



Chapter Summary

Chapter 10 *Investment Strategy*

- 10.1 Investment Strategy: Basic Concepts
 - Whose investment strategy?
 - The Dow Ten and dividend yield strategies
 - The hedge fund approach
- 10.2 Tactical and Strategic Asset Allocation
 - Strategic asset allocation and two fund separation
 - What is tactical asset allocation?
 - Cash management and dollar cost averaging
- 10.3 Investment Strategy for Value Investors
 - The Graham, Dodd and Cottle approach
 - The Warren Buffett approach
 - Practical investment strategy for individual value investors
- 10.4 Advanced Topics in Investment Strategy*
 - The interior decorator fallacy and other confusions*
 - The fallacy of large numbers and growth optimal portfolios*
 - Behavioral portfolio theory: Keynesian investment strategy?*

Questions

Notes

* Indicates section or sub-section contains advanced material

chapter 10 Investment Strategy

10.1 INVESTMENT STRATEGY: BASIC CONCEPTS

Whose investment strategy?

At numerous points in this book it has been observed that semantics, the meanings of words, are too often ignored in discussing topics in Finance. Important words – such as “fundamental analysis,” “value investing,” “growth stock” – can be interpreted in a number of ways. Even though specific words being used can have different meanings, the words are interpreted as having one precise meaning by the individual or text using the word.¹ A related semantic problem is encountered in other situations where more than one word is being used to identify the same general concept. This is the case with “investment strategy.” Alternative references to this concept include “investment policy,” “portfolio management” and “asset allocation strategy.” Well specified distinctions between these words are not available. The terminology selected to describe the underlying concept is often associated with a particular approach to Finance. For example, “portfolio management” or “asset allocation strategy” is the terminology commonly used in modern Finance, e.g., Campbell and Viceira (2002), Bodie et al. (1999), while “investment policy” is more commonly used in old Finance, e.g., Graham, Dodd and Cottle (1962) (GDC, ch. 5).

Because the term *investment strategy* is not commonly attached to any particular approach, this is the terminology adopted in this book to describe the various approaches to selecting combinations of securities that form an investment portfolio. Investment strategy is an essential adjunct to security analysis, which is concerned with determining the value of individual securities. Because investment strategy is a dynamic decision, determination of portfolio value weights at a point in time, and varying of the value weights over time, falls within the scope of investment strategy. As a consequence, there are both short-term and long-term elements of the investment strategy decision. The “modern portfolio theory” that is at the core of modern Finance is concerned almost exclusively with investment strategy. The closely related notions of tactical asset allocation and strategic asset allocation are other examples of such “top down” investment strategies. In contrast, a “bottom up” investment strategy is where security analysis

takes the central role, such as in the GDC approach. In bottom up approaches, the portfolio composition is determined through the incremental process of doing valuations on individual securities.

Whatever the terminology, the importance of the investment strategy decision is difficult to deny. For example, Lummer and Riepe (1994) observe: "Any security-specific selection decision is preceded, either implicitly or explicitly, by an asset allocation decision. Asset allocation is therefore the most fundamental of investment decisions." Brinson et al. (1991) estimate that for comparable portfolios, the strategic asset allocation decision – the long-term asset mix – is responsible for 91.5% of the variation in portfolio returns for a sample of 91 large US pension plans. The Brinson et al. result has generated considerable debate both within modern Finance, e.g., Statman (2001), and among practitioners, e.g., Jahnke (1997), Renkenthaler (1999), about various aspects of investment strategy. This involves assessing the relative importance of the long-term asset allocation mix versus portfolio rebalancing around that mix, subject to the cash management requirements of the investment portfolio.² Using a sample of UK pension plans, Blake et al. (1999) confirm and expand the results of Brinson et al., raising legitimate questions about the prevailing investment strategy practices of pension fund managers. There is a demonstrated need to extend these results to different institutional situations and wider asset mixes, e.g., including higher fractions of foreign assets.

While there is an ongoing, largely academic debate about the relative importance of the different aspects of investment strategy, Arnott (1998, p. 162) reflects the commonly held view of practitioners: "It is often said that asset allocation is the most important decision that an investment manager or fund sponsor faces. It is conventional wisdom, and is demonstrated in academic journals, that asset allocation has more influence on aggregate portfolio returns than any other single decision." Whether such statements are justified is difficult to determine in practice, if only because the effectiveness of a particular investment strategy decision cannot be accurately determined *ex ante*. Studies such as Blake et al. (1999) are limited by having to make assessments based on *ex post* performance. Conclusions such as security selection is a "zero expected-excess-return activity" (Blake et al. 1999, p. 459) have to be tempered by the specifics of the Blake et al. sample. In addition, the security selection strategies of, say, value investors also have implications in the realm of investment strategy, e.g., the process of buying undervalued securities and selling overvalued securities complements the tactical asset allocation decision. While specific pension plans may be ineffective at selecting securities, it is difficult to maintain that this is generally the case.

Analysis of investment strategy has progressed considerably from the early prognostications of the old Finance reflected in GDC. Though primarily concerned with security selection, GDC did give some attention to investment strategy. The essence of "investment strategy" as stated by GDC (p. 59) is: "the allocation of the available funds among the various types of securities, including variations in such proportions under changing conditions and decisions to hold cash for future commitments." This general statement still applies but the various aspects of investment strategy are now more explicitly identified and examined (see section 3.3). In particular, investment strategy is now considered to involve three related types of decisions (see section 10.2 for more precise definitions and detailed discussion): "*strategic asset allocation*," where long-term portfolio composition

chapter 10 Investment Strategy

10.1 INVESTMENT STRATEGY: BASIC CONCEPTS

Whose investment strategy?

At numerous points in this book it has been observed that semantics, the meanings of words, are too often ignored in discussing topics in Finance. Important words – such as “fundamental analysis,” “value investing,” “growth stock” – can be interpreted in a number of ways. Even though specific words being used can have different meanings, the words are interpreted as having one precise meaning by the individual or text using the word.¹ A related semantic problem is encountered in other situations where more than one word is being used to identify the same general concept. This is the case with “investment strategy.” Alternative references to this concept include “investment policy,” “portfolio management” and “asset allocation strategy.” Well specified distinctions between these words are not available. The terminology selected to describe the underlying concept is often associated with a particular approach to Finance. For example, “portfolio management” or “asset allocation strategy” is the terminology commonly used in modern Finance, e.g., Campbell and Viceira (2002), Bodie et al. (1999), while “investment policy” is more commonly used in old Finance, e.g., Graham, Dodd and Cottle (1962) (GDC, ch. 5).

Because the term *investment strategy* is not commonly attached to any particular approach, this is the terminology adopted in this book to describe the various approaches to selecting combinations of securities that form an investment portfolio. Investment strategy is an essential adjunct to security analysis, which is concerned with determining the value of individual securities. Because investment strategy is a dynamic decision, determination of portfolio value weights at a point in time, and varying of the value weights over time, falls within the scope of investment strategy. As a consequence, there are both short-term and long-term elements of the investment strategy decision. The “modern portfolio theory” that is at the core of modern Finance is concerned almost exclusively with investment strategy. The closely related notions of tactical asset allocation and strategic asset allocation are other examples of such “top down” investment strategies. In contrast, a “bottom up” investment strategy is where security analysis

takes the central role, such as in the GDC approach. In bottom up approaches, the portfolio composition is determined through the incremental process of doing valuations on individual securities.

Whatever the terminology, the importance of the investment strategy decision is difficult to deny. For example, Lummer and Riepe (1994) observe: "Any security-specific selection decision is preceded, either implicitly or explicitly, by an asset allocation decision. Asset allocation is therefore the most fundamental of investment decisions." Brinson et al. (1991) estimate that for comparable portfolios, the strategic asset allocation decision – the long-term asset mix – is responsible for 91.5% of the variation in portfolio returns for a sample of 91 large US pension plans. The Brinson et al. result has generated considerable debate both within modern Finance, e.g., Statman (2001), and among practitioners, e.g., Jahnke (1997), Renkenthaler (1999), about various aspects of investment strategy. This involves assessing the relative importance of the long-term asset allocation mix versus portfolio rebalancing around that mix, subject to the cash management requirements of the investment portfolio.² Using a sample of UK pension plans, Blake et al. (1999) confirm and expand the results of Brinson et al., raising legitimate questions about the prevailing investment strategy practices of pension fund managers. There is a demonstrated need to extend these results to different institutional situations and wider asset mixes, e.g., including higher fractions of foreign assets.

While there is an ongoing, largely academic debate about the relative importance of the different aspects of investment strategy, Arnott (1998, p. 162) reflects the commonly held view of practitioners: "It is often said that asset allocation is the most important decision that an investment manager or fund sponsor faces. It is conventional wisdom, and is demonstrated in academic journals, that asset allocation has more influence on aggregate portfolio returns than any other single decision." Whether such statements are justified is difficult to determine in practice, if only because the effectiveness of a particular investment strategy decision cannot be accurately determined *ex ante*. Studies such as Blake et al. (1999) are limited by having to make assessments based on *ex post* performance. Conclusions such as security selection is a "zero expected-excess-return activity" (Blake et al. 1999, p. 459) have to be tempered by the specifics of the Blake et al. sample. In addition, the security selection strategies of, say, value investors also have implications in the realm of investment strategy, e.g., the process of buying undervalued securities and selling overvalued securities complements the tactical asset allocation decision. While specific pension plans may be ineffective at selecting securities, it is difficult to maintain that this is generally the case.

Analysis of investment strategy has progressed considerably from the early prognostications of the old Finance reflected in GDC. Though primarily concerned with security selection, GDC did give some attention to investment strategy. The essence of "investment strategy" as stated by GDC (p. 59) is: "the allocation of the available funds among the various types of securities, including variations in such proportions under changing conditions and decisions to hold cash for future commitments." This general statement still applies but the various aspects of investment strategy are now more explicitly identified and examined (see section 3.3). In particular, investment strategy is now considered to involve three related types of decisions (see section 10.2 for more precise definitions and detailed discussion): "*strategic asset allocation*," where long-term portfolio composition

objectives are determined; "*tactical asset allocation*," that involves portfolio rebalancing and investment timing decisions; and, *cash management*, which often involves the satisfaction of minimum cash balance requirements and other liquidity considerations.³ Security analysis is considered to be an input to these three types of decisions.

Given this decomposition of investment strategy into three related decisions, modern Finance dedicates considerable attention to modeling the various aspects of the optimal investment strategy decision. Because there are a range of investors with different investment strategy objectives, there cannot be a unique optimal solution to the investment strategy decision problem applicable to all investors. However, by classifying investors into rough groupings, it is possible to identify significant differences in the decision problem that require solution. These differences impose constraints on the types of solutions that can be selected. For example: insurance companies are subject to restrictions on the quality of securities that can be purchased; or, due to capital and reserve requirements imposed by regulators, commercial banks have to hold a significant fraction of liquid or near liquid assets; or, due to the aggregate value of funds under management, large institutional investors have to purchase more than a small number of securities. Subject to such institutional constraints, the approach to investment strategy that is selected will depend on philosophical differences between fund managers, investors and the like about, say, the handling of uncertainty in security returns.

For purposes of analyzing the investment strategy decision, owners of securities can be roughly divided into two groups: individuals; and institutions. Institutions can be further subdivided according to *the degree of regulatory controls* on portfolio composition. Institutions subjected to wide ranging control include life insurance companies, depository institutions and certain types of trust funds. Fire and casualty insurance companies are subject to a lesser degree of regulatory control, with mutual funds, investment funds, commingled trusts and pension funds subject to the lowest level of regulatory control. This is not to imply that, say, a mutual fund is not subject to regulations on investment strategy. In addition to the legally binding fund prospectus that provides a general description of the investment strategy of the fund, the investment activities of the fund will be subjected to the range of legislation enforced by the SEC, state-blue-sky laws and so on. However, these controls are relatively light compared to the restrictions imposed on, say, depository institutions where there are restrictions on ownership of equity securities, fraction of assets invested in liquid securities needed to satisfy capital and reserve requirements and so on.

Turning to *investment strategy for individual investors*, there is also considerable diversity. At one extreme are the small individual investors dependent on the cash flows from an investment portfolio to pay day-to-day expenses. Such investors are driven by necessity to follow an investment strategy with characteristics that are not much different from that of, say, a life insurance company: conservative, primarily invested in high quality bonds that have little or no risk of capital loss.⁴ Another type of individual investor with a specialized investment strategy would be the high net worth individual seeking to gain a tax advantage from trading in securities with favorable tax treatment, such as municipal bonds. Ignoring these special cases, GDC (p. 66) suggest decomposing individual investors into *defensive investors* and *enterprising investors*:

It would be most useful, we think, to classify [individual] security buyers into two groups – defensive investors and aggressive or enterprising investors. Defensive investors are those who should place their chief emphasis upon freedom from effort, annoyance and the necessity for making frequent investment decisions. The great majority of individual investors belong in the defensive category. Their specific training and experience with bonds and stocks are not adequate to warrant making independent decisions about the selection of securities . . . The distinguishing feature of [enterprising investors] is their willingness and ability to devote time and care to the selection of sound and attractive investments. Their chief objective should be by training, intelligence and guidance to take advantage of the numerous opportunities in normal markets to buy securities for considerably less than they are worth.

For GDC, writing prior to the widespread availability of low management expense ratio index funds, the defensive investor with no cash management restrictions could hold a portfolio of equity investment (mutual) fund shares and US government bonds with the investment mix varying between (25% to 75%) equity and (75% to 25%) fixed income depending on the “subjective feeling” of the investor. The portfolios of the enterprising investor would follow the prescriptions of “intrinsic value” investing set out in chapters 7–9 of this book.

There has been considerable progress in the analysis of investment strategy since GDC. If modern Finance has made a single outstanding practical contribution beyond the realm of derivative security valuation, this contribution is asset allocation strategies. Starting from Markowitz and continuing to the present, the portfolio management problem, together with solutions derived using mean-variance optimization, has been a centerpiece for the teachings of modern Finance (see section 2.4 and chapter 3). More recently, this basic theory has been dressed up and repackaged in various forms under the guise of asset allocation strategies. Recognizing that there is a considerable diversity across investors leads to specific analysis of the long-term “strategic asset allocation” decision, e.g., Campbell and Viceira (2002). Because strategic asset allocation only deals with long-term solutions, considerations of market timing and portfolio rebalancing also need to be addressed. This falls within the scope of tactical asset allocation. Finally, the solutions to both strategic and tactical asset allocation depend on the types of asset classes that are being considered for inclusion in the portfolio, e.g., large capitalization (cap) stocks, small cap stocks, mortgage-backed securities, corporate bonds, and real estate. Allowing investment in international assets considerably complicates the asset allocation problem. These issues are examined in section 10.2.

The Dow Ten and dividend yield strategies

Investment strategies can take a wide variety of forms. In some cases, the investment strategy is intimately related to a strongly held investment philosophy. This could be the case with the “top down” investment strategies associated with modern Finance that systematically develop approaches to strategic and tactical asset allocation (see section 10.2). In this spirit, a top down investment strategy details an approach to security selection that either does not involve security analysis or incorporates security analysis

at a later stage in the decision process. This investment strategy category can also be expanded to include mechanical methods of dealing with the common stock investment decision that do not directly involve security analysis but leave strategic asset decisions, such as the split between stocks and bonds, unresolved. One example of such a strategy is the "*Dogs of the Dow*" or "Dow Ten" (Dow 10) strategy that recommends buying the ten highest dividend yielding stocks from the Dow-Jones Industrial Average.⁵ In this case, the investment strategy is driven by a security selection method that does not involve security analysis - other than calculation of the dividend yield. No consideration is given to the strategic asset mix, company earnings or whatever. The tactical allocation strategy is to change the portfolio whenever stocks enter and leave the Dow Ten list.

For purposes of assessing the Dow Ten strategy it is expedient to assume that the strategic asset allocation is 100% invested in equities, though this is not essential. Siegel (1998, p. 65) describes the history of the strategy:⁶

The Dow 10 strategy, which calls for investors to buy the ten highest-yielding stocks in the Dow-Jones Industrial Average, has been regarded as one of the most successful investment strategies of all time. James Glassman of the *Washington Post* claimed that John Slatter, a Cleveland investment advisor and writer, invented the Dow 10 system in the 1980's. Harvey Knowles III and Damon Perry analyzed and praised the system in their book, the *Dividend Investor* written in 1992, as did Michael O'Higgins and John Downes in *Beating the Dow*.

