Bond Pricing

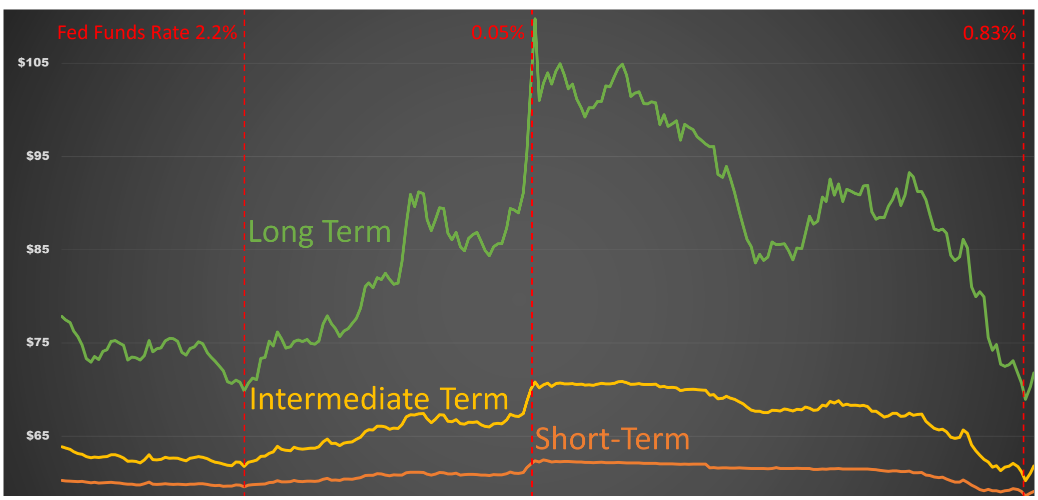
Bond prices are determined by 5 factors:

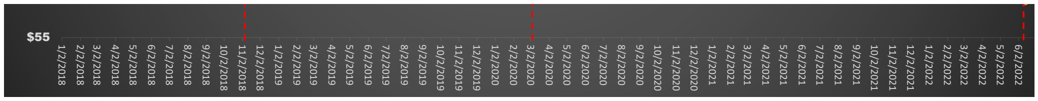
1. par value
2. coupon rate
3. market interest rates
4. accrued interest
5. credit rating of the issuer

Generally, the issuer sets the price and the yield of the bond so that it will sell enough bonds to supply the amount that it desires. The higher the credit rating of the issuer, the lower the yield that it must offer to sell its bonds. A change in the credit rating of the issuer will affect the price of its bonds in the secondary market: a higher credit rating will increase the price, while a lower rating will decrease the price. The other factors that determine the price of a bond have a more complex interaction.

When a bond is first issued, it is generally sold at **par**, which is the **face value** of the bond. Most corporate bonds, for instance, have a face and par value of $1,000. The par value is the principal, which is received at the end of the bond's term, i.e., at **maturity**. Sometimes when the demand is higher or lower than an issuer expected, the bonds might sell higher or lower than par. In the secondary market, bond prices are almost always different from par, because interest rates change continuously. When a bond trades for more than par, then it is selling at a **premium**, which will pay a lower yield than its stated coupon rate, and when it is selling for less, it is selling at a **discount**, paying a higher yield than its coupon rate. When interest rates rise, bond prices decline, and vice versa. Bond prices will also include accrued interest, which is the interest earned between coupon payment dates. **Clean bond prices** are prices without accrued interest; **dirty bond prices** include accrued interest.

This graph shows how interest rates even affect exchange-traded funds based on bonds. Note that the 3 Vanguard ETFs (VGSH < https://investor.vanguard.com/investment-products/etfs/profile/vgsh > , VGIT < https://investor.vanguard.com/investment-products/etfs/profile/vgit > , VGLT < https://investor.vanguard.com/investment-products/etfs/profile/vglt >) based on US Treasuries were affected similarly when interest rates changed from 2018 to 2022, but the long-term ETF, VGLT < https://investor.vanguard.com/investment-products/etfs/profile/vglt > , changed in price by a much larger percentage even though interest rates changed by only 2%. This reflects the greater interest-rate sensitivity of long-term bonds over short-term bonds.

