SECURITY ANALYSIS AND PORTFOLIO MANAGEMENT

Selected Solutions to Problem Set #3:

Section I: Portfolio Management

3) Portfolio Expected Return

The expected return on the portfolio $E[R_p]$ is the value weighted sum of the expected returns on the individual securities, the $E[R_i]$:

$$E[R_p] = \sum_{i=1}^{k} W_i \, E[R_i]$$

where k is the number of securities in the portfolio. To calculate the value weights, W_i:

$$W_i = \frac{\$A_i}{\sum_{i=1}^{k} A_i} \qquad where \qquad \sum_{i=1}^{k} W_i = 1$$

with A_i being the dollar value invested in security i.

Example:

If \$1 million is invested in a portfolio of 2 securities, and there is \$500,000 in each security, then each W_i = .5.

Portfolio Standard Deviation

As with calculating the risk for individual securities, calculations are done for the variance and the standard deviation is determined by taking a square root.

Various types of **EQUIVALENT** forms for the portfolio variance:

$$var[R_p] = \sigma_p^2 = \sum_{i=1}^{k} \sum_{j=1}^{k} W_i \, W_j \, \sigma_{ij}$$

$$= \sum_{i=1}^{k} W_i^2 \, \sigma_i^2 + 2 \sum_{i>j}^{k} W_i \, W_j \, \sigma_{ij}$$

$$= \sum_{i=1}^{k} W_i^2 \, \sigma_i^2 + 2 \sum_{j=1}^{k} \sum_{i=1,\, i>j}^{k} W_i \, W_j \, \sigma_{ij}$$

In the double sum expression, when i=j the covariance is a variance.

It is easiest to understand these results for the case where k = 2, when there are only two securities in the portfolio. In this case:

$$\sigma_{[}^2 = W_1^2 \sigma_1^2 + W_2^2 \sigma_2^2 + 2 W_1 W_2 \sigma_{1,2}$$

Similarly for 3 assets in the portfolio:

$$\sigma_p^2 = W_1^2 \sigma_1^2 + W_2^2 \sigma_2^2 + W_3^2 \sigma_3^2$$

$$+ 2 \{W_1 W_2 \sigma_{1,2} + W_1 W_3 \sigma_{1,3} + W_2 W_3 \sigma_{2,3}\}$$

The Minimum Variance Portfolio for Two Securities

Which portfolio has the smallest risk?

----> Minimize the variance--

Using the result that $W_1 + W_2 = 1$, for a portfolio with two securities:

$$\sigma_p^2 = W_1^2 \sigma_1^2 + (1 - W_1)^2 \sigma_2^2 + 2 W_1 (1 - W_1) \sigma_{1,2}$$

$$\frac{d\sigma_p^2}{dW_1} = 2 W_1 \sigma_1^2 - 2 (1 - W_1) \sigma_2^2 + 2 (1 - 2W_1) \sigma_{1,2} = 0$$

$$W_1^* = \frac{\sigma_2^2 - \sigma_{1,2}}{\sigma_1^2 + \sigma_2^2 - 2 \sigma_{1,2}}$$

The Portfolio Optimization Problem

Different combinations of securities produce different combinations of $E[R_p]$ and F_p

In general, when there are k securities in the portfolio, the rational investor will wnat to choose he optimal combination of securities which achieves the smallest possible portfolio risk for a given level of expected return. This leads to the optimization problem associated with calculating the **efficient frontier**

$$\underset{\{W_i\}}{Min} \sum_{i=1}^{k} W_i^2 \sigma_i^2 + 2 \sum_{i>j} W_i W_j \sigma_{i,j}$$

subject to:

$$\sum_{i=1}^{k} W_i E[R_i] = given\ E[R_p] \qquad \sum_{i=1}^{k} W_i = 1$$

The efficient frontier provides a set of portfolios all with the smallest variance but with different levels of expected return.

In various textbooks, the efficient set is also referred by other names, such as the investment opportunity set or the efficient set.

Key point: The shape and location of the efficient frontier depends on whether **short selling** of securities is permitted.

Special Case: In general where there are many possible securities, solution of the efficient set from the optimization problem is complicated. For purposes of illustration, it is usually assumed that there is only two **risky** securities. In this case it is possible to derive the efficient frontier directly and to illustrate the concepts.