In an odd twist, Siegel attempts to classify the Dow Ten theory as a form of value investing:

The basic theory behind the Dow 10 strategy is grounded in "value investing." Value investors are contrarians who believe that the swings of optimism and pessimism about the market and individual stocks are frequently unjustified, so buying out-of-favor stocks is a winning strategy. Since firms reduce cash dividend payouts infrequently, stocks with a high dividend yield are often those that have fallen in price and are out of favor with investors. For this reason, the Dow 10 strategy is often called the "dogs of the Dow."

Ignoring the apparent semantic confusion over what constitutes value investing, Siegel is compelled to provide an explanation for the success of the Dow 10 that is reported. This explanation is found in the modern Finance interpretation of "value investing."

According to Siegel (1998, pp. 66-7) the Dow 10 strategy has had a stellar historical performance: "The Dow 10 strategy has outperformed both the overall Dow and the S&P 500 Index in every decade except the 1930's, besting the Dow 30 by an average of 3.26 percent per year and the S&P 500 Index by an even larger 3.7 percent per year since 1939." Siegel is unambiguous about the empirical track record of the Dow 10 strategy:

You might think that these spectacular returns were achieved with higher risk, but that is not the case. The standard deviation of annual returns on the Dow 10 strategy was actually lower than the Dow 30 and only slightly more than the S&P 500 Stock Index. And the Dow 10 was spectacular during the 1973-74 bear market. During those two years when the Dow 30 was down by 26.5 percent and the S&P Index was down 37.3 percent, the Dow 10 strategy actually gained 2.9 percent!

Siegel also provides reasons for why the Dow 10 strategy "worked." This enthusiasm stands in stark contrast to the empirical results of Hirschey (2000) where the Dogs of the Dow strategy is referred to as a "myth." By finding evidence for a similar dividend-yield effect on the Toronto Stock Exchange ("Canadian dogs"), Visscher and Fillbeck (2003) provide evidence that the essence of the Dow 10 strategy is related to dividend payout by high market capitalization firms and not to specific characteristics of the Dow-Jones Industrial Index. Visscher and Fillbeck also report considerable market interest in "Euro dog" funds based on securities traded on the London, Frankfurt, Paris and Amsterdam exchanges.

The conflicting views over the past performance of the Dow 10 strategy would seem to be resolvable. The search for such a resolution motivated McQueen et al. (1997) but the goal was found to be illusive. Using a 50 year sample, results for the Dow 10 strategy were mixed. Though the crude return results of 3 percent plus per annum above the Dow described by Siegel are confirmed, when adjustment is made for transaction costs, tax implications and the increase in risk due to the loss of diversification inherent in comparing the Dow 10 with the 30 stock DJIA, the gain was less than 1 percent. Using a 1961-98 sample, Hirschey (2000) goes farther and finds that after adjustment for rebalancing costs and taxes there is no significant difference between the performance of the Dow 10 and the DJIA. Given the central role of empirical verification in logical positivism, the conflicting views over the performance of dividend-yield strategies, in general, and the Dow 10 strategy, in particular, is puzzling. Surely an empirical issue about the *ex post* performance of an investment strategy such as the Dow 10 is resolvable by an examination of past data?

The recognition that the Dow 10 strategy is a specific case of a *dividend yield strategy* is significant, if only because a connection is established to the empirical results in modern Finance concerning "value stocks." Along with a low price to book ratio and low *P/E* ratio, the dividend yield is one of the metrics that modern Finance uses to define a value stock. Visscher and Fillbeck (2003) provide the following summary of this research:

The effectiveness of dividend yield strategies in enhancing portfolio returns has been debated for many years. Some studies have suggested that little or no relationship exists between dividend yield and share price returns (Black and Scholes 1974; Goetzmann and Jorion 1993, 1995). Many other studies, however, have identified a positive relationship between dividend yield and share price (Fama and French 1988; Hodrick 1992; Grant 1995). Benjamin Graham (see Rea 1977) reported an average compound growth rate between 1925 and 1975 of 19.5 percent for US stocks that had a dividend yield greater than two-thirds of the AAA bond yield, compared with a 7.5 percent return for the DJIA.

This largely empirical debate is accompanied by a related theoretical debate over the appropriate functional form for the relationship between dividend yields and stock returns. This debate revolved around the question of whether there was a U-shaped relationship where companies with high dividend yields and companies paying no dividends outperform companies with dividend yields lying between the extremes, e.g., Elton et al. (1983), Christie (1990).

Putting aside the semantic confusion arising from differing interpretations of value stocks and value investing, the empirical evidence assembled in modern Finance concerning value stocks, including those with high dividend yield, indicates the rudiments of a potentially profitable investment strategy.⁷ Considerable attention in modern Finance has been given to the problem of designing practical investment strategies from core theories, such as those based on quadratic optimization techniques. For example, Grauer and Hakansson (1993, 1998) develop the *empirical probability assessment approach* where an appropriately specified quadratic optimization problem for portfolio returns is solved each rebalancing period using parameters that are re-estimated each period using a moving sample window. Similarly, the "value investing" approach of modern Finance (not to be confused with the value investing approach of old Finance) is based on a sorting approach. At each rebalancing point, the universe of common stocks is sorted and stocks that have a high weighted scoring based on the chosen value factors are selected. More precisely, the highest decile or quartile of stocks is chosen and held until the next rebalancing date. The scoring system is based on weights determined from empirical estimation of an appropriate multi-factor regression or factor analytic model

The hedge fund approach

The derivative securities industry, both in the US and globally, is immense. The industry continues to grow into new areas and offer a bewildering array of new products. Significant derivative security trading activity takes place in a variety of geographical locations around the globe and is subject to an ad hoc and sometimes competing collection of regulators and regulations. This situation is exacerbated by the competing venues in which derivatives are traded. Futures exchanges scattered around the globe compete for business while the OTC market continues to proliferate new derivative products. In addition to direct trading of derivative securities, the intellectual technology of derivatives trading has also permeated the cash markets, as evidenced by the introduction of trading techniques such as dynamic portfolio insurance (see section 3.4). In addition to dynamic trading in securities markets, a wide variety of possible distributional shapes for the portfolio return can be obtained by combining a securities position with a position in derivative securities. These positions are typically written with the security or some related security as the underlying commodity for delivery on the derivative security (e.g., Poitras 2002, ch. 6). Though of considerable interest, strategies combining securities positions with derivative positions are described in considerable detail elsewhere and will not be examined in any detail here.

Before the recent renaissance in derivative security trading, various forms of restrictions, legal and otherwise, were imposed on the use and availability of derivatives. Historical roots of these restrictions can be found in the speculative abuses associated with these products (Poitras 2000, ch. 9; Poitras 2002, ch. 1). The use of options has, at various times, been singled out for specific restrictions. In the absence of direct trading of derivative products, securities markets have often developed trading techniques that have essential features of a derivative security. *Short selling* is one such activity. While

a short position in, say, a forward contract on the DJIA involves entering into a forward contract, a short sale would involve entering into a short sale agreement with the lender of the 30 component stocks. The sale and later repurchase of the DJIA stocks will generate cash flows that will likely involve further securities trading. The essential feature of the forward contract and the short sale is the creation of a position that will generate profit from a fall in the security price. However, as a consequence of the cash flow differences between a short sale and a short forward contract, it is often not practical to use the short sale to directly hedge an underlying securities position.

The short sale has a checkered history. For example, various edicts in seventeenth-century Holland banned the sale of a commodity for forward delivery not owned by the seller at the time the forward contract was created. This included company shares. As a legacy of this history, a range of restrictions, such as the "up tick" rule, still restrict short sellers in US markets. Individual investors often face restrictions on the ability to access the funds earned from the sale of securities. Where such access is available, it is possible to pursue hedge-fund-like investment strategies. By being short in one security (commodity) and long in another security (commodity), the profit functions for such strategies are similar in intuition to inter-commodity futures spread trades (Poitras 2002, ch. 3). However, there is an additional component of the profit function for a hedge fund related to the cash flows from the securities positions and the amount of capital invested in the fund. In combination with the capital size, the degree of leveraging determines the aggregate size of fund assets. By explicitly introducing over-valued securities into the universe of available opportunities, the hedge fund would seem to be a conceptual advance over traditional long-only funds.

Though the hedge fund concept appears to be a natural outlet for the GDC approach to value investing, there are practical reasons why GDC use the long-only approach. For example, there are the informational requirements associated with researching securities that are short selling opportunities. Such securities will almost certainly lie outside the universe of "100 or so primary stocks" that GDC use in selecting a long-only portfolio. Restricting short selling opportunities to the same securities as are considered for the long-only portfolio presents problems. Because most of these securities are selected due to the positive prospects, there will not be many short selling opportunities. In addition, secular market movements may cause bunching where most of the securities appear as short selling opportunities at much the same time. Another complication occurs with the types of strategies that fall within the hedge fund concept. The basic idea of leveraging fund capital using short sales can be extended to other commodities than securities, though there are liquidity and other factors that make securities an attractive trading vehicle for hedge fund purposes.

In a sense, the essence of a hedge fund is inherent in the process of financial intermediation, though it is not helpful to stretch this analogy too far. For example, viewed as a hedge fund a commercial bank is short demand and time deposits and long commercial loans. Price changes appear over time as changes in interest rates on liabilities and assets. However, unlike many actual hedge funds, commercial banks primarily aim to generate profits off the cash flows associated with the appropriately adjusted interest rate spread on the assets and liabilities, rather than on changes in prices. This analogy with a commercial bank does illustrate the range of possible

strategies that could be pursued by hedge funds. Profitability could originate from a correct assessment of the long position, with the short position serving to leverage the fund's capital. As such, the class of hedge fund strategists would include a two fund separator that is borrowing at the risk free rate and purchasing additional units of the market portfolio. Bond traders that are speculating on changes in yield curve shape will typically short the expensive and buy the cheap parts of the yield curve. This is also a potential hedge fund strategy.

Caldwell (1995) traces *the beginnings of the modern hedge fund industry* to 1949 when Alfred Jones (1901–1989) set up a general partnership operating a fund with the requisite elements.⁸ Prior to establishing this fund, Jones had led a full life. In addition to earning a Ph.D. in sociology from Columbia University in 1941, Jones was an associate editor for *Fortune* and a business writer for *Time* and other publications. Prior to being in the publishing industry, Jones had traveled the world, including a tour as vice consul at the US embassy in Berlin during the early days of Hitler's reign. Apparently, it was the research that Jones did for an article published by *Fortune* in March 1949 that provided the foundation for the establishment of his hedge fund. The research involved Jones interviewing many of the important players on Wall Street. The basic strategy guiding the fund was to combine short selling and leverage to create a relatively conservative investment portfolio aimed at capturing stock picking opportunities. At the time, this was a novel approach, given that both leverage and short sales were generally used to increase return variability, not reduce variability. Jones went beyond this to develop a measure of market exposure for his fund. The result was the first of the *market neutral* hedge funds.

Caldwell (1995, p. 7) describes the Jones approach to fund management:

Jones regularly calculated the exposure of his capital to market risk... His method of quantifying market exposure is highly valued by traditional hedge fund managers for its intuitive relevance, yet it is largely ignored or misunderstood by academics and the financial media.

$$\text{Market exposure} = (\text{Long Exposure} - \text{Short Exposure}) / \text{Capital}$$

A typical asset allocation for Jones would look like this: Given \$1,000 in capital, he would employ leverage to purchase shares valued at \$1,100 and sell shares short valued at \$400. His gross investment of \$1,500 (150 percent of capital) would have a net market exposure of only \$700 (\$1,100 – \$400), making this portfolio “70 percent net long.” Although Jones valued stock picking over market timing, he increased or decreased the net market exposure of his portfolio based on his estimation of the strength of the market. Since the market generally rose, Jones was generally “net long.”

As with other modern hedge funds, the Jones fund had an uncommonly high management fee. Though there was no high water marks or loss pay-backs, the 20% of realized profits to the general partner is in the realm of more recent arrangements. Also similar to recent hedge funds, Jones kept his entire investment capital in the fund, providing a strong managerial incentive for positive performance.

Jones introduced another innovation in 1954, reducing fund risk by bringing in other fund managers to run part of the portfolio, effectively starting the fund-of-funds approach to hedge fund management. Though there was oversight to ensure that duplication and cancellation was not happening, managers were given wide latitude to make investment decisions. Over the years, Jones would have as many as eight managers working the fund-of-funds portfolio. With this move, the Jones fund became an incubator for new hedge fund creation. Two of his early fund managers, Dick Ratcliffe and Carl Jones, eventually moved on to start their own hedge funds, Ratcliffe establishing Fairfield Partners in 1965 and C. Jones establishing City Associates in 1964. Certain elements of hedge fund structure were adapted by other funds. For example, the notion of hedge fund management fees, i.e., incentive-based partnership agreements, was adopted by, among others, Warren Buffett with Buffett Partners and Walter Schloss with WJS Partners. Though possessing some elements of hedge funds, these two funds did not regularly use short sales to create market neutral positions.

Caldwell (1995, p. 9) identifies another turning point in hedge fund history as the publication of an article by C. Loomis in *Fortune* magazine in April 1966. This article detailed the performance of the Jones fund, demonstrating that this fund was easily the best performing fund over the previous five years, even when account was taken of the 20% management fees. Loomis also provided a reasonably accurate description of how the Jones fund was run. Fuelled on the demand side by sudden investor demand for such investment vehicles and on the supply side by fund managers attracted by the high management fees, the result was a wave of new hedge funds being created (Caldwell 1995, p. 10):

Although we don't know how many hedge funds were established in the three-year flurry following Loomis's article, estimates range from 140 to several hundred. Michael Steinhardt and George Soros were among those setting up funds at this time. The SEC found 215 investment partnerships in a survey for the year ending 1968 and concluded that 140 of these were hedge funds, with the majority formed that year.

The market contraction which started at the end of 1968 and continued until the end of 1974 demonstrated that there was considerably more to running a hedge fund than desire and marketing. Considering just the 28 largest US hedge funds at year end 1968, within two years there had been five funds shut down, with fund asset values down 70% due to losses and withdrawals.

Some hedge funds survived the tough years between 1968 and 1974 while others foundered. Why? There are no detailed empirical studies of hedge fund strategies during this period. Hedge funds are closely held and such information was and is difficult to obtain. Anecdotal evidence is strongly in favor of the hypothesis that most fund managers did not adequately establish sufficient short positions. Many so-called hedge funds were decidedly long, not market neutral. In effect, these "hedge funds" were more like traditional mutual funds, only with the high performance fees. Since the early 1970s, there has been a veritable explosion in the complexity, availability and variety of securities, including the introduction of a range of relatively complex derivative

securities associated with financial commodities. This technological progress in the security design arena has been accompanied by the perception that there has been similar progress in the portfolio management and risk management systems that use these products. However, as evidenced by a string of high profile corporate collapses, "derivative debacles," and the like, this progress has also been accompanied by a systemic and partially uncontrollable increase in a range of manipulations that have exploited the leveraging potential of derivative security contracts.

It is not surprising that the renaissance in derivative securities has seen a remarkable resurgence of hedge funds. The *Report* of the President's Working Group on Financial Markets (1999) (PWGFM) investigating the collapse of Long Term Capital Management (LTCM) in 1998 makes the following observations about the growth in the hedge fund industry:⁹

A 1968 survey by the Securities and Exchange Commission identified 140 hedge funds operating at that time. During the last two decades, however, the hedge fund industry has grown substantially. Although it is difficult to estimate precisely the size of the industry, a number of estimates indicate that as of mid-1998 there were between 2,500 and 3,500 hedge funds managing between \$200 billion and \$300 billion in capital with approximately \$800 billion to \$1 trillion in total assets. Collectively, hedge funds remain relatively small when compared to other sectors of the US financial markets. At the end of 1998, for instance, commercial banks had \$4.1 trillion in total assets; mutual funds had assets of approximately \$5 trillion; private pension funds had \$4.3 trillion; state and local retirement funds had \$2.3 trillion; and insurance companies had assets of \$3.7 trillion.

