Two Assets, One Liability: Two Equations in Two Unknowns

$$D_L = w_1 D_{A1} + w_2 D_{A2} \rightarrow w_1 + w_2 = 1 \rightarrow w_2 = 1 - w_1$$

$$D_L = w_1 D_{A1} + (1 - w_1) D_{A2} = (D_{A_1} - D_{A_2}) w_1 + D_{A_2} \rightarrow D_{A_2} - D_L = (D_{A_2} - D_{A_1}) w_1$$

$$w_1 = \frac{D_{A_2} - D_L}{D_{A_2} - D_{A_1}}$$

Bond Trade Example: $D_L = 5 \quad D_{A1} = 0 \quad D_{A_2} = 20 \qquad w_1 = \frac{20 - 5}{20 - 0} = .75$

Three Asset Case: Two equations in Three Unknowns

$$D_L = w_1 D_{A1} + w_2 D_{A2} + w_3 D_{A3} \rightarrow w_1 + w_2 + w_3 = 1$$

$$\rightarrow w_3 = 1 - w_1 - w_2 \rightarrow D_L = w_1 D_{A1} + w_2 D_{A2} + (1 - w_1 - w_2) D_{A3}$$

$$= (D_{A_1} - D_{A_3}) w_1 + (D_{A2} - D_{A3}) w_2 + D_{A_3} \rightarrow D_{A_3} - D_L = (D_{A_3} - D_{A_1}) w_1 + (D_{A3} - D_{A2}) w_2$$

$$w_1 = \frac{D_{A_3} - D_L}{D_{A_3} - D_{A_1}} - \frac{D_{A_3} - D_{A2}}{D_{A_3} - D_{A_1}} w_2$$

Example:

$$D_{A3} = 30 \quad D_{A2} = 7 \quad D_{A1} = 2 \quad D_L = 7$$

$$\textit{if} \quad w_2 = 0.5 \rightarrow w_1 = 0.41 \quad 1 - w_1 - w_2 = .09$$

Changing the long weight to $D_{A3} = 10$ produces for $w_2 = .5 \rightarrow w_1 = .1875 \qquad w_3 = .3125$