Measuring Expected Returns: Domestic and International Securities

In the **notation** I will be using this is expressed

$R = (P_1 - P_0 + Div.)/P_0 = [(P_1 - P_0)/P_0] + [Div./P_0]$
$= [P_1/P_0] + [Div./P_0] - 1$

= Capital Gain (Loss) + Dividend (or Coupon) Yield

R is often calculated over a specific interval, typically one year.

Two different methods of determining the average over time:

Arithmetic Average and Geometric Average

The arithmetic average (R_A) would sum the four R and divide by four to get the average. In general:

$$R_A = \frac{\sum_{t=1}^{T} R_t}{T}$$

The geometric average (R_G) uses the product of the growth rates:

$$R_G = \{\prod_{t=1}^{T} (1 + R_t)\}^{1/T} - 1$$

Problems with the Arithmetic Average:

Buy a stock for $100

First year stock falls to $50 (R = -50%)
Second year stock rises to $100 (R = +100%)

$R_A = (-50 + 100)/2 = 25\%$

$R_G = (1 + (-.5))(1 + 1) - 1 = 0\%$

Arithmetic average provide correct results when averaging across different securities at one point in time

Arithmetic average provides an unbiased estimate of the return over the next period, when using past history of past returns.

Calculating the Domestic Return (R^D) on a Security Denominated in Foreign Currency

To calculate the domestic return, have to convert the values in R to domestic currency

$R = ((P_1\ S_1) - (P_0\ S_0) + (Div.\ S_1))/(P_0\ S_0)$

$$R_D = \frac{(P_1\ S_1) - (P_0\ S_0)}{P_0\ S_0} + \frac{Div_1\ S_1}{P_0\ S_0}$$

$$= \{\frac{P_1 + Div_1}{P_0}\}\{\frac{S_1}{S_0}\} - 1$$

$$= (1 + R)\ (1 + e) - 1$$

International Investment

Because of the presence of the exchange rate risk, the variance of this return is more complicated than for a domestic asset

$$var[R_\$] = \sigma_\$^2 = \sigma_£^2 + \sigma_e^2 + 2\ \sigma_{£,e}$$

The introduction of foreign assets into the portfolio significantly complicates the calculation of the portfolio variance.

Example: Portfolio composed of one domestic and one foreign asset

Using the variance formula for two securities from Lecture 2

$$\sigma_p^2 = W_d^2\ \sigma_d^2 + W_\$^2\ \sigma_\$^2 + 2\ W_d\ W_\$\ \sigma_{d,\$}$$

$$= W_d^2\ \sigma_d^2 + W_\$^2\ \{\sigma_£^2 + \sigma_e^2 + 2\ \sigma_{£,e}\} + 2\ \{W_\$\ W_d\ (\sigma_{£,d} + \sigma_{e,d})\}$$

Presence of foreign assets significantly complicates the portfolio variance

Example: You are considering purchasing two portfolios. One portfolio is composed 50/50 of two domestic assets each with E[R] = .1 and F = .15 and with a .5 correlation between the asset returns. The other portfolio is also 50/50 and contains one of these domestic assets and a foreign asset. The foreign asset has $E[R_{\$}] = .1$ with $F_{£} = .15$ and $F_{e} = .03$. The correlations between the foreign and domestic asset returns and between all the asset returns and the exchange rate are zero.

a) Calculate the portfolio variance for these two portfolios.

b) What can you conclude about the risk reduction properties of including foreign assets in your portfolio?

Solution: a) The portfolio variance for the domestic assets is just the conventional result. To get the portfolio variance when there is a foreign asset observ that the return on a foreign asset when the return in denominated in domestic currency ($R_{\$}$) is given as:

$R_{\$} = (1 + R_{£})\ (1 + e) - 1$

Taking logs and observing ln(1 + x) is approximately equal to x when x is sufficiently small produces the result:

$$var[R_{\$}] = \sigma_{\$}^2 = \sigma_{£}^2 + \sigma_{e}^2 + 2\ \sigma_{£,e}$$

Using the variance formula for two securities and doing appropriate substitutions, it follows that for a portfolio containing a foreign asset:

$$\sigma_p^2 = W_d^2\ \sigma_d^2 + W_{\$}^2\ \sigma_{\$}^2 + 2\ W_d\ W_{\$}\ \sigma_{d\$}$$

$$= W_d^2\ \sigma_d^2 + W_{\$}^2\ \{\sigma_{£}^2 + \sigma_{e}^2 + 2\ \sigma_{£,e}\} + 2\ \{W_{\$}\ W_d\ (\sigma_{£,d} + \sigma_{e,d})\}$$

Evaluating the relevant formulas gives for the domestic portfolio $var[R_{dp}] = .016875$ ($F_{dp} = .13$) and $var[R_{\$dp}] = .011475$ ($F_{\$dp} = .107121$)

b) Due to the much lower correlation between domestic asset returns and foreign asset returns and the exchange rate (than with other domestic asset returns) including foreign assets enhances the diversification process considerably.