Armed with the new risk management technologies, such as value-at-risk, a number of new style hedge funds actively use derivatives to leverage underlying capital. There is a substantial number of hedge fund strategies in place, only a small fraction of which are similar to the Jones model. Others, such as LTCM, are creatures of the renaissance in derivative securities.

The growth in the hedge fund industry has increased both the visibility and availability of information about hedge funds. For example, MARhedge is an important source of information, data and news about the hedge fund industry.¹⁰ Data available through MARhedge has been thoroughly examined in Ackermann et al. (1999). In order to provide some degree of organization to the mis-mash of hedge fund strategies, MARhedge (www.MARhedge.com) classifies hedge funds into eight broad categories: *global macro funds* that take positions on changes in global economic conditions in equity, FX and debt markets using derivatives, including index derivatives, and leverage; *global funds* that are similar to macro funds but targeted at specific regions, often involving stock picking; *long-only (US opportunistic) funds* that are like traditional equity funds but with the hedge fund characteristics of leveraging and incentive fees for managers – listed strategies for these funds include the traditional value, growth and short-term trading approaches; *market-neutral funds* where the basic objective is to be long in one group of securities and short in another group, such that market risk is controlled or neutralized – this group would include the Jones funds as well as conversion arbitrages, derivative security arbitrages and, fixed income arbitrages, such as the long off-the-run

Treasuries and short on-the-run Treasuries strategy; *sectoral hedge funds* that have an industry focus including short-sale funds, which short sell over-valued securities, investing the balance in indexes or fixed income securities; *event-driven funds* that target special situations such as distressed securities of firms in reorganization or bankruptcy as well risk trading in takeovers that involve buying the target and selling the acquirer; *short sales funds* where the fund is positioned to benefit from market declines including funds that are index driven and those based on stock picking; and, the *fund of hedge funds* approach that may involve leveraging to acquire positions in the individual funds. Within each of these general groupings, a variety of different strategies could be pursued. Similarly, some funds may be involved in activities covering more than one fund category.

The large number of possible hedge fund strategies raises legitimate questions about how the term "hedge fund" is defined. As indicated, the term is generic, being used to describe a variety of different fund strategies that loosely share some similar characteristics. The PWGFM (p. 40) defines the term "to refer to a variety of pooled investment vehicles that are not registered under the federal securities laws as investment companies, broker-dealers, or public corporations." Both of these features, pooled investment vehicle and absence of registration, are important to identifying whether a given fund qualifies as a hedge fund. More precisely, in order to avoid the registration requirements specified under US federal securities laws for securities companies, hedge funds have to be privately structured and closely held. As such, the primary investors in hedge funds are high net worth individuals and institutional investors. Recently, certain fund operators have been successful in extending the market for hedge funds to smaller individual investors, e.g., minimum investment floor of \$25,000. This has usually been accomplished by pursuing a fund-of-funds strategy and then registering that fund with the SEC.

Hedge funds are not the only securities that seek *specific exemptions from US securities laws*. For example, venture capital pools, asset securitization vehicles, family estate planning vehicles and investment clubs can receive such treatment. While it would seem sensible for a defining feature of hedge funds to be the types of strategies which the funds pursue there is so much variation in this area that it is difficult to isolate specific elements that apply to all hedge funds other than to say that the strategies usually involve a combination of long and short positions, including leveraging. Hedge funds are not conventional investment vehicles. Investor liquidity is often compromised with "lock-up periods of one year for initial investors and subsequent restrictions on withdrawals to quarterly intervals" (Ackermann et al. 1999, p. 834). The regulatory exemptions that hedge funds work under severely restricts the ability of hedge funds to advertise. Another untypical feature of hedge funds concerns their management (Ackermann et al. 1999):

Hedge funds are... characterized by strong performance incentives. On average, hedge fund managers receive a 1 percent annual management fee and 14 percent of the annual profits. For most funds this bonus incentive fee is paid only if the returns surpass some hurdle rate or "high-water mark" - meaning there is no incentive fee until the fund has recovered from past losses. Although incentive fees and high-water marks could lead to

excess risk taking under some conditions, there are countervailing forces that may dampen risk. Hedge fund managers often invest a substantial amount of their own money in the fund. Furthermore, the managers of US hedge funds are general partners, so they may incur substantial liability if the fund goes bankrupt.

Perhaps the closest security comparable to a hedge fund are the shares in managed futures funds and commodity pools that have been traded for many years.

In contrast to mutual funds which have been and continue to be intensively studied, hedge funds have only started to receive attention relatively recently, though work on managed futures funds and commodity pools, which started somewhat earlier, is also applicable, e.g., Irwin and Brorsen (1985), Elton, Gruber and Rentzler (1987), Cornew (1988), Irwin et al. (1993), Edwards and Caglayan (2001). Some useful recent studies directly on hedge funds include Fung and Hsieh (2000, 2002), Brown et al. (1999), Schneeweiss and Spurgin (1998), Ackermann et al. (1999), Liang (2000), Brown et al. (2001), Gregoriou (2002) and Goetzmann et al. (2003). A useful observation about studies of hedge fund performance is given by Caldwell (1995, p. 13):

Considerable caution must be used when reviewing performance statistics for the hedge fund industry and its various segments. Even the best statistics are skewed by asset weighting (or lack thereof), voluntary selection and a strong survivorship bias. It's highly unlikely that hedge fund performance statistics accurately reflect the true, weighted average return to investors for any segment of this industry.

Though more recent studies have come some of the way to correcting these difficulties, e.g., Fung and Hsieh (2002), there is still considerable uncertainty about how to measure and assess hedge fund performance. Gregoriou (2002) also reports: "The median (half-life) survival time of all hedge funds is exactly 5.50 years. It is found, however, that millions managed, redemption period, performance fee, leverage, monthly returns and minimum purchase have an impact on the mortality of hedge funds. It is also found that certain hedge fund classifications experience higher survival times than others."

What role can hedge funds play in the investment strategy of an individual investor? For a number of reasons, this practical question is difficult to answer. Because a large number of hedge funds operate under exemptions in US securities laws that restrict the allowable number of investors, such funds are not feasible investments for small investors due to the large initial investment (in the millions of dollars) that is required. Hedge funds that are publicly traded and feature an initial investment as small as, say, \$25,000, are currently structured with the fund-of-funds approach. This creates difficulties for the small investor seeking to monitor the holdings of the various hedge funds contained in the mingled fund that is purchased. In addition, even for investors capable of making direct investment into individual hedge funds, due to the large number of hedge fund strategies it is difficult to make generalizations about hedge fund performance. However, intuition suggests that a form of MM1 (see section 2.4) will apply to hedge funds, significantly reducing the potential abnormal returns that would accrue to this type of investment.¹¹

10.2 TACTICAL AND STRATEGIC ASSET ALLOCATION

Strategic asset allocation and two fund separation

Asset allocation involves determining the asset mix for a specific portfolio. As discussed in section 10.1, the asset allocation decision is a precursor to the security selection decision. Strategic asset allocation is a key first step that professional investment advisors take in the process of establishing new account parameters for unsophisticated investors. The new client is asked a range of questions regarding income level, risk tolerance, age, expected retirement age and the like. The types of questions asked are much the same across investment advisory firms. The end result is typically some target percentages for equity and fixed income holdings that will likely be slowly changed as the individual investor ages and the contribution levels to the portfolio change. This process is guided by the intuition that bonds are “safe” and equities are “risky.” Investors approaching retirement with low levels of risk tolerance are directed to portfolios that are heavily weighted to fixed income while those investors that are younger and have a higher level of risk tolerance are funneled into portfolios heavily weighted to equities. Despite the practical importance of such asset allocation strategies, Elton and Gruber (2000, p. 27) observe: “almost no attention has been paid [by academics] to examining advice regarding the asset allocation decision.”

Canner et al. (1997) initiated a discussion about the inconsistency between central propositions of modern Finance, such as the capital asset pricing model and the two fund separation theorem (see chapter 2), and the strategic asset allocation advice conventionally provided by professional investment advisors. In particular, the *two fund separation* theorem requires that the asset composition of the risky tangency portfolio – the market portfolio – is the same for all investors. Differences in risk tolerance across investors are handled by altering the value weights allocated to the riskless asset and the risky tangency portfolio. Hence, the ratio of stocks to bonds will not change across investors. This is inconsistent with the conventional strategic asset allocation advice of professional investment advisors. However, as Elton and Gruber (2000) illustrate, a range of qualifications are required to accommodate the various possible specifications of “modern portfolio theory” which includes the two fund separation theorem as a special case. Included in these qualifications are: the presence of a riskless asset; whether short sales are permitted; and the number and type of risky assets permitted in the risky portfolio.

The two fund separation result goes by a number of related names, such as the “portfolio separation theorem,” “two mutual fund theorem” and the like, e.g., Ingersoll (1987, ch. 6), Elton and Gruber (1995).¹² The essence of the two fund separation result is that all investors will hold combinations of only two funds, the market portfolio and the riskless security. In the purest form, the investment strategy decision associated with two fund separation is solely a strategic asset allocation decision: how is the total amount of invested capital divided between the riskless asset and the market portfolio? Because two fund separation is predicated on the assumption of market efficiency, it is not feasible to engage in tactical asset allocation where the proportions invested in the

two funds varies according to market timing decisions. The investment strategy decision is based on the risk attitudes of the investor. While the composition of the market portfolio is fixed externally, the proportion of the portfolio held in the riskless asset can be either positive (lending) or negative (borrowing). If the proportion is negative then the investor has borrowed at the riskless rate and has a leveraged position in the market portfolio.

The theoretical conditions required for two fund separation to apply are quite restrictive (see section 3.2). If all other assumptions of the model are maintained, the basic two fund separation result is not affected by whether short sales are allowed. Given the assumed homogeneity of investors, the tangency portfolio is the market portfolio because all assets have to be held in equilibrium. Short sales of the riskless asset are used to move the investor along the capital market line. Dropping the assumption that there is a riskless asset requires a zero beta portfolio to be determined as a substitute for the riskless asset. This will require a short sales assumption for the risky assets. Using the zero beta portfolio in place of the riskless asset, a version of the two fund separation theorem still holds with the additional implication that "all portfolios on the efficient frontier are a linear combination of any two other efficient portfolios" (Elton and Gruber 2000, p. 28). It follows that "any recommended portfolio must be a linear combination of any two other recommended portfolios." This provides an additional restriction on the properties of portfolios recommended by professional investment advisors.

Dropping both the riskless asset assumption and the short-sales-allowed assumption is sufficient to undermine the two fund separation theorem. Elton and Gruber (2000, pp. 28–9) provide a summary of the implications arising from dropping these assumptions:

If short sales [and the riskless asset] are not allowed, the nature of the efficient frontier changes. The two-fund theorem no longer holds. Securities enter and leave the efficient frontier at different risk-return tradeoffs. The points where they enter or leave are called corner portfolios. Securities may be held in zero weight for a range of risk tolerance and some assets are never held. Generally, the maximum return portfolio on the efficient frontier will consist of one asset and the minimum risk portfolio will consist of multiple assets. Thus, if short sales are not allowed and advisors are rational, any allocation recommendation should not be a linear combination of any two others unless all three lie at or between adjacent corner portfolios.

It follows that the practical implications of modern portfolio theory depend on the assumptions made to generate the results of interest. Depending on the assumptions made and the empirical return data used to determine the associated portfolio allocations, a range of possible specifications have to be considered in determining whether specific professional investment advice is consistent with the prescriptions of modern portfolio theory.

The central impetus for Canner et al. (1997) was to develop tests of rationality consistent with modern portfolio theory that could be applied in assessing the validity of strategic asset allocation advice of professional investment advisors and financial planners. Though Canner et al. argue for the irrationality of a decrease in the ratio of

bonds to stocks as investors' risk tolerance increases, Elton and Gruber (2000, p. 40) demonstrate: "whether or not short sales are allowed, the sign of the relationship between the bond stock ratio and risk hypothesized in Canner et al. cannot be used as a rationality test." In general, it seems that simple theoretical tests derived from modern portfolio theory are problematic and empirical properties of bond and stock returns have to be considered. Yet, once empirical properties are introduced this raises the problem of using *ex post* parameter estimates to determine the *ex ante* values required to make the strategic asset allocation decision. Ultimately, an underlying feature of this discussion is the implicit assumption that modern portfolio theory is the appropriate measure of rationality. Even if the epistemological approach of modern Finance is accepted, the various possible specifications of the model admit a range of sometimes conflicting strategic asset allocation prescriptions.

What is tactical asset allocation?

Tactical asset allocation is a catchall expression that is aimed at capturing gains to *market timing and portfolio rebalancing* decisions. Tactical asset allocation strategies can take a variety of forms. A common format is reflected in Ragsdale and Rao (1994, p. 209):

In tactical asset allocation, the central question is: Which asset class will provide superior future returns? Because relative returns are more important than absolute returns in this particular setting, many tactical asset allocators have focused on expected return premiums or spreads. As a practical matter, the most important comparison is that between stocks and fixed income (either bonds or cash) and the forecast on which the most effort is expended is the expected return for stocks. The comparison between stocks and fixed income is crucial because these are the two largest pools of assets in institutional portfolios. Stock return forecasts are important because, historically, stocks have provided the highest and most volatile investment returns.

Though a number of methods can be used to determine tactical asset allocation decisions, the basic procedure involves starting from a *benchmark* asset mix derived from the strategic asset allocation decision and then employing tactical methods to systematically deviate from the benchmark mix. Precisely how much deviation from the benchmark is permitted depends on the specifics of the fund being managed. Using the studies in Lederman and Klein (1994) as a guide, it appears that tactical strategies based on mean-variance optimization techniques are an important component of the class of available strategies.

As portrayed in modern Finance, tactical asset allocation takes place prior to a security selection decision. As such, the composition of risky assets is usually associated with a stock fund, such as the S&P 500, and a bond fund. Following Fox (1999), this leaves two key elements in the tactical asset allocation decision: "the tilt" and *forecasting ability*. The *tilt* is concerned with the amount of deviation from the benchmark holdings of the risky assets. If the benchmark is, say, 60/40 stocks and bonds then a

"full tilt size" could be 20% deviation, i.e., maximum values of 80/20 and 40/60. Fox uses this full tilt value in combination with a forecasting ability variable to simulate a range of tactical asset outcomes. Fox (1999, p. 46) describes some results of the simulation relationship: "In the long run, managers with forecasting ability of 60% or better will virtually never underperform the benchmark. For managers who have superior forecasting skill, increasing tilt size improves the entire range of possible return outcomes." As with other tactical asset allocation studies, performance is measured relative to the benchmark portfolio. In a study of the *tracking error* that arises in tactical asset allocation, Ammann and Zimmermann (2001, p. 32) demonstrate that the higher returns to forecasting ability are not at the expense of benchmark tracking accuracy: "imposing fairly large tactical asset allocation ranges produces surprisingly small tracking errors."

Another element in the tactical asset allocation exercise is the *rebalancing frequency*.¹³ Fox (1999), for example, uses monthly rebalancing as do Ammann and Zimmermann (2001). Presumably, this fixed rebalancing interval is a restriction imposed by the requirements of the research design, rather than being reflective of actual fund manager practices. However, Fox (1999, p. 40) observes: "the relatively small size of the US [tactical asset allocation] universe. Other types of portfolio managers can be compared to fifty to eighty peers, each with a long investment history. [Tactical asset allocation] managers have many fewer counterparts and only a handful have long-enough observed histories for accurate assessment." It would appear that the relevance of the tactical and strategic asset allocation approach is due more to the connection to modern Finance than to the practical importance of the approach in the fund management industry. As such, there is little guidance from practicing fund managers as to the appropriate rebalancing interval. Perhaps a daily or weekly tilt would apply for some funds while a quarterly or annual tilt would apply for other funds? Perhaps an irregular rebalancing interval is most appropriate but, if so, the process for determining a rebalancing point needs to be adequately specified.

