f_{t, t+1}$). These values are used to calibrate the parameters used in the Monte Carlo process.
2. Identify the stochastic interest rate model that will be used in the Monte Carlo procedure to generate the one period Treasury spot rates. A range possible processes can be selected such as the self-calibrating one factor models of Black, Derman and Toy (1990) or Ho-Lee. The specification of the interest rate model will involve the selection of the volatility parameter.
3. Use the selected volatility and interest rate model to generate a sufficiently large number of interest rate paths. Each of the paths will be composed of a sequence of one period zeros that apply to future time periods. Treating the one period zeros as forward rates, construct the spot rates along each path.
4. Specify the decision rules used to determine the random cash flows. For example, a callable bond requires a rule for determining at what interest rate level the bond will be called.
5. Path by path, examine the relevant interest rate and determine when or whether the bond will be called along that path. Generate the contingent cash flow along that path.
6. Using the spot rates for each path, determine the present value of the cash flow along each path for a given level of the static spread.
7. For each static spread value, sum the present values across all the generated paths and divide by the number of paths. Repeat the process for a range of static spread values.
8. Compare the present values that were determined by averaging across the paths with the observed market price of the bond with the embedded option. When the observed bond price is equal to the value calculated from the Monte Carlo process, the associated value of the 'averaged-across-paths' static spread is the option adjusted spread (OAS).
9. By repeating steps 3)-8), recalculate the OAS for a different volatility assumption.



Constructing the Monte Carlo

Note: The example taken from the standard CFA Level 2 fixed income textbook uses a **semi-annual coupon 10 year callable bond** → 20 cash flow dates, interest rates given in Tables are **annualized** though the payments are semi-annual

- Table 6.6: generating the interest rate paths.
- Table 6.7: calculating the implied zeros
- Tables 6.8: specifying the cash flows
- Table 6.9: solving for the OAS



TABLE 9-2

**Monte Carlo Simulation of Ten Binomial Paths for Six-Month One Period Treasury Zeros;
Initial Zero Rate of 7% with 10% Binomial Rate Changes***

Semi-Annual Period	Short-term One Period Zero Paths									
	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	5	<u>6</u>	<u>7</u>	<u>8</u>	<u>9</u>	<u>10</u>
1	7.0%	7.0%	7.0%	7.0%	7.0%	7.0%	7.0%	7.0%	7.0%	7.0%
2	7.7	7.7	7.7	6.3	6.3	6.3	7.7	6.3	7.7	7.7
3	8.5	6.9	6.9	6.9	5.7	5.7	8.5	5.7	8.5	8.5
4	7.6	6.2	6.2	7.6	5.1	6.2	9.3	6.2	9.3	9.3
5	6.9	6.9	5.6	8.4	4.6	6.9	8.4	6.9	8.4	8.4
6	7.5	7.5	5.1	9.2	4.1	6.2	7.5	7.5	7.5	7.5
7	8.3	8.3	4.5	10.1	4.5	5.6	6.8	8.3	6.8	6.8
8	9.1	9.1	5.0	11.2	5.0	5.0	6.1	9.1	6.1	6.1
9	10.0	10.0	5.5	12.3	5.5	4.5	5.5	8.2	6.7	6.7
10	9.0	9.0	6.1	13.5	6.1	4.1	5.0	7.4	6.1	7.4
11	9.9	8.1	6.7	12.2	6.7	4.5	4.5	8.1	5.4	6.7
12	8.9	8.9	7.3	10.9	7.3	4.0	4.0	8.9	4.9	6.0
13	8.1	9.8	8.1	9.8	8.1	4.4	3.6	9.8	4.4	5.4
14	8.9	8.9	7.2	8.9	7.2	4.0	3.2	10.8	4.0	4.9
15	9.7	8.0	6.5	8.0	6.5	4.4	2.9	9.7	4.4	5.3
16	8.8	7.2	7.2	8.8	7.2	3.9	3.2	10.7	4.8	4.8
17	7.9	6.5	7.9	9.6	7.9	3.5	3.5	9.6	5.3	4.3
18	7.1	7.1	8.7	10.6	8.7	3.2	3.9	10.6	5.8	4.8
19	7.8	7.8	7.8	11.7	9.6	2.9	4.3	11.7	6.4	5.2
20	8.6	7.0	8.6	10.5	8.6	3.2	4.7	12.8	7.0	4.7