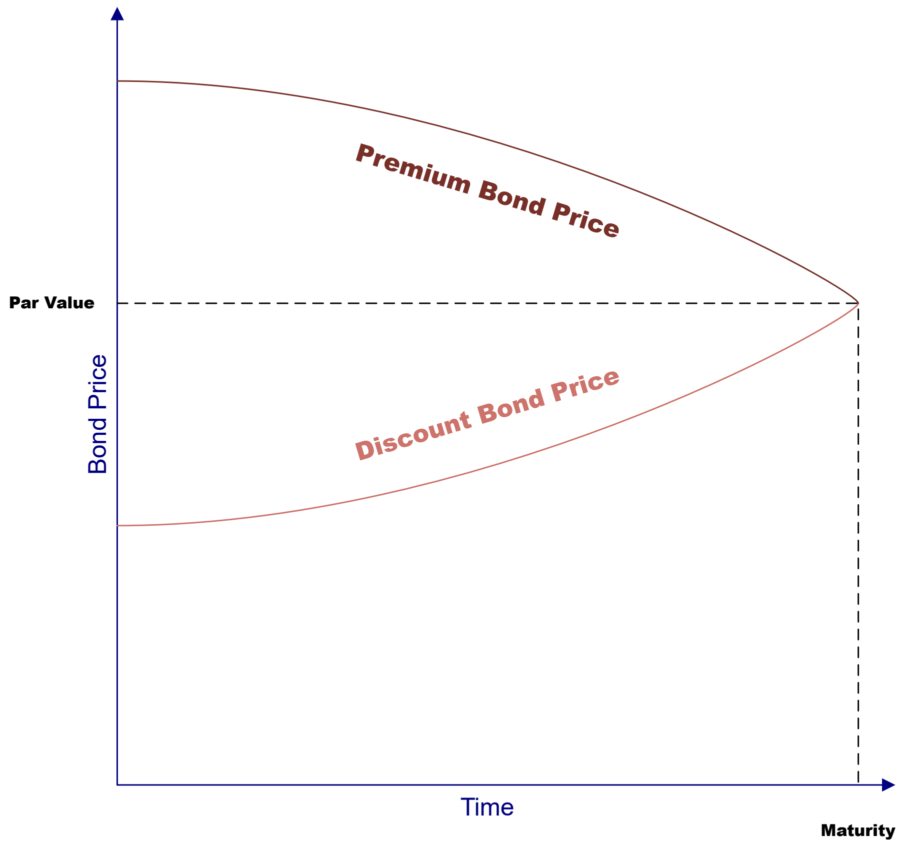




Tips

Investment Tip. Buy long-term bonds when interest rates are highest. You can usually determine when interest rates are near the top by monitoring the economy. The Federal Reserve increases interest rates when inflation is high or increasing. If inflation is subdued, then the Federal Reserve will not increase interest rates further since that would depress the economy. Never buy long-term bonds when interest rates are near 0% unless you intend to keep the bonds until maturity because interest rates can only rise! For instance, if you had bought VGLT < https://investor.vanguard.com/investment-products/etfs/profile/vglt > at the end of 2018 and held until March, 2020, you would have earned a capital gain of more than 40% while earning a nice, guaranteed interest rate. VGLT < https://investor.vanguard.com/investment-products/etfs/profile/vglt > spiked when the Fed funds rate dropped to near zero for the Covid-19 pandemic, That's when you sell since interest rates can only rise from there, and by mid-2022, they were rising fast because inflation was high. Naturally, this causes bond prices to drop, including VGLT < https://investor.vanguard.com/investment-products/etfs/profile/vglt > , as you can see in the graph.