Though there are decided similarities with other approaches to investment strategy, the terminology "tactical asset allocation" is intimately tied to strategic asset allocation and, in turn, to the modern Finance approach to investment strategy. As for similarities with other approaches, tactical asset allocation strategies can be contrasted with, say, the Dow theory (see section 9.2) where: the tilt is usually a "bang-bang" solution, i.e., 100% stocks during bull markets shifting to 100% fixed income or cash during bear markets; the forecast is generated by interpreting the Dow theory signals; and the rebalancing period is determined by the movements of the market rather than by a fixed time interval such as monthly or quarterly frequency. Within the conventional tactical asset allocation framework, industry rotation and country rotation strategies are considered to be too close to the realm of security analysis. There are some practical reasons for this. For example, certain institutional investors, such as pension funds and life insurance companies, have restrictions on holdings of foreign assets. A similar comment applies to the allowable tilt, where there are also various restrictions on the tilt imposed on institutional investors.

As a consequence of being narrowly structured within the modern Finance approach, the connection between security analysis and tactical asset allocation is left largely

unexplored by proponents. Yet, when security analysis is eventually added to the process, there are a number of potential feedbacks into the tactical asset allocation decision that have to be considered. For example, the tracking error associated with deviations from the benchmark portfolio could become substantial if the actual risky assets are small collections of individual securities (or even individual securities), as opposed to broadly diversified passive index portfolios. Another potential feedback concerns the rebalancing frequency. Using, say, a monthly rebalancing interval would require that securities be purchased and sold on specific days while security analysis typically requires that transactions be determined by value calculations. In effect, the tilt is determined by the number and variety of securities that satisfies the security analysis selection criteria at a given point in time. It is not obvious how to integrate security analysis, which involves decisions about a significant number of individual securities, into the tactical asset allocation framework, where stylized forecasts are made for a passive index.

Despite the connections between tactical asset allocation and modern Finance, there is a lack of agreement among modern Finance adherents about the efficacy of the practice. To purists, tactical asset allocation is another form of market timing strategy. Even when the potential for market timing is recognized, modern Finance purists usually observe that the range of optimal changes in portfolio weights is much less than typically recommended by tactical asset allocators. For example, Samuelson (1990) observes:

If you do have timing ability, flaunt it! But in the absence Napoleonic pretensions to clairvoyance, your rational flauntings are more likely to involve switches of a few percent in your equity fraction around some optimal intermediate level rather than the swings from 100% in stocks to 10% in stocks that characterized many asset allocation systems.

The basic intuition behind the attack on tactical asset allocation is embedded in the terminology used by Samuelson (1990) to describe the practice: "across-time deviations from diversification." Market timing requires the investor to temporarily deviate from the allocations that provide optimal, long-term diversification. Hence, unless there are sufficient gains in expected returns, tactical asset allocation will produce a sub-optimal amount of diversification resulting in a lower measured performance using, say, the Sharpe ratio (see below). While this conclusion is strongest for iid returns and log utility, the basic result carries through to other situations.

Cash management and dollar cost averaging¹⁴

Despite being an integral part of investment strategy, little attention is given to the cash management element of the decision. For example, in the CAPM, which provides the theoretical basis for two fund separation, the cash management decision is modeled as a problem in riskless lending and borrowing. Yet, unless the investment horizon of the portfolio is exactly equal to the term to maturity of the default-free, zero coupon fixed income security used as the riskless asset, then there will be a reinvestment risk

associated with rolling the security over at maturity.¹⁵ In addition to confusions associated with specifying the riskless asset, there are also complications associated with the cash flow requirements of the portfolio over the investment horizon. The basic optimization model underlying the CAPM and two fund separation assumes that there is a fixed level of initial wealth that is invested at the beginning of the investment period. Depending on the specification of the optimization problem there may be rebalancing along the time path. In addition, in the consumption-investment form of the problem, allowance could be made for cash inflows or outflows along the time path, in the form of consumption that can take positive or negative values. In a practical context, this leads to consideration of trading strategies concerned with buying along the time path, such as dollar cost averaging, e.g., Leggio and Lien (2003).

Adherents of modern Finance have long recognized the *practical limitations of the riskless asset* concept, e.g., Roll (1978). This concept of "cash" as a riskless security that pays the riskless rate of interest differs from actual "cash" which is legal tender – Federal Reserve bank notes in the US – that do not pay interest. Even this type of cash is subject to the erosion of purchasing power associated with price level inflation. Ignoring inflation, it is possible to view cash as a "riskless" security but this requires the riskfree rate of interest to be set equal to zero. Perhaps an interest bearing chequeable bank account is a more appropriate security to use as cash? What about using a money market mutual fund? Both of these securities feature changing interest rates, violating the CAPM condition that the security is riskless unless the frequency of interest rate changes equals the portfolio rebalancing frequency. In models with a fixed rebalancing frequency of, say, 3 months, then it is possible to use a 3 month US Treasury bill as the riskless asset. In any event, the issue of defining cash is a significant complication in assessing the appropriate cash management strategy to pursue. However, because the resolution of this complication diverts attention from more germane issues, it will be assumed that, somehow, an appropriate cash asset has been identified.¹⁶

It is conventional in the modern Finance approach to investment strategy to unbundle the cash management decision from the investment management decision. There are a number of reasons given for this simplification. For example, the previous discussion illustrated the point that the cash management decision requires "cash" to be defined. Even when an acceptable definition for the cash asset(s) is given, the cash requirements for the portfolio decision are usually given exogenously. Over time, there may be cash inflows to the portfolio from net labor income or cash outflows due to drawdowns during retirement. However, such cash flows do not typically impact the optimal solution to the theoretical decision problem in a transparent fashion. Interpretation usually involves making sense of intertemporal marginal rates of substitution, correlations between labor income and investment returns, rates of time preference and the like. To avoid such complications, it is conventional to simplify the cash inflow/cash outflow component of the portfolio decision by assuming that a lump sum of initial wealth is invested at the beginning of the investment horizon and is held until the end of the horizon. This framework can be adjusted to permit the initial lump sum to be invested over a sequence of time periods, as in dollar-cost-averaging strategies.

Unbundling the cash management decision from the larger investment optimization problem permits a number of possible approaches to be pursued. Unfortunately, this

unbundling does not necessarily lead to useful and practical recommendations. Following the theoretical approach used in economics, the optimal cash management solution will depend on the individual investor's supply and demand for cash. The supply of cash will depend on the initial capital and, possibly, other factors such as the cash flow requirements, investment returns and labor income over the time path. The demand for cash will have three basic elements: precautionary demand; transactions demand; and speculative demand. Unfortunately, using this approach opens a hornet's nest of conflicting opinions about: the specification of the different demand functions; the various definitions of money, from "high powered money" to "near-money" to "outside money"; the empirical properties of the possible demand function specifications and so on. Following Tobin (1958), the speculative demand for money can be modeled using much the same theoretical framework as that employed in the derivation of the Sharpe-Lintner CAPM. This approach reduces the solution of the cash management decision problem to that contained in the CAPM.

In the CAPM framework, optimal holding of the riskless asset (cash) depends on a combination of investor risk attitudes and the spread between the riskfree rate and the expected return on the risky portfolio. When appropriate, this optimal holding is adjusted at each of the potentially diverse number of rebalancings that occur over the investment horizon. In practice, this process reduces to deciding the optimal method of purchasing risky assets over the rebalancing period (investment horizon). Cash assets are held as a buffer to support such purchases. A number of possible methods are available. *Lump sum investing* involves purchasing the desired asset allocation at the beginning of the rebalancing period (investment horizon). The disadvantage of this approach is that the purchase decision may occur at a time when the risky assets are selling at "high" prices, somehow defined. In addition, the change in asset prices over time will cause the portfolio weights to deviate from the desired asset allocation. If no rebalancing is permitted along the path, or if the optimal lump sum allocation is 100% in risky assets, then the lump sum approach reduces to the *buy-and-hold* approach. In such cases, the tactical and strategic asset allocation decisions are identical.

At least since Merton (1973) it has been recognized that the buy-and-hold approach is generally sub-optimal (see end of chapter questions). A widely recommended alternative approach to buy-and-hold is *dollar cost averaging* (DCA) where the investor makes a fixed dollar investment at regular intervals during the rebalancing period. The underlying rationale for DCA is that the risk of buying at a market high associated with lump sum investing is avoided. In addition, for risky assets with volatile prices, the larger number of shares purchased at low prices will more than offset the smaller number of shares purchased at high prices, resulting in a net gain. An alternative to DCA is *value averaging* where, instead of investing a fixed dollar amount each period, the size of the investment is set to maintain a constant increase in the value of the portfolio each rebalancing period. For example, assume the objective is to increase the portfolio value \$100 per month (the rebalancing window). If the portfolio increases 5% in the first month, then investment in the second month would be \$95 (+ \$105 = \$200). If there is a 2% drop in the second month then the investment in the third month would be \$104 (+ 196 = \$300) and so on. Both value averaging and DCA are aimed at capturing the gains of "buy low, sell high."

In comparing these different approaches, it is useful to identify the various types of investment situations being compared. The conventional structure is associated with an inheritance. The investor acquires a sum of money at the beginning of the investment horizon and has to determine the strategy for achieving the highest level of expected utility at the end of the investment horizon, allowing for possible rebalancing within the horizon. This basic structure would also apply to, say, a retired investor trying to determine the optimal method for withdrawing funds from an investment account to use for consumption expenses, e.g., Vora and McGinnis (2000). In this case the relevant question being determined can be described as: is it optimal for the retired investor to make one large withdrawal at the beginning and hold cash over the horizon or is it optimal to make regular withdrawals? To address these types of questions, Leggio and Lien (2003) set the rebalancing frequency at one month and the investment horizon at one year. Distributional information, such as average returns and standard deviation of returns, are then calculated over the sample of annual returns. This general approach tends to mask other possible aspects of the typical investment decision. For example, by working with the arithmetic average of returns, the growth in the value of the portfolio over time is not directly incorporated (see sections 1.1 and 10.4).

In addition to the structure of the underlying investment decision, to determine the optimal investment purchasing strategy some method of performance measurement is required. It has long been recognized that any claim about the optimality of this or that investment strategy also involves an implicit claim about *the method of performance measurement*. In turn, because optimality is usually situated within an expected utility framework, it follows that performance measurement be similarly situated. As such, it is conventional to take both the risk and return of the strategy into account, except in situations where the objective function imposes risk neutrality. The various limitations and failings of the CAPM have made the subject of performance measurement an active research area within modern Finance, e.g., Ferson and Schadt (1996). Included in the various issues that have arisen are: the method of measuring risk, e.g., beta vs. standard deviation; the structure of the investment problem, e.g., are cash inflows and outflows permitted; the method of measuring return, e.g., geometric return vs. arithmetic average vs. CAPM alpha; and, the types of portfolios being compared.

Perhaps the most widely used method of performance measurement in modern Finance is the *Sharpe ratio* (SR):

$$SR_p = \frac{E[R_p] - r}{\sigma_p}$$

where $E[R_p]$ and σ_p are the expected return and standard deviation of portfolio return (see section 3.1 for definitions) and r is the riskfree interest rate. Being the empirical representation of the capital market line, the Sharpe ratio is directly related to the CAPM. This measure is closely related to a number of other measures that are also based on the CAPM such as: Treynor's measure, which adjusts the SR by substituting the beta of the portfolio for σ_p ; and, Jensen's alpha, which uses the alpha estimate from the market model expressed in excess return form. Because these measures are all based on the CAPM, it is possible to develop precise theoretical relationships between these

measures (see end of chapter questions). Significantly, the reliance on the CAPM to measure performance raises some questions about these estimators.

In addition to portfolio performance measures associated directly with the CAPM, a number of other measures have been proposed. The primary insight of the CAPM measures is that both the risk and the return of the portfolio have to be considered in assessing performance, not just return alone. Though it is possible to consider higher moments of the distribution (see section 3.1), this is not conventional. The focus on risk and return leads to consideration of the various methods of estimating the relevant parameters. For example, the $E[R]$ could be estimated using the arithmetic or geometric averages.¹⁷ Sortino et al. (1999) provide an illustration of a performance measure that does not employ standard estimators. More precisely, the *upside potential ratio* (UPR) is calculated by taking the ratio of the Sortino downside risk measure (δ) (Sortino and Price 1994) to an upside expected return measure (θ):

$$\delta^2 = \int_{-\infty}^r (r - x)^2 f[x] dx \quad \theta = \int_r^{\infty} (x - r) f[x] dx$$

where r is the riskfree interest rate, x represents the random variable for security return and $f[x]$ is the return distribution.

As an illustration involving some of these performance measures, consider the Leggio and Lien (2003) empirical comparison of dollar cost averaging, value averaging and lump sum investing (see table 10.1). This study uses the monthly returns from the Ibbotson and Associates *Yearbook* from four different asset classes: large cap stocks, with returns calculated using the S&P composite index, small cap stocks, which is measured with the Ibbotson small cap index, with US corporate bonds and US government bonds measured using total return indexes for long-term maturities. The riskfree interest rate is US Treasury bills. The study uses the Sharpe ratio, the *Sortino ratio* ($\{E[R] - r\}/\delta$) and the upside potential ratio (θ/δ) to compare the strategies. In doing the calculations, the strategies are assumed to have an investment horizon of one year. It is assumed that a fixed amount of capital is received at the beginning of each period. Dollar cost averaging would then involve investing 1/12 of the capital in the risky asset each month while lump sum investing would invest all the capital in the risky asset at the beginning of the period. The $E[R]$ is then estimated as the arithmetic average of the annual returns over the sample, with other parameter estimates following appropriately. In calculating the results in table 10.1, a 1970–99 sample is used. Leggio and Lien also report results for other samples, such as 1950–1999 and 1926–1999, with some differences in results.

Table 10.1 is useful for comparing the trading strategies, as well as the properties of the performance measures. Leggio and Lien (2003, p. 219) describe the results:

For the sample period of 1970–99... the Sharpe ratio and Sortino ratio lead to identical rankings of investing strategies for all asset classes. For corporate bonds, the Upside Potential ratio also ranks the investing strategies consistently with the other ratios. However, for both large-cap stocks and government bonds, Upside Potential ratio ranks the preferred investing strategy as reported by the Sharpe ratio to be the least preferred strategy.

Table 10.1 Annualized excess returns and risk measures, 1970–99

Strategy	Mean	Standard deviation	Sharpe ratio	Sortino ratio	Upside potential ratio
<i>A Asset: large stocks</i>					
Lump Sum	7.76	16.18	0.48	4.95	2.93
Dollar Cost Average	3.60	8.88	0.41	4.46	3.46
Value Average	3.46	9.38	0.37	3.73	3.28
<i>B Asset: small stocks</i>					
Lump Sum	9.70	22.84	0.42	4.49	4.02
Dollar Cost Average	1.83	12.17	0.15	1.87	3.44
Value Average	6.96	13.35	0.52	4.93	3.93
<i>C Asset: corporate bonds</i>					
Lump Sum	3.01	11.91	0.25	3.79	5.69
Dollar Cost Average	2.79	6.85	0.41	6.24	6.19
Value Average	-0.13	5.22	-0.02	-0.34	4.60
<i>D Asset: government bonds</i>					
Lump Sum	2.90	12.33	0.24	3.91	4.87
Dollar Cost Average	3.00	6.71	0.45	7.07	4.78
Value Average	-0.40	5.96	-0.07	-1.15	5.20

Source: Leggio and Lien (2003)

As for the different trading strategies, the relative rankings reveal that, using the Sharpe and Sortino ratios, lump sum did best for large stocks, while value average did the best for small stocks – a case where DCA did significantly worse than the other two strategies. For corporate and government bonds, DCA had the best performance. Value averaging was disastrous for the two bond cases. Using the upside potential ratio to measure performance produced generally contrary results, e.g., value averaging had the best performance for government bonds. Such results raise substantive questions about the different possible methods used to measure portfolio performance. In addition, claims about the superior performance of DCA often made by practitioners are also brought into question.