*Each interest rate in a given path is the (annualized) 6 month zero coupon interest rate simulated for that path. Each value is calculated by randomly multiplying the interest rate for the previous period by either $(1.1 = 1 + .1)$ or $(.9 = 1 - .1)$. Each path provides a different random result for the six month interest rate process.

Steps 1 and 2 – see Table 6.6/z

- Calculate the sequence of one period implied zero coupon interest rates from current yield curve – method discussed in Lecture 4 – these are needed to ‘calibrate’ the stochastic model.
- Generating the future one period Treasury spot rates (zero coupon interest rates)
 - A number of different methods available – usually in the form of a one factor or two factor – when only the one period ahead spot rates are generated, this is a **one factor model**
 - Two factor models generate both the short and long spot interest rates
 - This allows the stochastic model to capture a wider range of term structure shapes at the expense of increased complexity



Calculated Treasury Spot Rates for the Ten Paths in Table 6-z*

Semi-Annual Period	Paths									
	1	2	3	4	5	6	7	8	9	10
1	7.00%	7.00%	7.00%	7.00%	7.00%	7.00%	7.00%	7.00%	7.00%	7.00%
2	7.35	7.35	7.35	6.65	6.65	6.65	7.35	6.65	7.35	7.35
3	7.72	7.21	7.21	6.74	6.32	6.32	7.72	6.32	7.72	7.72
4	7.70	6.97	6.97	6.96	6.02	6.30	8.12	6.30	8.12	8.12
5	7.53	6.95	6.69	7.25	5.73	6.41	8.17	6.41	8.17	8.17
6	7.53	7.05	6.42	7.57	5.46	6.37	8.07	6.60	8.07	8.07
7	7.64	7.22	6.15	7.94	5.33	6.26	7.89	6.84	7.89	7.89
8	7.83	7.46	6.01	8.34	5.29	6.10	7.66	7.13	7.66	7.66
9	8.07	7.75	5.95	8.77	5.31	5.92	7.42	7.25	7.56	7.56
10	8.17	7.88	5.96	9.24	5.39	5.73	7.17	7.26	7.41	7.54
11	8.33	7.90	6.02	9.51	5.50	5.62	6.93	7.34	7.23	7.46
12	8.38	7.99	6.13	9.62	5.65	5.48	6.68	7.48	7.03	7.34
13	8.36	8.13	6.28	9.64	5.84	5.40	6.44	7.66	6.83	7.19
14	8.39	8.18	6.35	9.59	5.94	5.30	6.21	7.88	6.63	7.02
15	8.48	8.17	6.36	9.48	5.98	5.24	5.99	8.01	6.47	6.91
16	8.50	8.11	6.41	9.43	6.05	5.15	5.82	8.17	6.37	6.78
17	8.46	8.01	6.50	9.45	6.16	5.06	5.68	8.26	6.31	6.63
18	8.39	7.96	6.62	9.51	6.30	4.95	5.58	8.39	6.28	6.53
19	8.36	7.95	6.68	9.62	6.47	4.84	5.51	8.56	6.28	6.46
20	8.37	7.90	6.78	9.67	6.58	4.76	5.47	8.77	6.32	6.37

* Annualized semi-annual zero coupon interest rates calculated from the one period zeros in Table 6-z by assuming the local expectations hypothesis and observing that the hypothesis implies: $z_{t,t+1} = f_{t,t+1}$.