Although prevailing interest rates are usually the main determinants of bond prices in the secondary market, as a bond approaches maturity, the present value of its future payments converges to the par value; therefore, the par value becomes more important than the prevailing interest rates since the bond price, whether at a premium or discount, converges to the par value, as can be seen in the diagram below.



Bond Value = the Sum of the Present Value of Future Payments

A bond pays interest either periodically or, in the case of zero coupon bonds, at maturity. Therefore, the value of the bond = the sum of the present value of all future payments — hence, it is the present value of an annuity, which is a series of periodic payments. The present value is calculated using the prevailing market interest rate for the term and risk profile of the bond, which may be more or less than the coupon rate. For a coupon bond that pays interest periodically, its value can be calculated thus:

Bond Value

- **= Present Value (PV) of Interest Payments**
- **+ Present Value of Principal Payment**

Bond Value

- **= PV(1st Payment)**
- **+ PV(2nd Payment) + ... + PV(Last Payment)**
- **+ PV(Principal Payment)**

Bond Price Formula

$$\text{Clean Bond Price} = \frac{C_1}{(1+r/k)^1} + \frac{C_2}{(1+r/k)^2} + \ldots + \frac{C_n}{(1+r/k)^{k\,n}} + \frac{P}{(1+r/k)^{k\,n}}$$

- **C** = coupon, or interest, payment per period
- **k** = number of coupon periods in 1 year
- **n** = number of years until maturity
- **r** = annualized market interest rate
- **P** = par value of bond

Examples

Example: Calculating Bond Value as the Present Value of its Payments

Suppose a company issues a 3-year bond with a par value of $1,000 that pays 4% interest annually, which is also the prevailing market interest rate. What is the present value of the payments?

The next table shows the amount received each year and the present value of that amount. As you can see, the sum of the present value of each payment equals the par value of the bond.

Year	Payment	Amount Received	Present Value
1	Interest	$40	$38.46
2	Interest	$40	$36.98
3	Interest + Principal	$1040	$924.56
	Totals	**$1120**	**$1,000.00**

The above formula can be simplified by using the formula for the present value of an annuity, and letting k=2 for bonds that pay a semiannual coupon:

Simplified Bond Price Formula for Semiannual Coupon Bonds

$$\text{Clean Bond Price} = \frac{C}{r}\left[1 - \frac{1}{(1+r/2)^{2n}}\right] + \frac{P}{(1+r/2)^{2n}}$$

- **C** = Annual payment from coupons
- **n** = number of years until maturity
- **r** = market annual interest rate
- **P** = par value of bond

Note that the above formula is sometimes written with both C and r divided by 2; the results are the same since it is a ratio.

Examples

Example: Using the Simplified Bond Pricing Formula

Given:

- **Par Value: 100**
- Nominal Yield: 5%
- **Annual Coupon Payment: $5**
- **Maturity: 5 years**
- **Market Interest Rate = 4%**

Case 1: 2 Annual Coupon Payments

With **10 semiannual payment periods**, the **market interest rate is divided by 2** to account for the shorter period:

$$\text{Bond Price} = \frac{5}{.04}\left[1 - \frac{1}{(1.02)^{10}}\right] + \frac{100}{(1.02)^{10}} = 104.49$$

Case 2: 1 Annual Coupon Payment, resulting in **5 payment periods** at the market interest rate:

$$\text{Bond Price} = \frac{5}{.04}\left[1 - \frac{1}{(1.04)^5}\right] + \frac{100}{(1.04)^5} = 104.45$$

In the primary bond market, where the buyer buys the bond from the issuer, the bond usually sells for par value, which = the bond's value using the coupon rate of the bond. However, in the secondary bond market, bond price still depends on the bond's value, but the interest rate to calculate that value is determined by the market interest rate, which is reflected in the actual bids and offers for bonds. Additionally, the buyer of the bond must pay any accrued interest on top of the bond's price unless the bond is purchased on the day it pays interest.