Milevsky and Posner (2003) seek a theoretical resolution to the popularity of DCA among practitioners and individual investors. Working within a continuous time framework using Brownian bridges, Milevsky and Posner model DCA as a path dependent claim and develop a mathematical proposition that “proves”: “the expected return from . . . the DCA strategy – conditional on knowing the final value of the security will uniformly exceed the return from the underlying security for all sufficiently large volatilities.” In effect, DCA outperforms lump-sum investing. This leads to the conclusion that rational investors using DCA are working with target prices. In effect, adherents of DCA are working with conditional subjective expectations. In this framework, the

more volatile is the underlying security price, "the greater is the benefit to dollar-cost averaging – conditional on knowing the final value." This result is in sharp contrast to what has been generally accepted wisdom about DCA, based on Constantinides (1979) and later theoretical studies where DCA is shown to be a dynamically inefficient trading strategy. Whether the Milevsky and Posner proposition clarifies or muddies the overall picture on DCA is, at present, unclear.

10.3 INVESTMENT STRATEGY FOR VALUE INVESTORS

The Graham, Dodd and Cottle approach

Writing just prior to the widespread acceptance of modern Finance in academic circles, GDC (p. 439) make the following revealing observation: "There is nothing to prevent the investor, whether an individual or an institution, from making actual investment purchases of a unit such as the Dow-Jones industrial group when the composite valuation indicates he is getting good value for his money. Such a policy may be unusual, but it is not illogical." Though there is an element of the old Finance intrinsic value approach in this statement, GDC capture the essence of the investment strategy inherent in the two fund separation theorem, albeit with the qualification that the Dow-Jones industrial average is only a crude proxy for the "market portfolio." What is revealing in the statement is how "unusual" an investment strategy that favored diversification over security selection was in the early 1960s. Though there were widely diversified investment funds available at the time, the concept of index funds was relatively undeveloped. The value of the "professionally managed investment fund," so strongly promoted by Irving Fisher and others three to four decades earlier (see section 2.3), was still the prevailing model.

Given the different possible approaches that fall within the "value investing" category (see section 7.4), the identification of a specific investment strategy for value investors is not possible. There is so much diversity that it is even possible for strategies similar to two fund separation to be included within the value-investing rubric. This point is reflected in GDC (p. 446):

Those who practice [the intrinsic value approach] will use their conclusions as a basis for selecting issues to make up a common-stock portfolio, and for recommending the sale of holdings that appear definitely overvalued or the replacement of less by more attractive stocks... Each security analyst, however, will develop his own approach to these problems in accordance with his investment philosophy, his interests and his individual capabilities. We have no desire to prescribe dogmatically any one course of procedure in this area.

As such, all that can be concluded about the GDC approach to value investing is that it involves "buy low, sell high" with intrinsic value, diversification and margin of safety being the guides for specific decisions. This approach does differ from, say, the value investing approach of Philip Fisher where a long-term investment horizon is used and

outstanding stocks are retained because there are an insufficient number of opportunities for alternative investment and timing the market is too difficult. In other words, GDC visualize trading a portfolio of "primary stocks" while Philip Fisher has an aversion to trading.

Within this context, GDC do recognize that the intrinsic value approach imposes restrictions on the "model investment policy." The cumulative effect of determining whether the estimated intrinsic value of a security satisfies the margin of safety principle requires a dynamic security trading strategy. Restrictions on this trading strategy are imposed by the diversification principle, but this is insufficient guidance. This point is not lost on GDC: "Many of our readers would like as much guidance as we are able to give them in this highly important and intensely practical aspect of the work of the security analyst." Though GDC are largely concerned with security analysis, i.e., techniques for determining the value of individual securities, some attention is given to the problem of managing the aggregation of securities that are being bought and sold. However, as indicated, the motivation for this was largely driven by the desires of "many of our readers" as opposed to being included as an element of expertise or profound insight. What emerges is a suggested approach to investment strategy that has a certain incongruity with other elements of the GDC approach, but this is what is proposed so this is what is reported here.

GDC (p. 447) visualize a *three step investment strategy* process for an individual with sufficient funds to "make possible adequate diversification." This case is selected because it is the easiest to determine. Qualifications would be required for other cases. The *first step* in the process is to determine the proportion of the portfolio to be held in common stocks "at varying levels of the stock market." In more modern terminology, this is the "asset allocation" decision. The *second step* is to establish a process for selecting the common stocks to be included in the portfolio; this is one aspect of the "security analysis" decision. The *third step* relates to "rules for selling a security held or replacing one stock by another"; this is partly a security analysis decision and partly an asset allocation decision. To adherents of the Warren Buffett or Philip Fisher approach to "value investing," the first step in the GDC investment strategy process may seem to be at odds with the prescriptions of: not following day-to-day fluctuations in the stock market; and, not analyzing or worrying about the general economy. The description also seems to be at odds with decidedly negative statements about market timing made by GDC in chapter 53 that explicitly compares market analysis with security analysis.

Is it possible to take the view that step one does not involve predicting the direction of the stock market but, rather, is an artifact of steps two and three? To answer this question consider the GDC description of the first step:

we shall assume that the typical individual investor would always have a fixed-value component in his portfolio. Accordingly, we would set an upper limit of 75 percent to be held in common stocks and a lower limit of 25 percent. This minimum proportion, which would be kept in common stocks irrespective of the level of the market or the outlook for equities, is in accord with the view . . . regarding the inclusion of common stocks in the ordinary investment portfolio.

This relatively innocuous statement is followed by:

The proposed common stock proportions would be put into effect pursuant to a formula plan, which in turn would be geared to the analyst-investor's valuation of the DJIA, Standard & Poor's Composite Index, or some other measure of the market. Assume, for example, that the analyst accepts our unpopular valuation of the DJIA at between 540 and 570 at the beginning of 1962. The actual market level of 730 would then call for holding at or close to the minimum figure of 25 percent in common stocks. This percentage would not be increased until the level fell to the top of the *then* valuation range – not necessarily 570. It would be brought up to 50 percent of the portfolio within the new valuation range, and to the maximum of 75 percent at various levels below the range.

GDC qualify this with the statement: "Very few professionals or nonprofessionals in finance would at this time accept or follow the policy outlined above." The reasons for this are as expected: a formula-timing approach is not considered to be desirable; the assumption that the market has an estimable "central value" is "obsolete"; and, even if there is a central value, this value cannot be determined from "long-term past experience." Despite these qualifications, step one of the "model investment policy" still appears to have a tactical asset allocation component.

GDC recognize that step one can be separated from steps two and three. It is possible that application of the intrinsic value method plus "margin of safety" principle used in step two will provide an automatic adjustment of the portfolio proportion held in common stocks that has much the same effect as the "formula-timing" approach of step one. In particular, when the market is "overvalued" then there will be relatively few opportunities for investment in common stocks and vice versa when the market is "undervalued." Exogenous maximum and minimum bounds on the proportion invested in common stocks could be achieved by altering the margin of safety as the bounds are approached. Given this, step two involves picking 20–30 stocks from a "basic portfolio list" of not more than 100 "large, prosperous, soundly capitalized and well-known" common stocks. Stocks not on the basic list would only be considered if the estimated intrinsic value indicated the stock was "at least 25 percent" cheaper than the most "overvalued" stock on the primary list. Restrictions on the amount invested in any one industry would also be imposed.

The GDC (pp. 448–9) guidance on step three is somewhat vague. There is a reference to a formula-timing-motivated reduction in the fraction of the portfolio in common stocks: "The investor with a full 75 percent common-stock portfolio at a low market level would begin to sell off, on some graduated basis, after the market has again advanced to at least the upper limit of the (new) appraisal range." The reduction could occur by reducing all stocks proportionately "or by selling those issues which at the time of sale bear the least attractive relation to their then appraised [estimated intrinsic] value." This could be an implicit recognition by GDC of the inability that the intrinsic value plus margin of safety method has in providing a complete ranking for purposes of selling (and buying) stocks. In particular, the method does not rank stocks according to the size of the difference between the estimated intrinsic value and market price – this would put too much pressure on the precision of estimating intrinsic value. Rather, all that is provided by the method is a buy/no buy and sell/no sell decision.

Finally, GDC recommend that any replacement of one stock with another not be done unless there is a large advantage, "say, at least, one third."

In the end, the limited advice on asset-allocation-type investment strategy provided in GDC is largely incongruent with the rest of the text and does not rise substantially above the mundane. This is hardly surprising when it is recognized that GDC is dedicated to security analysis. The essential elements of the GDC approach – intrinsic value, margin-of-safety and diversification – do implicitly impose restrictions that generate a coherent tactical asset allocation strategy. In effect, GDC have a *security-analysis-driven investment strategy* (also referred to as a "bottom-up" strategy). However, this connection is not developed or even explicitly recognized by GDC. Presumably, investors adhering to the GDC approach would recognize the overvaluation (undervaluation) of almost all common stocks around market tops (bottoms) and would sell (buy) stocks and buy (sell) bonds resulting in a rapid tilt away from (toward) stocks during periods of declining (rising) stock prices. In technical terms, by rebalancing along the time path using intrinsic value as a selection criteria, GDC are implicitly pursuing a growth-optimal-like investment strategy (see section 10.4). If this outcome appears as wishful thinking, anecdotal evidence indicates otherwise.

The Warren Buffett approach

As observed in section 7.4, the Warren Buffett approach to value investing is not easily replicated by individual investors. Whereas Buffett, through Berkshire Hathaway, is able and willing to purchase whole companies, both publicly traded and privately held, individual investors are typically restricted to investing in companies by purchasing publicly traded corporate securities with investments in private companies not appearing on the menu of available choices. The advantage that Buffett has over the individual investor is apparent in the Berkshire-Hathaway annual reports where numerous purchases of whole companies are reported. For example, the 2002 Report (Buffett 2003) details direct purchases of Albecca and Fruit of the Loom as well as purchases through subsidiaries such as the purchase of pipelines by MEHC. In addition to positions in whole companies, Berkshire Hathaway has substantial positions in the insurance and re-insurance sector. These companies are also involved in the management of asset portfolios. Buffett makes no secret that the investment management activities of the insurance segment are incorporated into the investment decisions made by Berkshire Hathaway.

As discussed in section 7.3, Buffett is a strong adherent of the use of the GDC principles of intrinsic value and margin-of-safety. Application of these concepts in the US stock market during 1999–2002 leads Buffett to conclude that purchases of common stocks were not generally warranted during this period. More precisely, Berkshire Hathaway (2003, pp. 15–16) takes the following view of common stocks:

Despite three years of falling prices which have significantly improved the attractiveness of common stocks, we still find *very* few that even mildly interest us. That dismal fact is testimony to the insanity of valuations reached during the Great Bubble. Unfortunately, the hangover may prove to be proportional to the binge.

Table 10.2 Common stock positions of Berkshire Hathaway, larger than \$500 million, 12/31/02

Shares	Company	Cost (\$ millions)	Market
151,610,700	American Express Company	\$1,470	\$5,359
200,000,000	The Coca-Cola Company	1,299	8,768
96,000,000	The Gillette Company	600	2,915
15,999,200	H&R Block, Inc.	255	643
6,708,760	M&T Bank	103	532
24,000,000	Moody's Corporation	499	991
1,727,765	The Washington Post Company	11	1,275
53,265,080	Wells Fargo & Company	306	2,497
	Others	<u>4,621</u>	<u>5,383</u>
	Total common stocks	<u>\$9,164</u>	<u>\$28,363</u>

The aversion to equities that Charlie and I exhibit today is far from congenital. We love owning common stocks – If they can be purchased at attractive prices. In my 61 years of investing, 50 or so years have offered that kind of opportunity. There will be years like that again. Unless, however, we see a very high probability of at least a 10% pre-tax return (which translates to 6%–7% after corporate tax), we will sit on the sidelines. With short-term money returning less than 1% after tax, sitting it out is no fun. But occasionally successful investing requires inactivity.

As indicated in table 10.2, this statement is not meant to imply that Berkshire Hathaway has no position in equities. Quite the contrary, the company has long standing common stock investments in a number of high profile companies, including Coca-Cola, Gillette and the Washington Post.¹⁸ At the least, the view expressed about common stocks would have increased the tilt of Buffett's aggregate investment position towards fixed income securities. This tilt is further confirmed in the Report where a "sextuple" increase to \$8.3 billion in Berkshire Hathaway junk bond holdings during 2002 is reported.

The actual experience of Berkshire Hathaway relates directly, if anecdotally, to the implementation of the GDC security-analysis-driven investment strategy, with the qualification that the value investing approach of Warren Buffett is not precisely the same as GDC. For example, Buffett has a stronger bias to buy-and-hold than GDC. The replacement of overvalued securities with undervalued securities is a more important part of the security analysis process for GDC than for Buffett. Berkshire Hathaway has some practical reasons for this buy-and-hold bias. Large positions in common stocks are difficult to unwind and repurchase. If there is a positive long-term view on the security of a company then the gains to trading, particularly in trading range markets, will likely be illusory. In addition, though the security selection methodology is similar, Buffett has adapted the GDC approach to include the Philip Fisher great-companies model. This synthesis reinforces the buy-and-hold bias of Buffett's investment strategy. There are only so many great companies in the world, and the securities, particularly the common stock, of such companies are scarce. In this regard, Buffett goes well beyond Fisher in extending the universe of securities to include purchases of whole companies.

It is one of the intellectual oddities of modern Finance that, despite rafts of studies on arcane and incidental issues, so little attention has been given to anecdotal study of the security market activities and writings of Warren Buffett. The bulk of information and discussion about Buffett is to be found in the trade press. While there are some academic efforts along this line, e.g., Statman and Scheid (2002), it is difficult for proponents of modern Finance to reconcile the inconsistency with efficient markets that Warren Buffett presents. In addition, there is also the general epistemological unwillingness to consider anecdotal evidence. Perhaps Buffett has just been lucky? In any game there will be winners. In big games, there will be big winners. As Stickney (1997) observes (see section 1.3): "academic research focuses on the average relation between selected accounting information and stock prices across a large number of firms." The intellectual oddity arises because a potentially important source of insights into securities markets is being ignored. It appears that the observations of Frank Knight (1925) still apply: "One who aspires to explain or understand human behavior must be, not finally but first of all, an epistemologist." In this context, the logical positivism of modern Finance is not structured to generate knowledge from the type of anecdotal evidence provided by Warren Buffett, despite the potential practical significance of such knowledge.

Some evidence on the performance of Berkshire-Hathaway has already been provided in section 7.4. Table 7.4 documents the remarkable performance of the company. Table 7.5 lists the major operating companies owned by Berkshire Hathaway with Table 7.6 listing the acquisition criteria. All of this information was obtained from the 2002 Annual Report (Buffett 2003). These Berkshire Hathaway annual reports are an essential source on the views of Warren Buffett. As such, it is surprising that the most recent annual Report is not required reading in introductory investment classes, if not in more advanced classes. Yet, courses with such a requirement are the exception rather than the rule in the academic curriculum. Those perusing the 2002 Report will find the thirteen "Owner Related Business Principles" that Buffett includes to provide a precise description of the managerial approach at Berkshire Hathaway. These principles were originally developed in 1983 and were initially circulated to shareholders in 1996. In addition to such gems, the annual reports can also be examined to address a range of important questions such as: how does Berkshire Hathaway make money? and, what are the current views of Buffett and Charles Munger on securities markets?

From the Report it is possible to partially unravel the complex current financial composition of Berkshire Hathaway, and determine sources of profit. As illustrated in tables 10.3 and 10.4, the insurance business plays a key role in the overall revenue and earnings of Berkshire-Hathaway. The insurance business is particularly attractive for an operator such as Buffett because of the ability to gain competitive advantage from superior performance of the investment portfolio. This is another aspect of the Buffett approach that is not accessible to individual value investors. This element goes back to the creation of Berkshire Hathaway in 1967. The life and times of Warren Buffett and Berkshire Hathaway have been examined in detail elsewhere (e.g., Hagstrom 1995, 2000; Lowenstein 1995), so only a brief excerpt will be given here. The company was formed from a merger of Berkshire Cotton Manufacturing, a textile company, and Hathaway Manufacturing. Also included in the merger were two insurance companies.