Calibration, Monte Carlo and the Interest Rate Model

- In Table 6.6 a 'naïve' binomial model is used where the short term interest rate can move up or down by 10%
 - As in 6.6 a sufficient number of paths are generated
 - In actual Monte Carlo the number of paths is much larger
 - Is this a 'recombining' binomial process?
- Generate the term structure of interest rates (Table 6.7)
 - Sum over all the paths and average – 'calibration' requires that the average reproduces the current term structure from Step 1 → if not 'calibrate'
 - More sophisticated 'self-calibrating' processes define the steps in the binomial process to ensure that , when summing over all the paths, the initial term structure is captured, e.g., Black-Derman-Toy & Ho-Lee.



Step 3: Table 6.7/y

Generating the term structure from the one-period spot rate paths

Recall the calculation of implied forward rates, treat the generated future one period rates as implied forward rates (see SAIS, ch.4, p.229); *recall comment about use of semi-annual coupon bond*

$$(1 + z_2)^2 = (1 + z_1)(1 + f_{1,2}) = (1 + z_1)(1 + r_{1,2})$$

$$(1 + z_3)^3 = (1 + z_1)(1 + f_{1,2})(1 + f_{2,3}) = (1 + z_1)(1 + r_{1,2})(1 + r_{2,3})$$

$$(1 + z_4)^4 = (1 + z_1)(1 + f_{1,2})(1 + f_{2,3})(1 + f_{3,4}) = (1 + z_1)(1 + r_{1,2})(1 + r_{2,3})(1 + r_{3,4})$$

$$\left(1 + \frac{z_2}{2}\right)^2 = \left(1 + \frac{z_1}{2}\right)\left(1 + \frac{f_{1,2}}{2}\right) = \left(1 + \frac{.07}{2}\right)\left(1 + \frac{.077}{2}\right) = 1.03675 \quad \rightarrow \quad z_2 = 7.35\%$$



Steps 4-5: Specifying the Cash Flows (Table 6.8/v-w)

- Key difficulty of OAS is the need to specify the decision rule applicable to the security
 - The example in Table 6.8 is for a **callable bond**.
 - Recall previous discussion about the empirical aspects of callable bond exercise.
 - The (not so realistic) decision rule used to generate the cash flows in the lower tableau of Table 6.8 is described at the bottom of the Table (see copy below)
 - The top part of Table is produced by adding 100 bp to Table 6.6. Then look for values 5.8% below to produce cash flows in bottom of table

* Refinancing rate paths are determined by adding 100 basis points to the one period Treasury zero paths in Table 6-z. Cash flow for each refinancing path is for an 8.8% semi-annual coupon 10 year corporate bond callable at 103. The call rule used is that if the refinancing rate falls to 5.8% or less and bond has at least three years to maturity then the bond is called on the coupon payment date at 103 to produce a payment of $103 + 4.4 = 107.4$