Bond Price Listings

When bond prices are listed, the convention is to list them as a percentage of par value, regardless of what the face value of the bond is, with 100 being equal to par value. Thus, a bond with a face value of $1,000 selling for par, sells for $1,000, and a bond with a face value of $5,000 also selling for par will both have their price listed as 100, meaning their prices are equal to 100% of par value, or $100 for each $100 of face value.

This pricing convention allows different bonds with different face values to be compared directly. For instance, if a $1,000 corporate bond was listed as 90 and a $5,000 municipal bond was listed as 95, then it can be easily seen that the $1,000 bond is selling at a bigger discount, and, therefore, has a higher yield. To find the actual price of the bond, the listed price must be multiplied as a percentage by the face value of the bond, so the price for the $1,000 bond is 90% × $1,000 = 0.9 × $1,000 = $900, and the price for the $5,000 bond is 95% × $5,000 = .95 × $5,000 = $4,750.

A **point** = 1% of the bond's face value. Thus, a point's actual value depends on the face value of the bond. Thus, 1 point = $10 for a $1,000 bond, but $50 for a $5,000 bond. So a $1,000 bond that is selling for 97 is selling at a 3 point discount, or $30 below par value, which equals $970.

Brokers profit from bonds either by charging a set commission or by charging a markup, a certain percentage over and above what the broker paid for the bond. Only a small portion of the more than 1 million bonds available are sold on public exchanges like the New York Stock Exchange, where pricing is transparent. Instead, most bonds are traded over the counter. Most prices listed by brokers do not include any markup that they may charge, but some brokers, such as Fidelity, may charge a set commission, such as $1 per bond.

You can compare prices by comparing listed prices by different brokers if you have more than 1 brokerage account. Brokers may not have the same bonds with the same CUSIP number, but they will have comparable bonds that should have the same yield, such as a 5- year corporate bond with an AAA rating.

You can also check trade reporting data provided by the Municipal Securities Rulemaking Board

(MSRB) < https://www.msrb.org/ > for municipal bonds and by the Trade Reporting and Compliance Engine (TRACE) < https://www.finra.org/filing-reporting/trace > for fixed income securities traded over-the-counter.

Accrued Interest

Listed bond prices are **clean prices** (aka **flat prices**), which do not include accrued interest. Most bonds pay interest semi-annually. For settlement dates when interest is paid, the bond price = the flat price. Between payment dates, accrued interest must be added to the flat price, which is often called the **dirty price** (aka **all-in price**, **gross price**):

Dirty Bond Price = Clean Price + Accrued Interest

Accrued interest is the interest that has been earned, but not paid, and is calculated with this formula:

Formula for Calculating Accrued Interest

$$\text{Accrued Interest} = \text{Interest Payment} \times \frac{\text{Number of Days Since Last Payment}}{\text{Number of days between payments}}$$

Graph of the purchase price of a bond over 2 years, which = the flat price + accrued interest. (It is assumed that the flat price remains constant over the 2 years, but would actually fluctuate with interest rates, and from other factors, such as changes in the credit rating of the issuer.) The flat price is what is listed in bond tables for prices. The accrued interest can be calculated by the above formula. Note that the bond price steadily increases each day until reaching a peak the day before an interest payment, then drops back to the flat price on the day of the payment.

When you buy a bond on the secondary market, you must pay the former owner of the bond the accrued interest. If this were not so, you could make a fortune buying bonds right before they paid interest then selling them afterward. Because the interest accrues every day, the bond price increases accordingly until the interest payment date, when it drops to its flat price, then starts accruing interest again.

Day-Count Conventions

In calculating the accrued interest, the actual number of days was counted from the last interest payment to the value date. Most bonds use this day-count basis, called **actual/actual basis**, because the actual number of days are used in the calculations. However, some bonds use a different day-count basis, which will cause the accrued interest to be slightly different from that calculated using the actual/actual convention. Closely related to actual/actual are these conventions, which are only used for bonds with 1 annual coupon payment:

Actual/360:

- **Accrued Interest = Coupon Rate × Days/360**

Actual/365:

- **Accrued Interest = Coupon Rate × Days/365**

Note that the accrued interest calculated under the actual/360 convention is slightly more than the interest calculated under the actual/actual or the actual/365 method.