Table 10.3 Berkshire Hathaway Inc. and subsidiaries, selected financial data (\$ millions except per share data)

	2002	2001	2000	1999	1998
<i>Revenues</i>					
Insurance premiums earned	19,182	17,905	19,343	14,306	5,481
Sales and service revenues	17,347	14,902	7,361	5,918	4,675
Interest, dividend and other investment income	3,061	2,815	2,725	2,314	1,049
Revenues of finance and financial products businesses	2,126	1,658	1,505	987	394
Realized investment gains ^a	637	1,363	3,955	1,365	2,415
Total revenues	<u>42,353</u>	<u>38,643</u>	<u>34,889</u>	<u>24,890</u>	<u>14,014</u>
<i>Earnings</i>					
Net earnings ^{a,c,d}	<u>4,286</u>	<u>795</u>	<u>3,328</u>	<u>1,557</u>	<u>2,830</u>
Net earnings per share ^d	<u>2,795</u>	<u>521</u>	<u>2,185</u>	<u>1,025</u>	<u>2,262</u>
<i>Year-end data^b</i>					
Total assets	169,544	162,752	135,792	131,416	122,237
Notes payable and other borrowings of non-finance businesses	4,807	3,485	2,663	2,465	2,385
Notes payable and other borrowings of finance businesses	4,481	9,019	2,116	1,998	1,503
Shareholders' equity	64,037	57,950	61,724	57,761	57,403
Class A equivalent common shares outstanding, in thousands	1,535	1,528	1,526	1,521	1,519
Shareholders' equity per outstanding Class A equivalent common share	<u>41,727</u>	<u>37,920</u>	<u>40,442</u>	<u>37,987</u>	<u>37,801</u>
	2002	2001	2000	1999	1998
Net earnings as reported	4,286	795	3,328	1,557	2,830
Goodwill amortization, after tax	—	636	548	476	111
Net earnings as adjusted	<u>4,286</u>	<u>1,431</u>	<u>3,876</u>	<u>2,033</u>	<u>2,941</u>
Earnings per class A equivalent common share:					
As reported	2,795	521	2,185	1,025	2,262
Goodwill amortization	—	416	360	313	88
Earnings per share as adjusted	<u>2,795</u>	<u>937</u>	<u>2,545</u>	<u>1,338</u>	<u>2,350</u>

^a The amount of realized investment gains and losses for any given period has no predictive value, and variations in amount from period to period have no practical analytical value, particularly in view of the unrealized appreciation now existing in Berkshire's consolidated investment portfolio. After-tax realized investment gains were \$383 million in 2002, \$842 million in 2001, \$2,392 million in 2000, \$886 million in 1999, and \$1,553 million in 1998.

^b Year-end data for 1998 includes General Re Corporation acquired by Berkshire on December 21, 1998.

^c Net earnings for the year ending December 31, 2001 includes pre-tax underwriting losses of \$2.4 billion in connection with the September 11th terrorist attack. Such loss reduced net earnings by approximately \$1.5 billion and earnings per share by \$982.

^d Effective January 1, 2002, Berkshire adopted Statement of Financial Accounting Standards ("SFAS") No. 142

"Goodwill and Other Intangible Assets." SFAS No. 142 changed the accounting for goodwill from a model that required amortization of goodwill, supplemented by impairment tests, to an accounting model that is based solely upon impairment tests.

A reconciliation of Berkshire's Consolidated Statements of Earnings for each of the five years ending December 31, 2002 from amounts reported to amounts exclusive of goodwill amortization is shown below. Goodwill amortization for the years ending December 31, 2001 and 2000 includes \$78 million and \$65 million, respectively, related to Berkshire's equity method investment in MidAmerican Energy Holdings Company.

Table 10.4 Berkshire Hathaway, sources of profit (\$ millions)

	Revenues		
	2002	2001	2000
<i>Operating businesses</i>			
Insurance group:			
Premiums earned:			
GEICO	6,670	6,060	5,610
General Re	8,500	8,353	8,696
Berkshire Hathaway Reinsurance Group	3,300	2,991	4,712
Berkshire Hathaway Primary Group	712	501	325
Investment income	<u>3,067</u>	<u>2,844</u>	<u>2,796</u>
Total insurance group	22,249	20,749	22,139
Apparel	1,619	726	678
Building products	3,702	3,269	178
Finance and financial products	2,126	1,658	1,505
Flight services	2,837	2,563	2,279
Retail	2,103	1,998	1,864
Scott Fetzer Companies	899	914	963
Shaw Industries	4,334	4,012	—
Other businesses	<u>1,983</u>	<u>1,488</u>	<u>1,436</u>
	41,852	37,377	31,042
<i>Reconciliation of segments to consolidated amount</i>			
Realized investment gains	637	1,363	3,955
Other revenues	29	35	54
Eliminations	(56)	(65)	(26)
Purchase-accounting adjustments	<u>(109)</u>	<u>(67)</u>	<u>(136)</u>
	42,353	38,643	34,889
<i>Operating Profit before taxes</i>			
	2002	2001	2000
<i>Operating businesses</i>			
Insurance group operating profit:			
Underwriting profit (loss):			
GEICO	416	221	(224)
General Re	(1,393)	(3,671)	(1,254)
Berkshire Hathaway Reinsurance Group	534	(647)	(162)
Berkshire Hathaway Primary Group	32	30	25
Net investment income	<u>3,050</u>	<u>2,824</u>	<u>2,773</u>
Total insurance group operating profit (loss)	2,639	(1,243)	1,158
Apparel	229	(33)	6
Building products	516	461	34
Finance and financial products	1,016	519	530
Flight services	225	186	213
Retail	166	175	175
Scott Fetzer Companies	129	129	122
Shaw Industries	424	292	—
Other businesses	<u>691</u>	<u>377</u>	<u>320</u>
	6,035	863	2,558
<i>Reconciliation of segments to consolidated amount</i>			
Realized investment gains	603	1,320	3,955
Interest expense ^a	(86)	(92)	(92)
Corporate and other	2	8	22
Goodwill amortization and other			
purchase-accounting adjustments	<u>(119)</u>	<u>(630)</u>	<u>(856)</u>
	6,435	1,469	5,587

^a Amounts of interest expense represent interest on borrowings under investment agreements and other debt exclusive of that of finance and financial products businesses and interest allocated to certain other businesses.

An initial key to the future success of Berkshire Hathaway can be found in the substantial profit generated by Buffett after assuming management of the insurance company asset portfolios.

During the 1970s, the insurance aspect of the business was expanded considerably with the purchase of three additional insurance companies and the creation of five more. The textile segment was closed and a holding company was created. Management of the aggregate Berkshire Hathaway asset portfolio led naturally to the acquisition of non-insurance related businesses. The role of these businesses in the company has gradually risen from about 1/3 to 40% of revenue in the early 1990s to almost 50% currently (see table 10.4). Included in the asset purchases of Berkshire Hathaway have been a number of large common stock purchases in Coca-Cola, Gillette, American Express and other high profile companies. Table 10.2 reveals the success of these particular purchases. Given the size of these positions, even when common stock purchases are considered, the Buffett approach is not directly accessible for the typical individual investor. A position of, say, 200,000,000 shares in Coca-Cola differs substantively from a 500 share position, if only in terms of access to senior management. The upshot is that the typical individual value investor would have considerable difficulty in formulating a complete investment strategy from the experience of Warren Buffett.

Practical investment strategy for individual value investors

What are appropriate investment strategy concerns for an individual investor with a relatively small capital base seeking to pursue the value investing approach? The potential for smaller investors to pursue such an approach has been greatly enhanced in recent years with the explosion in investment information both on-line and in a range of financial media. If used with sufficient skepticism, these information sources permit the individual investor to obtain a depth and breadth of information about publicly traded companies that was available only to professional investors even a couple of decades previously. Legislative changes in the US have permitted IRAs, 401 (k) and other tax advantaged retirement savings plans (RRSPs in Canada) that now provide a pool of investment funds representing a significant fraction of the individual investor's net worth. Many of these plans provide sufficient flexibility for the individual investor to, say, pursue strategic and tactical asset allocation strategies that are consistent with personal investment philosophy. The accumulated evidence about the below benchmark performance of the average actively managed mutual fund provides considerable incentive for the individual investor to formulate and implement individual investment strategies based on the value investing approaches suggested by the likes of GDC and Philip Fisher, and popularized by Warren Buffett.

For reasons discussed previously, it is difficult for individual value investors to directly pursue the Warren Buffett investment strategy. Similarly, the guidance of GDC appears to be aimed at the practicing professional security analysts and portfolio managers that are involved in trading securities on a regular basis. Consideration also has to be given to historical changes in securities markets that have taken place since

GDC considered investment strategy. Yet, despite the historical evolution, there are certain constants that gives the GDC approach to value investing a timeless quality. One constant is the emphasis on distinguishing between *speculation and investment*. The individual investor has to determine the amount and degree of speculation that will be undertaken. The value investing approach is aimed at capturing superior investment returns. In this process, there is an element of speculation involved in predicting future security price performance. However, speculative activities such as the short-term trading of securities to earn profit are not directly employed. GDC do discuss "market analysis," a category that would include technical analysis and the Dow theorists. However, the discussion is superficial. As such, value investors have to ignore the returns accruing to predominately speculative situations when making investment decisions.

Sorting out the distinction between speculation and investment involves the identification of an individual *investment philosophy*. By taking the value investor route, the individual implicitly accepts elements that are contained in the investment philosophy of leading value investors, such as GDC. Even though there is some variation in certain aspects of philosophy across different value investors, where speculative securities are concerned there is general agreement. Such securities fail to conform to the criteria required to undertake an accurate assessment of intrinsic value. Given this, there is scope for honest disagreement over which particular securities are speculative. However, such disagreements revolve around factors concerned with the valuation methodology rather than with the criteria required to identify speculative situations. For example, some value investors may be attracted to the debentures of a bankrupt corporation such as Worldcom, based on an assessment that the potential payout of, say, 35 cents on the dollar is sufficiently in excess of the current price of 21 cents on the dollar. Other value investors may feel that, given their knowledge about bankruptcy law, Worldcom and the telecom industry, it is not possible to make an accurate enough assessment of the asset values and other factors to make a decision that is not overly speculative in nature.

Issues of investment philosophy are at the core of many investor decisions. Value investing requires the future cash flows of a security to be estimated. In turn, this requires the individual investor to acquire sufficient information to make reasoned decisions about the estimated cash flows. This imposes substantial demands regarding information collection, processing and analysis. Given that there are thousands of potential securities domestically and ten of thousands if international securities are included, this poses a quandary for individual investors. In the face of potential information overload, what are the appropriate methods to use in arriving at an investment decision for a given security? Faced with this quandary, value investors have used a variety of different methods. For example, Warren Buffett recommends only investing in securities of businesses that the investor can understand. This approach considerably narrows the field of potential securities. GDC divide investors into *defensive investors* and *enterprising investors* (see section 10.1) where "[t]he great majority of individual investors belong in the defensive category." In general, such investors are well served by the two-fund separation theorem.

The individual enterprising investor requires guidance well beyond that of the individual defensive investor. For example, while the defensive investor will automatically

achieve a high level of diversification associated with having a position in a broadly-based stock index such as the S&P 500, the enterprising value investor needs to consider the treatment of diversification. As discussed above, though the *benefits of diversification* are explicitly recognized by value investors, this is expressed only in terms of, say, not concentrating invested capital in a small number of companies in one industry. Close reading of the editions of Graham and Dodd (1934) up to GDC reveals that modern issues of investment strategy – strategic and tactical allocation – are largely irrelevant. In the value investing approach, the asset allocation decision is determined by available security investment opportunities. The decision process starts with security analysis and works backward to tactical and strategic asset allocation considerations. Following Philip Fisher, the enterprising investor may well be confident in holding only a small number of securities, subject to the restriction that there be diversification across industries: “any holding of over twenty different stocks is a sign of financial incompetence.” In contrast, GDC visualize picking the best 20–30 stocks from a pool of 100.

Casual consideration of the diversification issue leads to a natural question for value investors to consider: what is the desired frequency of trading for the portfolio? A sound and widely recommended rule-of-thumb is, if possible, *avoid transactions costs and management fees*. Defensive investors can partially achieve this outcome by using low management fee, high liquidity index funds. However, even defensive investors have to decide on a long run allocation between stocks, bonds and cash and whether there will be rebalancing of the portfolio at regular intervals. This rebalancing decision will depend on whether the investor wants to make a market call. As discussed in chapter 10, there are good reasons to engage in such prognostications, even if the call is weakly supported. Various factors come into play, such as: whether the market is in a trading range or a trend; and, whether it is in a secular bull or secular bear phase. Forecasting ability does not have to be that acute in order to undertake a shift to bonds and other fixed income securities in a stock market downdraft. As the Dow theorists suggest, look for the signs of speculative excess and do not be too worried about getting out early.

It seems that a defensive investor pursuing an active rebalancing strategy may engage in more securities trading – albeit in index funds and bonds – than an enterprising investor following a Philip Fisher, buy-and-hold approach. In pursuing this approach, the defensive investor is motivated by the observation that, while stocks may outperform bonds in the long run, bonds can outperform stocks over investment horizons of significant length. General results about asset classes, such as the equity premium that is of concern in modern Finance, can belie actual results for individual investors trading specific securities over a given investment horizon. Properly executed, a strategy of rebalancing between index funds and fixed income securities would provide attractive returns. This strategy also has a grounding in the principles of modern Finance. If security analysis is judged to be a zero or negative expected value activity (due, for example, to the perception that markets are efficient) then two-fund separation is a rational investment strategy. Recognizing that two fund separation involves an optimal allocation between the risky market portfolio and the riskless asset, rebalancing along the path to maintain the optimal weights is indicated.

While there is attractive potential returns in the rebalancing strategy of the defensive investor, such returns are dwarfed by the possible gains for the enterprising investor. It is not surprising that, in practice, such gains are almost always accompanied by an enhanced element of speculation. In the technical analysis context (see chapter 10), speculation is typically associated with active trading strategies. In contrast, the enterprising value investor following, say, the *Philip Fisher approach* (see section 7.4) would have the buy-and-hold strategy of a long-term investor. Only a limited amount of rebalancing is admitted. The speculative element comes primarily from narrowing the number of securities in the portfolio. "There are a relatively small number of truly outstanding companies." The difficulty with this approach centers on the *ex ante* identification of the truly outstanding companies. As indicated in chapter 9, it is not necessary or practical to have knowledge of the full set of securities. With the thousands of common stocks and bonds traded on securities markets, it is only necessary to develop a comfortable knowledge about a smaller universe of securities. This raises natural questions about the construction of this smaller universe from the larger, unmanageable universe with tens of thousands of stocks in hundreds of industries and countries.

In selecting this universe, a sensible first step is to follow Warren Buffett's dictum (see sections 7.4 and 10.3): *concentrate on the securities of companies that are understandable*. This dictum is implied in the essence of value investing because purchasing securities in companies that are not understood is speculative. Other dictates of Warren Buffett about the characteristics of these understandable companies follow appropriately: look for companies that have a consistent operating history and favorable long-term prospects. Included in the consistent operating history will, in most cases, be a strong and sustained profit margin. Company management needs to be closely scrutinized and desired rules of conduct verified. Though Buffett tends to favor companies with a franchise that is needed by consumers and has strong brands, such as Coca-Cola or Gillette, companies that compete primarily on a price/net cost basis, such as oil and gas producers, can also be acceptable as long as these companies satisfy the criteria for a value investment. One such criterion is that management has been able to use past retained earnings to generate a more than compensating change in market value.

Each value investor needs to specify the particular criteria that is to be applied in identifying feasible companies for investment. Though there will be considerable overlap in these criteria across value investors, there will be significant variation in the application across investors. Given this, the amount of time and energy an individual investor can devote to gaining and maintaining information about a group of stocks will determine the size of the universe. Individual investors with relatively little time can pursue *hybrid strategies* that mix defensive and enterprising styles. While maintaining a sizeable position in the passive market index, the hybrid investor will also have a small stable of securities in individual companies. The objective of the hybrid investor is to incrementally outperform the market index return. The dedicated individual enterprising investor is assisted by the securities market information that can be rapidly found on the internet, cable television and in modern print sources such as the *Wall Street Journal*. Even with effortless information access, it is unlikely that there is enough time for an individual to exceed the 100 stock universe identified in

GDC. Probably, the total number of stocks that could carefully be followed would be much less.

