Macaulay Duration of a Zero Coupon Bond

$$P[y] = \frac{1}{(1 + y)^T} \qquad \frac{dP}{dy} = -\frac{T}{(1 + y)^{T+1}} \qquad -\frac{(1 + y)}{P}\frac{dP}{d(1+y)} = T = D^*$$

Modified Duration and Convexity of a Zero Coupon Bond

$$P[y] = \frac{1}{(1 + y)^T} \qquad \frac{dP}{dy} = -\frac{T}{(1 + y)^{T+1}} \qquad -\frac{1}{P}\frac{dP}{dy} = \frac{T}{(1+y)} = D$$

$$\frac{d^2P}{dy^2} = \frac{T(T+1)}{(1 + y)^{T+2}} \qquad \frac{1}{P}\frac{d^2P}{dy^2} = \frac{T(T+1)}{(1 + y)^2} = CON$$

Duration and Convexity for Bond Trader Example from Chapter 5, p.276 (p.29)/Lecture 5

Liability: $D^* = T = 5$ $\qquad$ $CON_L = (5 * 6) / (1.05)^2 = 27.21$

Asset: D* = .75 Cash + .25 (20) = 5 $\qquad$ $CON_A = 0 + ((.25)(20 * 21)) / (1.05)^2 = 95.24$
Note: 376,890/1,504,370 = .25

$CON_A > CON_L$

Observe that the theta for the barbell is .75 (0) + .25 (5%) = 1.25%

Duration of a Term Annuity

See p. 212-3 in SIAS and question 7 on p.243 for duration of a par bond.
See also Macaulay duration of term annuity and par bond on class webpage.