Other methods also count each month as 30 days, regardless of the days in the month, and each year counts as 360 days. Though these methods are rarely used nowadays to calculate accrued interest, they did simplify counting the number of days between a coupon date and the value date, a time-saver before calculators and computers, especially since the figured interest differed little from that calculated with the actual/actual method. So, under these methods, 3 days separate February 28 and March 1, because each month counts as 30 days, including February, even though February has either 28 or 29 days. By the same reasoning, 25 days separate January 15 and February 10, even though 26 days separate those dates. When figuring accrued interest using any day-count convention, the 1st day is counted but not the last day. So in the previous example, January 15 is counted but not February 10.

30/360 and 30E/360 Day-Count Conventions

Start Date:	M1/D1/Y1
End Date:	M2/D2/Y2
Day Count Fraction	= Day Count/360
Day Count	= (Y2 − Y1) × 360 + (M2 − M1) × 30 + (D2 − D1)

30/360 Day-Count Convention (aka US 30/360)

If (D1 = 31)	Set D1 = 30
If (D2 = 31) and (D1 = 30 or 31)	Set D2 = 30

30E/360 Day-Count Convention (aka European 30/360)

If D2 = 31	Set D2 = 30

So the number of days between December 29 and January 31 is 32 under the 30/360 convention, but 31 days under the 30E/360 convention. This is determined thus:

- 1 month × 30 = 30 days +
    - Under 30/360, January 31 is not changed since the 1st date was not 30 or 31, so 2 additional days come after January 29, yielding a total of 30 + 2 = 32 days.
    - Under 30E/360, the January 31 date is automatically changed to January 30, yielding a total of 30 + 1 = 31 days.

The number of days are then divided by 360, then multiplied by the coupon rate to determine the accrued interest:

30/360 and 30E/360:

- **Accrued Interest = Coupon Rate × Days/360**

Day Count Conventions
Used in US Bond Markets

Bond Market	Day-Count Basis
Treasury Notes and Bonds	Actual/Actual
Corporate and Municipal Bonds	30/360
Money Market Instruments	Actual/360 • So a 1% bond would earn 365/360 × 1% of interest in 365 days.

As already stated, most bond markets outside of the U.S. use the actual/actual convention except:

Bond Markets Not Using Actual/Actual

Bond Market	Day-Count Basis
Eurobonds	30/360
Denmark, Sweden, Switzerland	30E/360
Norway	Actual/365

Examples

Example: Calculating the Purchase Price for a Bond with

Accrued Interest

You purchase a corporate bond with a settlement date on September 15 with a face value of $1,000 and a nominal yield of 8%, that has a listed price of **100-08**, and that pays interest semi-annually on February 15 and August 15. Accrued interest is determined using the actual/actual convention. How much must you pay?

The semi-annual interest payment is $40 and there were 31 days since the last interest payment on August 15. If the settlement date fell on a interest payment date, the bond price would equal the listed price: **100.25% × $1,000.00 = $1,002.50** (8/32 = 1/4 = .25, so 100-08 = 100.25% of par value). Since the settlement date was 31 days after the last payment date, accrued interest must be added. Using the above formula, with 184 days between coupon payments, we find that:

$$\text{Accrued Interest} = \$40 \times \frac{31}{184}$$

$$= \$6.74$$

Therefore, the actual purchase price for the bond will be **$1,002.50 + $6.74 = $1,009.24**.