The question of the number of securities and companies to follow is closely related to the number of industries that these companies fall within. From this point, it is difficult to provide general guidance. Some value investors may favor the "*buy-low-sell-high*" approach of GDC and take the *ex ante* view that such opportunities arise because of across industry differences in valuation. By rotating the portfolio across industries, these differences in valuation can be obtained. Such a value investor would, almost certainly, cover a larger number of industries than an enterprising investor seeking to follow, say, the Philip Fisher approach. In this case, there would only be a cursory examination of cross-industry valuations. Because considerable effort is expended in finding only a small number of securities, a paring process takes place that quickly reduces the list of potential candidates. If an attractive opportunity appears in an unfamiliar industry or area to the investor, time and effort is expended in acquiring the understanding. Additions and, if necessary, subtractions from the portfolio take place at glacial speed. As a consequence, the time and effort given to each trading decision is considerable. Fisher would be disappointed to find that any investor following his approach has as many as twenty securities in the portfolio.

10.4 ADVANCED TOPICS IN INVESTMENT STRATEGY

*The interior decorator fallacy and other confusions**

The investment strategy decision for institutional investors is often constrained by regulatory considerations. In addition, institutions usually have substantial internal resources: to determine the long-term portfolio allocation between stocks, bonds and cash within allowable limits; to make short-term, market timing adjustments to the long-term portfolio composition; and, to make security selection decisions designed to enhance overall portfolio returns. Some institutions may tolerate only slight deviations from long-term portfolio composition targets, preferring to enhance returns through reliance on security selection within defined asset groupings. Other institutions may permit significant deviation from long-term targets in order to enhance returns through market timing and security selection strategies. These decisions are guided by the constraints facing the specific institution. For example, mutual funds will be driven by the requirements set out in the fund prospectus and the need to obtain competitive returns relative to competing funds in the same grouping. Due to the diversity of institutions and institutional investment objectives, combined with the considerable internal expertise available within most institutions, it is difficult to briefly make helpful generalizations about the investment strategy decisions of institutions. As a consequence, in what follows discussion focuses on the investment strategy for individual investors.

Relative to the variation in investment strategies across institutions, the investment strategy decision for individual investors is more homogeneous. Due to the perceived

lack of sophistication, the investment strategy decision for many individual investors is guided by recommendations provided through the advisory services of brokerage firms and investment banks, either directly or through mutual funds offered by these firms. These advisory services are an important linchpin of revenue generation for many securities firms. Fees are obtained for providing services required to design and implement "customized" portfolios for specific investors. These bundled and unbundled services include brokerage, in-house mutual funds, accounting management, investment research and the like. Since the inception of securities markets, financial sector firms have demonstrated considerable ingenuity and adaptability in generating revenue from the investment advisory process. In recent years, challenges have been presented by the deregulation of brokerage fees, the emergence of discount brokers, the rise of online trading and the enhanced accessibility of index funds with low management fees. In the face of these challenges, the advisory services of securities firms have thrived based, it seems, largely on the perception of the typical individual investor that, somehow, the investment strategy decision is "too complicated" and specialized advice is required.

Is the individual investment strategy decision sufficiently complicated to warrant the compensation that financial planners, investment advisors and securities firms are paid to facilitate this decision? In many cases, there are reasons to expect that the compensation is excessive. As discussed in section 3.2, the two fund separation theorem suggests that, for the typical investor, a strategy of holding the market portfolio and the riskless asset is optimal. While this solution is idealized, the basic intuition suggests that holding low management fee, passively managed stock market index fund(s) combined with long-term government bonds and a sufficient level of cash is an appropriate strategy for a wide range of investors. The long-term weights of bond, index fund and cash can be determined by the individual investor after due reflection, without the need to pay sizeable fees for advisory services. This view is supported, for example, by numerous empirical studies that have found the *ex post* performance of the "average" actively managed mutual fund underperforms the appropriate index benchmark fund after expenses, e.g., Daniel et al. (1997). As Campbell and Viceira (2002, p. 3) observe: "Financial planners have traditionally resisted [this] simple investment advice... This resistance may to some extent be self-serving."

Despite the apparent overcharging for services exhibited by various investment advisory services, "almost no attention [in academic studies] has been paid to examining advice regarding the asset allocation decision" (Elton and Gruber 2000, p. 27). As evidenced by Bernstein (1992), Canner et al. (1997), Campbell and Viceira (2002) and others, this lack of attention is surprising given that the asset allocation advice dispensed by investment advisory services is seemingly in conflict with the tenets of the investment strategy proposed in modern Finance: "as Peter Bernstein points out in his 1992 book *Capital Ideas*, many financial planners and advisors justify their fees by emphasizing the need for each investor to build a portfolio reflecting his or her unique personal situation" (Campbell and Viceira 2002, p. 3).¹⁹ For example, younger investors and aggressive investors are encouraged to hold "riskier" portfolios than older investors and conservative investors, usually by investing a larger fraction of the portfolio in stocks versus bonds plus cash (see table 10.5). This need to tailor investment portfolios to the specific characteristics of individual investors is referred to as the *interior deco-*

Table 10.5 Asset allocations recommended by selected financial advisors

Advisor and investor type	% of portfolio			Ratio of bonds to stocks
	Cash	Bonds	Stocks	
A Fidelity				
Conservative	50	30	20	1.50
Moderate	20	40	40	1.00
Aggressive	5	30	65	0.46
B Merrill Lynch				
Conservative	20	35	45	0.78
Moderate	5	40	55	0.73
Aggressive	5	20	75	0.27
C Jane Bryant Quinn				
Conservative	50	30	20	1.50
Moderate	10	40	50	0.80
Aggressive	0	0	100	0.00
D The New York Times				
Conservative	20	40	40	1.00
Moderate	10	30	60	0.50
Aggressive	0	20	80	0.25

Source: Canner et al. (1997)

rator fallacy by Bernstein and others.²⁰ This terminology makes a connection to the strong role that personal preferences and tastes play in interior decoration. The fallacy arises because much of the advice is either unnecessary or inconsistent with the portfolio theory of modern Finance.

The interior decorator fallacy begs the question: can sound reasons be provided for recommending distinct investment strategies for different investors? Theoretically, there appears to be considerable support for the need to tailor investment portfolios. Included in the possible reasons that have been proposed for differences between individual investors are: tax status, usually associated with income, capital gains and estate taxes; the length of the investment horizon, where young persons typically have long horizons and retired persons have short horizons; the cash payout requirements; the randomness of labor income and stock market returns, including the correlation between these two variables; different investor attitudes towards risk, often characterized as "aggressive" or "conservative"; short sales constraints, that are more costly for non-institutional investors; the level of current wealth; and, illiquidity of assets in the aggregate investor portfolio, such as real estate or privately held businesses. The diversity and number of possible factors makes it difficult to develop a general framework for individualized asset allocation.²¹ Instead, considerable effort has been dedicated to studying specific aspects of the problem. The resulting diversity of possible solutions gives the appearance

of a complicated decision problem when, for most investors, a rational investment strategy is not difficult to determine.

While security analysis is viewed as a zero expected return activity and is largely ignored in modern Finance, the subject of investment strategy – albeit in the guise of “portfolio management” – has a long history with numerous substantive contributions. The bulk of these largely theoretical contributions involve, in some fashion, the risk aversion properties of expected utility functions (see section 3.3).²² In particular, the relative risk aversion property of the expected utility function determines the fraction of wealth held in risky assets as the level of investor wealth increases. Relative risk aversion plays a key role in many of the solutions to the portfolio management problem proposed in modern Finance. An early example is the solution to the *time diversification puzzle* provided by Samuelson (1963) and re-examined in Samuelson (1994), Bodie (1995), Kritzman and Rich (1998), Fisher and Statman (1999) and Kritzman (1994, 2000). This puzzle arises from the conventional recommendation that investors with longer investment horizons allocate a higher fraction of the portfolio to riskier assets, i.e., common stocks. Time diversification requires that, somehow, the riskiness of stocks is “diversified” as the investment horizon increases. The logic of this view is described by Siegel (1998, p. 26): “Although it might appear to be riskier to hold stocks than bonds, precisely the opposite is true [for long investment horizons]; the safest long-term investment for the preservation of purchasing power has clearly been stocks, not bonds.”

Does the length of the investment horizon matter to the portfolio allocation decision? Is the optimal portfolio for short-term and long-term investors the same? Samuelson (1963) states a set of *three sufficient theoretical conditions* for expected utility maximizing investors to hold a constant fraction of a portfolio in risky assets as the investment horizon increases, e.g., Kritzman (2000, p. 48):

1. Investors have constant relative risk aversion, which means that they maintain the same percentage exposure to risky assets regardless of changes in wealth.
2. Investment returns are independently and identically distributed, which means that they follow a random walk.
3. Future wealth depends only on investment results and not on human capital or consumption habits.

Being based on logical deduction, these three conditions can be used to assess the rationale for making different asset allocation recommendations based on the length of the investment horizon. The first point is sensible if investors are expected utility maximizers; constant relative risk aversion requires that the investor keep the fraction of the portfolio invested in risky assets constant as wealth increases.²³ Recognizing that investor wealth changes over the life cycle, this condition is needed to ensure that time diversification does not apply as wealth levels change over time. However, the use of expected utility theory in this context may be undermined by, say, the *Friedman-Savage puzzle* where the expected utility function does not display a uniform attitude towards risk across wealth levels, e.g., Shefrin and Statman (2000).²⁴

Condition two is more complicated to interpret, if only because the statistical implications of a random walk are subject to misunderstanding. It is helpful to reference

a result concerning the standard deviation for a sum of standard normal random variables. More precisely, the discrete time random walk can be specified as:

$$X(t) = X(t-1) + Z(t) \quad \text{where } X(0) = 0, \text{ and } t \in \{1, 2, 3 \dots\}$$

where $Z(1), Z(2), Z(3) \dots$ form a stochastic process of *independent* random variables with the standard normal probability distribution: $Z(t) \sim N[0, 1]$ (see section 1.3).²⁵ This requires the $Z(t)$ to be identically, independently distributed (iid) random variables. Over any time interval 0 to T , the variance of $\Delta X(t)$ can be evaluated by determining the variance:

$$\text{var} \left\{ \sum_{t=1}^T Z(t) \right\} = E \left[\sum_{t=1}^T Z(t) \right]^2 = \sum_{t=1}^T \sigma_Z^2 = T \sigma_Z^2 = T$$

Hence, when $\Delta t = T$, the Z is $N[0, T]$. Now, consider what happens when the time interval Δt shrinks. Because $Z(t)$ is $N[0, \Delta t]$ over any arbitrary time interval, the random walk now has the form:

$$X(t + \Delta t) = X(t) + Z(t + \Delta t) \sqrt{\Delta t}$$

In Samuelson's condition two, the variable $X(t)$ is the investment return. It follows that if investment returns are iid then the variance will increase proportionally with time while the standard deviation will increase with the square root of time.

What is the relevance of this result for time diversification? In general, time diversification requires that the expected utility of terminal wealth increases as the length of the investment horizon increases. If returns follow a random walk, then the riskiness of the investment – as measured by the standard deviation of returns – will increase with the square root of time. Whether this increases or decreases expected utility will depend on the connection between the increase in risk and the weight attached to risk in the expected utility function. For example, where the expected utility functions is risk loving, then time diversification holds by construction when returns are iid. For functions displaying risk aversion, then time diversification will depend on the desired holding of risky assets as the wealth level increases. When returns are not iid, the picture is much cloudier and results can depend on the specific process selected. For example, as discussed below, Siegel (1998) relies on the assumption of mean reversion to motivate time diversification.²⁶

Given that returns are iid, the application of condition two now requires the evaluation of a risky investment as time increases *for an investor with a constant relative risk aversion utility function*. Though it is possible to demonstrate this result in general terms, it is expedient to illustrate this point using an example. Consider the expected log utility function, $E[\ln[W]]$, that displays the constant relative risk aversion property (see end of chapter 1 questions). Condition two states that if the return generating process is iid then an investor making decisions using expected log utility will be indifferent between a certain outcome and a wealth process that grows according to the iid return process. For illustrative purposes, let the iid process be binomial. Following

Kritzman and Rich (1998), construct a certainty equivalent comparison between a certain outcome, say \$100, and a one-step ahead binomial process for wealth. Choosing the probability of the up and down moves to be 50%, let the outcome in the up state be \$133.33, a one-third gain, and the down state to be \$75, a one-quarter loss. It is possible to show that $E[\ln W(1)] = .5[\ln[133.33] + \ln[75]] = 4.6052 = \ln[100]$. Hence, the appropriate return generating (recombining) binomial process has a one-third gains on the upside and a one-quarter loss on the downside, with equal probabilities of up and down moves (see end of chapter questions).

Given initial wealth, $W(0)$, of \$100, it follows that $W(1) = .5[133.33 + 75] = 104.17$, implying an expected return of 4.17%. In this example, time diversification requires that the expected utility will increase as the risky expected wealth process evolves through time. In other words, comparing the expected utility of terminal wealth for a one period investor and a T period investor ($T > 1$), the expected utility will be equal across the horizon. To see this, consider the recombining binomial process taken one more period ahead where $E[\ln W(2)] = .25\ln[177.28] + .25\ln[56.25] + .5\ln[100] = E[\ln W(1)] = .5[\ln[133.33] + \ln[75]] = 4.6052 = \ln[100]$. The expected utility of terminal wealth is unchanged as the investment horizon gets longer. Time diversification does not apply as there is no incentive to change the investment in the risky asset based on the length of the investment horizon. Kritzman (2000, p. 53) observes: "this result is not restricted to investors with log-wealth utility functions. It applies to all investors who have constant relative risk aversion as long as returns follow a random walk. If [time diversification was] true, we would observe an increase in expected utility as the number of periods of the investment increases."

One of the strongest academic proponents of time diversifications is Siegel (1998). The rationale provided in favor of "stocks for the long-run" focuses on the empirical properties of the "standard measures of risk" (Siegel 1998, pp. 31-2):

stocks are riskier than fixed-income investments over short-term holding periods. But once the holding period increases to between 15 and 20 years, the *standard deviation* of average annual returns, which is the measure of the dispersion of returns used in portfolio theory, become *lower* than the standard deviation of average bond or bill returns. Over 30-year periods, equity risk falls to only two-thirds that of bonds or bills. As the holding period increases, the standard deviation of stocks falls nearly twice as fast as that of fixed-income assets.

Siegel explicitly denies the validity of the random walk hypothesis: "data show that the random walk hypothesis cannot be maintained and that the risk of stocks declines far faster when the holding period increases more than predicted. This is a manifestation of the *mean reversion* of equity returns." For Siegel, time diversification is predicated on an empirical foundation. Condition two in Samuelson (1963) is denied and an alternative empirical model for stock prices, based on *mean reversion* (see section 5.2), is proposed. Kritzman (2000, pp. 57-8) claims that when the stock price process is mean reverting, then constant relative risk aversion still does not exhibit time diversification. However, Barberis (2000, p. 227) argues that mean-reversion will lower the variance of cumulative returns over long horizons: "This makes stocks appear less risky to long-horizon investors and leads them to allocate more to equities than would investors with

shorter-horizons" (see end of chapter questions).²⁷ In any event, time diversification with mean-reversion could still follow if condition one is dropped and replaced by an increasing relative risk aversion utility function, such as the quadratic.

Some care has to be taken in interpreting the Siegel arguments in favor of time diversification. For example, similar to a host of other academics that approach Finance from an economist's perspective, Siegel uses "real" returns to measure performance. As this involves dividing one random variable, the nominal return, by another random variable, the inflation rate (see section 7.3), the statistical properties and the practical intuition of this variable are unclear.²⁸ During investment time periods where the inflation rate is low, the differences between real and nominal returns will be small. However, for long time horizons, there will likely be periods where real and nominal returns diverge significantly due to the level and volatility of inflation rates. For a number of reasons, it is more meaningful to assume that investors are concerned with actual returns, not "real" returns. The process by which an individual investor makes adjustments for changes in price levels varies considerably. For example, some investors, such as retired investors with income streams that are fixed in nominal terms, will experience an erosion in real income as prices rise. In contrast, other investors with, say, labor income streams that increase with inflation will have an offset to potential "real" investment income losses.

