Assume mean-variance expected utility: $EU[\pi] = E[\pi] - b\ var[\pi]$ where $b > 0$

EQUATION 1 **Optimal Speculative Position**

$$\max_Q \quad EU[\pi_{spec}] = Q\ \{E[F(1,T)] - F(0,T)\} - b\ Q^2\ \sigma_F^2$$

$$\frac{dEU}{dQ} = \{E[F(1,T)] - F(0,T)\} - 2b\ Q\ \sigma_F^2 = 0 \quad \rightarrow \quad Q^* = \frac{E[F(1,T)] - F(0,T)}{2b\ \sigma_F^2}$$

EQUATION 2 **Minimum Variance Hedge Ratio Solution**

$$var[\pi_H] = E[(\pi_H - E[\pi_H])^2] = Q_S^2\ \sigma_S^2 + Q_H^2\ \sigma_F^2 - 2Q_S\ Q_H\ \sigma_{S,F}$$

$$\frac{d\ var[\pi_H]}{dQ_H} = 2Q_H\ \sigma_F^2 - Q_S\ \sigma_{S,F} = 0 \quad \rightarrow \quad \frac{Q_H^*}{Q_S} = \frac{\sigma_{S,F}}{\sigma_F^2} = \frac{\sigma_S}{\sigma_F}\ \rho_{S,F}$$

EQUATION 3 **Optimal (Short) Hedge Ratio Solution**

$$\max_Q \quad EU[\pi_H] = E[\pi_H] - b\ var[\pi_H]$$

$$\frac{dEU[\pi_H]}{dQ_H} = \{F(0,T) - E[F(1,T)]\} - 2b\ \{Q_H\ \sigma_F^2 - 2Q_S\ \sigma_{S,F}\} = 0$$

$$\rightarrow \quad \frac{Q_H^*}{Q_S} = \frac{\sigma_{S,F}}{\sigma_F^2} + \frac{F(0,T) - E[F(1,T)]}{2b\ Q_S\ \sigma_F^2}$$