Tip: It may be more convenient to use a spreadsheet, such as Excel, that provides several functions for determining the number of days or the dirty bond price, with the settlement and maturity dates expressed as either a quote (e.g., "12/11/2012") or as a cell reference (e.g., B12):

Number of Days since Last Payment = COUPDAYBS(settlement, maturity, frequency, basis)

Number of Days Between Payments = COUPDAYS(settlement, maturity, frequency, basis)

Bond Price = PRICE(settlement,

maturity,

rate,

ytm,

redemption,

frequency,

basis

)

Search Help for more information. Below is another example of getting a bond's price by using Excel's PRICE function:

15-Feb-24	**Settlement Date**
15-Nov-37	**Maturity Date**
5.75%	**Coupon Rate**
6.50%	**Yield to Maturity**
100	**Redemption value**
2	**Number of Interest Payments per Year**
1	**Day Count Basis (Month/Year = Actual/Actual)**
=	PRICE(**settlement**,**maturity**,**rate**,**ytm**,**redemption**,**frequency**,**basis**)
$93.24	% of Par Value of Actual Price for Corporate Bond, $1,000 Face Value
$932.39	**Actual Price for Corporate Bond, $1,000 Face Value**

To calculate the accrued interest on a **zero coupon bond**, which pays no interest, but is issued at a deep discount, the amount of interest that accrues every day is calculated by using a **straight-line amortization**, which is found by subtracting the discounted issue price from its face value, and dividing by the number of days in the term of the bond. This is the interest earned in 1 day, which is then multiplied by the number of days from the issue date.

Steps to Calculate the Price of a Zero Coupon Bond

1. **Total Interest Paid by Zero Coupon Bond**
    - **= Face Value − Discounted Issue Price**

2. 1 Day Interest
   - = Total Interest / Number of Days in Bond's Term
3. Accrued Interest
   - = (Settlement Date − Issue Date) in Days × 1 Day Interest
4. Zero Coupon Bond Price
   - = Discounted Issue Price + Accrued Interest

Bonds with Ex-Dividend Periods may have Negative Accrued Interest

Interest accrues on bonds from one coupon date to the day before the next coupon date. However, some bonds have a so-called **ex-dividend date** (aka **ex-coupon date**), where the owner of record is determined before the end of the coupon period, in which case, the owner will receive the entire amount of the coupon payment, even if the bond is sold before the end of the period. The **ex-dividend period** (aka **ex-coupon period**) is the time during which the bond will continue to accrue interest for the owner of record on the ex-dividend date. (The ex-dividend date and the ex-dividend period are misnomers since bonds pay interest and not dividends, but the terminology was borrowed from stocks since the concept is similar. Although ***ex-coupon*** is more descriptive, ***ex-dividend*** is more widely used.) If a bond is purchased during the ex-dividend period, then accrued interest from the purchase date until the end of the coupon period is subtracted from the clean price of the bond. In other words, the accrued interest is negative. Only a few bonds have ex-dividend periods, which are usually 7 days or less. The UK gilt, for instance, has an ex-dividend period of 7 days, so if the bond is purchased at the beginning of that 7-day period, then the amount of interest subtracted from the clean price = the coupon rate × 7/365.

Most bond markets do not have ex-dividend periods except:

- Australia
- Denmark
- New Zealand
- Norway
- Sweden
- United Kingdom

PRICE, PRICEDISC, PRICEMAT, and DISC Functions in Microsoft Excel for Calculating Bond Prices and Other Securities Paying Interest

Microsoft Excel has several formulas for calculating bond prices and other securities paying interest, such as Treasuries or certificates of deposit (CDs), that include accrued interest, if any.

Microsoft Excel Functions: PRICE, PRICEDISC, PRICEMAT, and DISC

- Calculates the price, given the yield.
  - **Bond Price** (per $100 of face value)
    - = **PRICE(**
      - **settlement,**
      - **maturity,**
      - **rate,**