Refinancing Rate Paths and Callable Bond Cash Flow for Each Refinancing Path*

Semi-annual	Refinancing rate paths									
Period	1	2	3	4	5	6	7	8	9	10
1	8.0%	8.0%	8.0%	8.0%	8.0%	8.0%	8.0%	8.0%	8.0%	8.0%
2	8.7	8.7	8.7	7.3	7.3	7.3	8.7	7.3	8.7	8.7
3	9.5	7.9	7.9	7.9	6.7	6.7	9.5	6.7	9.5	9.5
4	8.6	7.2	7.2	8.6	6.1	7.2	10.3	7.2	10.3	10.3
5	7.9	7.9	6.6	9.4	5.6	7.9	9.4	7.9	9.4	9.4
6	8.5	8.5	6.1	10.2	5.1	7.2	8.5	8.5	8.5	8.5
7	9.3	9.3	5.5	11.1	5.5	6.6	7.8	9.3	7.8	7.8
8	10.1	10.1	6.0	12.2	6.0	6.0	7.1	10.1	7.1	7.1
9	11.0	11.0	6.5	13.3	6.5	5.5	6.5	9.2	7.7	7.7
10	10.0	10.0	7.1	14.5	7.1	5.1	6.0	8.4	7.1	8.4
11	10.9	9.1	7.7	13.2	7.7	5.5	5.5	9.1	6.4	7.7
12	9.9	9.9	8.3	11.9	8.3	5.0	5.0	9.9	5.9	7.0
13	9.1	10.8	9.1	10.8	9.1	5.4	4.6	10.8	5.4	6.4
14	9.9	9.9	8.2	9.9	8.2	5.0	4.2	11.8	5.0	5.9
15	10.7	9.0	7.5	9.0	7.5	5.4	3.9	10.7	5.4	6.3
16	9.8	8.2	8.2	9.8	8.2	4.9	4.2	11.7	5.8	5.8
17	8.9	7.5	8.9	10.6	8.9	4.5	4.5	10.6	6.3	5.3
18	8.1	8.1	9.7	11.6	9.7	4.2	4.9	11.6	6.8	5.8
19	8.8	8.8	8.8	12.7	10.6	3.9	5.3	12.7	7.4	6.2
20	9.6	8.0	9.6	11.5	9.6	4.2	5.7	13.8	8.0	5.7



Period	Cash Flow Paths									
	1	2	3	4	5	6	7	8	9	10
1	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4
2	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4
3	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4
4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4
5	4.4	4.4	4.4	4.4	107.4	4.4	4.4	4.4	4.4	4.4
6	4.4	4.4	4.4	4.4	0.0	4.4	4.4	4.4	4.4	4.4
7	4.4	4.4	107.4	4.4	0.0	4.4	4.4	4.4	4.4	4.4
8	4.4	4.4	0.0	4.4	0.0	4.4	4.4	4.4	4.4	4.4
9	4.4	4.4	0.0	4.4	0.0	107.4	4.4	4.4	4.4	4.4
10	4.4	4.4	0.0	4.4	0.0	0.0	4.4	4.4	4.4	4.4
11	4.4	4.4	0.0	4.4	0.0	0.0	107.4	4.4	4.4	4.4
12	4.4	4.4	0.0	4.4	0.0	0.0	0.0	4.4	107.4	4.4
13	4.4	4.4	0.0	4.4	0.0	0.0	0.0	4.4	0.0	4.4
14	4.4	4.4	0.0	4.4	0.0	0.0	0.0	4.4	0.0	4.4
15	4.4	4.4	0.0	4.4	0.0	0.0	0.0	4.4	0.0	4.4
16	4.4	4.4	0.0	4.4	0.0	0.0	0.0	4.4	0.0	4.4
17	4.4	4.4	0.0	4.4	0.0	0.0	0.0	4.4	0.0	4.4
18	4.4	4.4	0.0	4.4	0.0	0.0	0.0	4.4	0.0	4.4
19	4.4	4.4	0.0	4.4	0.0	0.0	0.0	4.4	0.0	4.4
20	104.4	104.4	0.0	104.4	0.0	0.0	0.0	104.4	0.0	104.4

* Refinancing rate paths are determined by adding 100 basis points to the one period Treasury zero paths in Table 6-z. Cash flow for each refinancing path is for an 8.8% semi-annual coupon 10 year corporate bond callable at 103. The call rule used is that if the refinancing rate falls to 5.8% or less and bond has at least three years to maturity then the bond is called on the coupon payment date at 103 to produce a payment of $103 + 4.4 = 107.4$



Steps 6-7: Calculating Prices for each Path (Table 6.10/u)