The Efficient Frontier with International Assets

Recall the equally weighted portfolio from Tutorial 5, #1. It was demonstrated that as the number of securities in the equally weighted portfolio increases, the variance of the portfolio is reduced to the covariance between the individual securities.

Key Point: International assets differ from domestic assets in that the returns have a much lower covariance with domestic assets

Foreign assets have low covariance with domestic assets for primarily two reasons:

1) The presence of exchange rate risk, which typically has low correlation with domestic and foreign security risks.

2) Foreign stock returns, denominated in foreign currency terms, have lower covariance with domestic

assets than domestic assets have with other domestic assets.

Example: The US stock market has been on a bull run over the past four-five years while the Japanese stock market has been doing poorly.

It is not always the case that foreign and domestic markets have low covariance. Both the Canadian and Hong Kong markets have historically had high covariance with US stocks.

Because of the lower covariance with domestic assets, as well as by increasing the number of assets available for portfolio selection, international assets will produce an outward shift in the efficient frontier.

Section II: Derivative Securities

1) i) Warrants are issued by the company issuing the stock. As a result, when an investor exercises a warrant and buys the stock, the company issues new stock increasing the number of shares outstanding. The proceeds from the sale of stock represents a cash inflow to the company. Exercise of a call option involves only a transaction between two parties with either a cash settlement or an exchange of stock taking place. There is no impact on the number of shares outstanding.

ii) Terms and conditions for call options are governed by standardized contracts, in the US specied by the Options Clearing Corporation. Standard options contracts have features such as regular expiration dates, the absence of dividend payout protection, and possibility of early exercise (American option). Most call option contracts have term to maturity when originally written of nine months and less. Warrant contracts are non-standardized and can have a wide variation in possible features (see page 298-9 of readings for a list of possible variations). When originally offered warrants are almost always long dated, with maturities around five years being common.

2) This question is answered on p.299-300 of the readings. An alternative derivation is given here. Assume for simplicity the firm is all equity financed. (If there is debt there are extra terms to write down but nothing much is changed). If the firm has N shares outstanding with a current market price of S(t) then the current market price is V(t) = N S(t). Now let the firm issue n European-style warrants with each warrant entitling the holder to purchase one share of stock at X at time T. Assume that these warrants are distributed on a pro-rata basis to the firm's shareholders of record.

At time T, if the warrants are exercised then:

$$V(T)^* = N\,S(T) + n\,X = N\,(S(T) + q\,X)$$

where q = n/N and S(T) is the stock price immediately before the warrants are exercised. The warrant holders, being smart investors, realize that for exercise to be profitable, it is not the price immediately prior to exercise which is relevant, but rather the stock price immediately after exercise S(T)* which is relevant. (Why?) It follows that:

$$S(T)^* = V(T)^*/(N + n) = [S(T) + qX]/(1 + q)$$

In other words, warrant exercise will occur if S(T)* > X which means:

$$\{[S(T) + qX]/(1 + q)\} - X > 0$$

Manipulating gives:

$$[S(T) + qX - (1+q)X]/(1+q) = [S(T) - X]/(1+q) > 0$$

Observing that the value of the warrant on the expiration date is:

$$W(T) = \max[0, S(T)^* - X] = \max[0, S(T) - X]/(1+q) = C(T)/(1+q)$$

Because the current price of the warrant is the discounted expected value of the payout of the warrant at maturity, it follows: $W(t) = C(t)/(1+q)$

3)a) Question II.2 gave the adjustment to the Black-Scholes call price required to value the warrant: $W = C/(1 + (n/N))$. To determine the value of the warrant in this case requires the value of the call option to be estimated using the values provided. This requires an assumption about the dividends on the stock which, because none are stated will be assumed to be zero. Using the Black-Scholes formula for a non-dividend paying stock gives $C = 9.20$ and doing the adjustment for the warrant price gives: $W = 8.36$

b) Assuming all the warrants are exercised, there will be 110,000 shares outstanding. Taking the $5.2 million to be the market value before the warrants have been exercised, the market value of the firm will rise by the $500,000 amount received from warrant exercise to give a stock price of $51.82.