        - **yield,**
        - **redemption,**
        - **frequency,**
        - **basis)**
  - **Discounted Bond Price**
    - = **PRICEDISC(**
      - **settlement,**
      - **maturity,**
      - **discount,**
      - **redemption,**
      - **basis)**
- Calculates the yield, given the price.
  - **Discount Rate of Security**
    - = **DISC(**
      - **settlement,**
      - **maturity,**
      - **price,**
      - **redemption,**
      - **basis)**
- Calculates the price of a security that pays interest only at maturity, such as a negotiable Certificate of Deposit:
  - **Security Price**
    - = **PRICEMAT(**
      - **settlement,**
      - **maturity,**
      - **issue,**
      - **rate,**
      - **yield,**
      - **basis)**
- **Dates are expressed as cell references (e.g., A1), or as DATE functions [format Date(year,month,day)]. (Note that Microsoft states that problems may arise if dates are entered as text):**
  - **Settlement = Settlement date.**
  - **Maturity = Maturity date.**
  - **Issue = Issue date.**
- **Rates are listed in decimal form (5%=.05):**
  - **Rate = Nominal annual coupon interest rate.**
  - **Yield = Annual** yield to maturity**.**
  - **Discount = Percentage of discount/**
- **Price = Price of security as a % of par value (but without the % or $ sign, so if $1,000 par-value bond is selling for $857.30, then the corresponding percentage value is 85.73).**

- **Redemption = Value of security at redemption per $100 of face value, usually = 100.**
- **Frequency = Number of coupon payments / year.**
  - **1 = Annual**
  - **2 = Semiannual (the most common value)**
  - **4 = Quarterly**
- **Basis = The number of days counted per year.**
  - **0 = 30/360 (This U.S. basis is the default, if omitted)**
  - **1 = actual days in month/actual days in year**
  - **2 = actual days in month/360**
  - **3 = actual days in month/365 (even for a leap year)**
  - **4 = European 30/360**

Examples

Examples: Using Microsoft Excel for Calculating Bond Prices and Discounts

These listed variables — where they apply — will be used for each of the example calculations that follow, for a 10-year bond originally issued in 1/1/2024 with a par value of $1,000:

- **Settlement date = 1/2/2024**
- **Maturity date = 12/15/2033**
- **Issue date = 12/15/2023**
- **Coupon rate = 6%**
- **Yield to maturity = 8%**
- **Price (per $100 of face value) = 21.99**
- **Redemption = 100**
- **Frequency = 2 for most coupon bonds.**
- **Basis = 1 (actual/actual)**

Price of a bond with a yield to maturity of **8%**:

Bond Price

- = PRICE(**Date(2024,1,2)**,
  - **Date(2033,12,15)**,
  - **0.06**,
  - **0.08**,
  - **100**,2,**1**)
- = 86.44858
- = **$864.49**

The discount price of a zero coupon bond with a $1,000 par value yielding **8%**:

Price Discount

- = PRICEDISC(**Date(2024,1,2)**,
  - **Date(2033,12,15)**,
  - **0.06**,
  - **0.08**,
  - **100**,**1**)
- = 20.39420
- = **$203.94**

The interest rate of a discounted zero coupon bond paying $1,000 at maturity, but that is now selling for $219.90:

Interest Rate of Bond Discount

- = DISC(**Date(2024,1,2)**,
  - **Date(2033,12,15)**,
  - **21.99**,
  - **100**,**1**)
- = 0.78396
- = **7.84%**
- Note that the PRICEDISC function value has been rounded, with the results used in the DISC function to verify the results. (21.99 = $219.90 for a bond with a $1,000 par value).

PRICEMAT calculates the prices of securities that only pay interest at maturity:

What is the price of a negotiable, **90**-day CD originally issued for **$100,000** on **3/1/2024** with a nominal yield of **8%**, a yield to maturity of **6%** and a settlement date of **4/1/2024**? Using the Microsoft Excel **Date function**, with format **DATE(year,month,day)**, to calculate the **maturity date** by adding **90 days** to the **issue date** and choosing the banker's year of 360 days by omitting its value from the formula, yields these results:

- **Market Price of CD**
  - = PRICEMAT(**Date(2024,4,1)**,
    - **DATE(2024,3,1)** + **90**,,
    - **Date(2024,3,1)**,
    - **0.08**,
    - **0.06**)
  - = **99.65916** (per $100 of face value) × **1,000**
  - = **$99,659.16**

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