- Recall the formula for a bond price using the spot rates and static spread (see previous slide)
 - Using the spot rate along each path calculate prices for a given level of static spread (60 bp highlighted)
 - These form the rows in Table 6.10/u
 - For example, the first row calculates all the bond prices using $ss = 40$ bp
 - Calculate the average price for each row by summing the prices and dividing by the number of paths, this is the price given in the last column (*Average PV*)
- **Compare the averages in the last column with the observed market price, if the price is \$103.3 then the OAS is 80 bp**



TABLE 9-11
Present Value of Each Path and Average Present Value Using Various Spreads

Trial OAS	Present Value of Path										Average PV
	1	2	3	4	5	6	7	8	9	10	
40 bp	100.6	103.5	107.5	93.73	108.5	111.5	107.9	100.4	107.7	112.4	105.4
50 bp	100.0	102.9	107.3	93.14	108.3	111.1	107.4	99.77	107.2	111.6	104.9
60 bp	99.35	102.2	107.0	92.55	108.0	110.7	106.9	99.13	106.7	110.8	104.3
70 bp	98.70	101.5	106.7	91.97	107.8	110.3	106.5	98.49	106.2	110.1	103.8
80 bp	98.06	100.8	106.4	91.40	107.5	109.9	106.0	97.86	105.7	109.3	103.3
90 bp	97.43	100.2	106.1	90.83	107.3	109.5	105.5	97.24	105.3	108.6	102.8
100 bp	96.80	99.58	105.9	90.26	107.1	109.1	105.1	96.62	104.8	107.9	102.3
110 bp	96.18	98.93	105.6	89.70	106.8	108.7	104.6	96.00	104.3	107.1	101.8
120bp	95.56	98.29	105.3	89.14	106.6	108.3	104.2	95.39	103.8	106.4	101.3



Step 8: Determining the OAS

Formula given below describes the calculation of the OAS in a sophisticated closed form

- $s = 1$ to S refers to the number of states (paths in the example)
- OAS is used below because the **OAS is determined by comparing the *Average PV* from Table 6.10/u with the actual callable bond price observed in the bond market**
- Denominator uses product operator

$$P_{OB} = \frac{1}{S} \sum_{s=1}^S \sum_{t=1}^T \frac{CF_t^s}{\prod_{i=1}^t (1 + z_{i-1, i}^s + OAS)}$$



Step 9: Recalculating OAS for different volatilities

- OAS depends on assumption about volatility of underlying state variable (interest rates in the example)
 - Changing the volatility assumption involves changing the step moves in the binomial process
 - For example, instead of up-down by 10%, could use 5% or 15%
- See examples in Table 6-t at end of Tables 6-10



Calculating the Cost of the Option

- $P_{OB} = P_B \pm P_{OP}$
- Only P_{OB} is observed P_B and P_{OP} have to be estimated
 - (-) is used because in this example, the contingency is for a callable bond
- To calculate the straight bond price use OAS for the static spread and price the bond as though there was no contingency
 - Determine $P_{OP} = P_B - P_{OB}$ for the callable bond
 - Because $OAS < ss$ it follows $P_{OPT} > 0$



OAS Pitfalls (SAIS, p.338-41)

- An OAS calculation is not like a yield to maturity calculation.
 - Different OAS values can be calculated for the same bond, depending on the procedure employed to estimate the OAS, see Kupiec and Kah (1999).
 - OAS is model dependent – model involves stochastic process assumption and exercise decision assumption
- The widespread use of OAS in assessing value for bonds with embedded options has led to a number of contributions aimed at identifying pitfalls in the implementation and interpretation of OAS,
 - e.g., Babbel and Zenios (1992), Kopprasch (1994).



More on OAS Pitfalls

- OAS aims at providing a number that can be used in the same manner as a traditional yield spread → esp., a basis point difference between a riskless (Government) straight bond a corporate bond.
 - This involves additive ss – this ignores the more realistic cases of multiplicative ss or ss the differs with term to maturity of the z
- OAS assumes only one source of randomness
 - What are the sources of randomness for MBS and CDO's?
 - What causes prepayments for MBS?