Another issue arising from the Siegel mean-reversion-based argument concerns the process of implementing a time diversification agenda as a portfolio management strategy. The basic Samuelson (1963) result, which appears to be carried forward by Siegel, is predicated on a *buy-and-hold investment strategy*. This involves purchasing stocks at the beginning of the holding period and selling at the end. Alternatively, it is possible to pursue a rebalancing strategy where the portfolio is adjusted at regular or irregular intervals according to some set of criteria. There are also hybrid strategies where the buy-and-hold strategy involves the purchase of investments that have a predetermined rebalancing mechanism. For example, if the stocks in the buy-and-hold portfolio are a broadly diversified value-weighted market index fund, such as the S&P 500, this implicitly involves passive rebalancing to reflect changes in index value weights as relative stock prices change. Yet, if mean reversion is the basis for the claim of time diversification, this implies that some process of active rebalancing is being used to exploit the time series properties of security returns. The precise connection between the rebalancing and the empirical assumption of mean-reversion is unclear.

As modern portfolio theory evolved, the significance of portfolio rebalancing was explicitly recognized. However, the analytical apparatus required to incorporate *optimal rebalancing* is considerably more involved than the buy-and-hold approach where the optimization problem is modeled using only a decision date and a terminal date. Techniques such as stochastic dynamic programming are required to theoretically solve the optimization problem when more than one decision date is permitted. For example, in continuous time this leads to the derivation of a partial differential equation (PDE) that can be solved for the optimal weights.²⁹ Depending on the specification of the problem, the PDE may or may not admit closed form solutions. Simplifying conditions required to obtain a solution were provided in early contributions by Samuelson (1969) and Merton (1969). These studies demonstrated that combining the assumptions of iid

security returns with the constant relative risk aversion power utility function is sufficient to generalize the one decision date result. In other words, using the simplifying conditions permits the portfolio time path to have the same asset allocation across possible investment horizons.

As the study of the optimal portfolio management progressed, theoretical features were introduced into the problem that altered the basic structure of the solutions. Consistent with concerns arising from microeconomic theory, the portfolio optimization problem was formulated as a *consumption-investment problem*. This involves an explicit concern with consumption along the time path. Following Merton (1973), this transforms the focus of the problem to measuring asset return risk using the covariance with the marginal utility of the investor. As Campbell (1996, p. 299) observes: "In an intertemporal setting, this need not be the same as covariance with the market return, because innovations in marginal utility can be driven by changing expectations of future returns, which determine the marginal productivity of wealth, as well as by increments to wealth itself." The upshot is that concern shifted from a theoretical framework in which asset allocations were readily identifiable to a framework where investor marginal utilities and hedging demands take center stage, e.g., Ingersoll (1987, ch. 13). While the elegance and rigor of the solutions was considerably enhanced by these changes, the practical applicability of the optimal solutions was correspondingly reduced.

The final condition given by Samuelson (1963) anticipates these sorts of theoretical complications by explicitly ruling out the potential impact of human capital and consumption effects on the portfolio allocation decision. Yet, much of the theoretical development of modern portfolio theory since Samuelson (1963) has taken place in a theoretical framework that incorporates such effects. Merton (1969) and Samuelson (1969) were able to deal with one aspect of this issue by demonstrating that "retirement is irrelevant for portfolio decisions if investment opportunities are constant and human capital is tradable." However, Merton (1971) "shows that time-varying investment opportunities result in portfolio rules with an intertemporal hedging component whose magnitude depends on the investment horizon of the investor" (Viceira 2001, p. 433). Merton (1973a) expands this notion in the context of an intertemporal capital asset pricing model to demonstrate that investors with long investment horizons will allocate more to equities if the utility function is more risk averse than log utility. This result provided further impetus to the theoretical shift from utility functions defined using terminal wealth, to consumption-portfolio problems using utility functions that included the time path of consumption as well as terminal wealth. As evidenced by recent studies such as Campbell and Viceria (2002), Constantinides (2002), Viceria (2001), and Lettau and Ludvigson (2001) this theoretical framework is still the convention, though there are some exceptions that work only with terminal wealth, e.g., Barberis (2000).

The fascination with dynamic asset pricing and optimal consumption-portfolio rules that was initiated by Fama (1970a), Hakansson (1970) and Merton (1971, 1973a) has continued to the present producing a bewildering array of theoretical contributions. However, while inherently interesting to economists with a theoretical orientation, the concern with the consumption-savings-portfolio decision masks essential elements that are of interest in practical Finance. To see this, consider the rhetoric of the following description provided by Viceria (2001, p. 435):

[this] paper builds a stationary model in which it is possible to explore life-cycle effects on portfolio choice and savings while preserving the analytical advantage of infinite-horizon models. Retirement is defined as a permanent zero-labor income episode . . . by comparing the optimal allocations of investors with different retirement horizons, it is possible to understand portfolio allocations over the life cycle because, if discount rates and the expected growth rate of labor income are constant over the life cycle, the investor's retirement horizon is relevant for her portfolio decisions only in that it determines her remaining human capital.

In this time separable power utility of consumption framework, Viceria (2001, p. 433) concludes: "With idiosyncratic labor income risk, the optimal allocation to stocks is unambiguously larger for employed investors than for retired investors, consistent with the typical recommendations of investment advisors." Though this result seems to favor time diversification, this conclusion is elusive: "Increasing idiosyncratic labor income risk raises investor's willingness to save and reduces their stock portfolio allocation towards the level of retired investors." Ultimately, as is common in economic theorizing, conditions for a contrary conclusion are also provided: "Positive correlation between labor income and stock returns has a further negative effect and can actually reduce stockholdings below the level of retired investors."

In defense of Viceria (2001), this "on-the-one-hand-and-on-the-other-hand" theorizing establishes a number of key issues that can possibly be subjected to empirical analysis. Is the correlation between labor income and stock returns positively and significantly correlated? Is labor income tradable? To what extent is it possible to borrow against future labor income? To what extent is labor income idiosyncratic? Unfortunately, empirical analysis of these types of questions raises a range of additional problems. For example, consider the Siegel rationale for time diversification that arises from mean reversion in stock returns. This empirical model restricts the future expected returns from wandering too far from the long run mean, thereby lowering the variance of cumulative returns over long horizons compared to the random walk model.³⁰ In an intertemporal portfolio management framework that permits optimal rebalancing, mean reversion allows investors to "predict" expected returns from realized returns and to adjust portfolios appropriately.³¹ Theoretically, this will result in investors holding "substantially more equities at longer horizons, but only when they are more risk-averse than log utility investors" (Barberis 2000, p. 227). However, such results require accurate empirical estimates for the process generating stock returns.

A key difficulty with using an empirical assumption, such as mean reversion of stock returns, to justify time diversification is that implementation requires parameter estimates. While this may not present much difficulty in a theoretical setting, when it comes to making actual trades based on these estimates, the resulting parameter uncertainty poses real problems. Barberis (2000, p. 227) finds: "in both the static buy-and-hold and the dynamic rebalancing problem, incorporating parameter uncertainty changes the optimal allocation significantly . . . In some situations, we find that uncertainty about parameters can be large enough to reverse the direction of the results. Instead of allocating more to stocks at long horizons, investors may actually allocate *less* once they incorporate parameter uncertainty properly." Barberis (2000) derives these results

using a utility model with constant relative risk-aversion power utility defined over terminal wealth. The upshot is that the issue of time diversification is too complicated to provide a definitive solution with practical validity. By implication, it is not possible to resolve other theoretical issues that arise from the interior decorator fallacy. Consistent with the interior decorator approach, it may be safe to say that the allocation between stocks, bonds and cash will vary across individuals. However, a theoretical resolution of precisely what those individual allocations are is currently unavailable.

*The fallacy of large numbers and growth optimal portfolios**

Though widely accepted in modern Finance, the mean-variance optimization model is not the only model of portfolio choice that has been explored and proposed. One important alternative is the portfolio that is "growth optimal," i.e., the portfolio that provides the highest expected (log) return over the investment horizon. Casual reflection about the properties of this portfolio raises questions about the riskiness of the return. The highest expected return will likely come with downside as well as upside. Investors that are risk averse will not find such a portfolio to be as attractive as some other "optimal" portfolio with less upside and downside. It follows that analysis of the growth optimal portfolio will have to account for the investor's expected utility function. At this point in the analysis confusion arises because, if the investment horizon is long enough, then the strong law of large numbers (see section 1.3) suggests that the growth optimal portfolio will outperform, with probability one, all other portfolios. In other words, in the limit, the actual cumulative return on the portfolio will converge to the expected value. Hence, for long term buy-and-hold investors, it would seem to be irrational for investors with any sensible expected utility function to select any other portfolio than the growth optimal portfolio.

As it turns out, the use of the law of large numbers to justify selection of the growth optimal portfolio has led to description of this argument as the *fallacy of large numbers*. Following Samuelson (1963), this fallacy is often motivated by referring to an individual accepting a sequence of bets involving an iid random variable with positive expected value when an individual bet is unacceptable. A possible example would be a coin flipping game where the individual loses \$9 for heads but gains \$10 for tails. One trial of such a game may be unacceptable while a sufficiently long sequence of trials for such a game is acceptable. For Samuelson, this represents a misunderstanding of the law of large numbers, which applies to averages, because both the variance and expected return from the game apply to a sum that will increase with the number of trials. It is possible, though highly unlikely, for there to be a long string of heads consuming most of the sequence, resulting in a loss that cannot be sustained. A connection to the St Petersburg paradox is suggested as the solution for that game involved determining a cost of play that depends on the number of trials. Though the number of trials also enters the fallacy of large numbers the connection is weak because of the additional connections to the growth optimal portfolio decision problem.³²

The fallacy of large numbers assumes an important place in modern Finance because of the implications for expected utility theory, e.g., Ross (1999), Peköz (2002).

Considerable debate has surrounded the implications of accepting a sequence of good bets when a single bet would be rejected. For example, Pratt and Zeckhauser (1987) define "proper risk aversion" as a property of expected utility functions where, if a single bet is rejected, then combinations of such single bets will also be rejected. Various studies have developed conditions on expected utility functions where a single good bet would be rejected but a sequence of such bets would be accepted, e.g., Nielson (1985). The basic intuition of these studies is provided by Peköz (2002, p. 2): "if the utility function decreases faster than exponentially in the negative direction, the small risk of loss can be magnified to overwhelm the benefits of a gain even for an arbitrary sequences of good bets." Attention has focused on properties of the independent random variables forming the sequence such as bounded or unbounded sample paths; and, identical or nonidentical distributions. The connection of these studies to mathematical statistics is illustrated by Peköz (2002) where the implications of permitting optional stopping – quitting the game early – are examined.

The debate surrounding the growth optimal portfolio is related to the fallacy of large numbers and, indirectly, to the time diversification puzzle because of a connection to the solution for the expected utility maximization problem with log utility. To see this consider the basic maximization problem for an investor with log utility defined for terminal wealth:

$$\max E[\ln[W(T)]] \rightarrow \max E\left[\ln\left[\frac{W(T)}{W(0)}\right]\right] = \max E[\ln[1 + R(0, T)]]$$

The division of $W(T)$ by $W(0)$ and resulting conversion of the problem into maximizing the expected return $E[R(0, T)]$ follows because the expected utility function is unique up to a linear transformation. (The maximization is a choice problem involving the portfolio value weights for the securities that determine $W(T)$.) Assuming a discrete time and state model, the expectation taken over N possible states can be written as:

$$\max E[\ln[1 + R(0, T)]] = \max \sum_{\xi=1}^N Pr_{\xi} \ln[1 + R(0, T)_{\xi}] = \max \sum_{\xi=1}^N \ln[1 + R(0, T)_{\xi}]^{Pr_{\xi}}$$

From this point, it can be observed that the sum of the logs is equal to the log of the products. In addition, ergodicity (see section 1.3) can be invoked to convert states to equally likely time periods to get:

$$\max \sum_{\xi=1}^N \ln[1 + R_{\xi}(0, T)]^{Pr_{\xi}} = \max \ln \left[\prod_{\xi=1}^N [1 + R_{\xi}(0, T)]^{Pr_{\xi}} \right] = \max \ln \left[\prod_{t=1}^T (1 + R(0, t)) \right]$$

It follows that maximizing the expected log utility of terminal wealth corresponds to a strategy that maximizes the geometric mean return, e.g., Ingersoll (1987, pp. 255–8).

The debate surrounding the growth optimal portfolio is theoretically connected to the underlying portfolio strategy that is being pursued and to the specification of the

expected utility function. The argument connecting the maximization of the expected log utility of terminal wealth with the growth optimal solution is structured in a portfolio choice problem that involves no additions or withdrawals from invested wealth until the investment horizon is reached. The consumption-investment problem along the time path is suppressed. Given this, though the growth optimal portfolio will have the highest *expected return* in the long run, it does not follow that this portfolio will have the highest *expected utility* for all investors. Ross (1999, p. 323) captures the point:

At the root of a contentious literature on economic behavior in the face of repeated random choices is the observation that maximizing the geometric growth rate of a portfolio will, with probability one, asymptotically outperform any other choice. This result has led some to suggest that any other choice would be irrational. Samuelson (1971), however, argued that convergence in probability is too weak to support such a strong behavioral conclusion. Indeed, since maximizing the geometric growth rate is equivalent to maximizing the expected log of wealth, the position of its advocates is tantamount to judging all other utility functions as irrational choices.

Ultimately, the debate over the growth optimal portfolio is not theoretically resolvable because the issues involved are too complex. As the solutions become more sophisticated the connection with the underlying portfolio management problem is lost and replaced with concerns about, say, the properties of convergent sequences and whether there is an option to quit the game early.

Behavioral portfolio theory: Keynesian investment strategy?

Behavioral finance has gradually emerged as a potential alternative to the approach of modern Finance (see sections 1.2, 2.4 and 9.3). This raises the possibility that an alternative investment strategy can be developed that incorporates the essential elements of behavioral finance while still retaining methodological features that are familiar to adherents of modern Finance. Given the central role the von Neumann-Morgenstern expected utility approach plays in the development of the central propositions of modern Finance, it is not surprising that the starting point for a *behavioral portfolio theory* begins with an alternative method for modeling decision making under uncertainty.³³ Such an alternative approach to portfolio theory is likely to produce theoretical results that are at odds with the mean-variance optimization model that is used to generate the CAPM and two fund separation results. Such an extension is provided in Shefrin and Statman (2000) where the *prospect theory* of Kahneman and Tversky (1979) is combined with Security-Potential/Aspiration theory of Lopes (1987) to produce a "behavioral portfolio theory" (BPT).

Prospect theory was proposed by Kahneman and Tversky to address failings identified in expected utility theory, such as those proposed by Allais (1953). The theory was extended to a version referred to as *cumulative prospect theory* in Tversky and Kahneman (1992). Following Levy and Levy (2002, pp. 1334–5), cumulative prospect theory is characterized by four propositions:

The main features of prospect theory are: (a) Investors make decisions based on *change* of wealth rather than *total* wealth, in contrast to what is advocated by expected utility theory ... (b) Investors maximize the expectation of value function $V(x)$ where x stands for the *change* in wealth (rather than total wealth). $V(x)$ is S-shaped $V'(x) > 0$ for all $x \neq 0$, $V''(x) > 0$ for $x < 0$ and $V''(x) < 0$ for $x > 0$. The parameters of the value function may change with wealth ... but the S-shaped property is general to all initial wealth levels. (c) Investors subjectively distort probabilities. They make decisions based on the subjective cumulative distribution F^* which is given by $F^* = T(F)$ where F denotes the objective cumulative distribution and T is some subjective transformation such that $T' > 0$, $T(0) = 0$ and $T(1) = 1$ (this is the main modification of cumulative prospect theory in comparison to prospect theory). (d) The "framing" of alternative outcomes may strongly affect subjects' choices.

Though prospect theory is supported in a series of experimental studies conducted by Kahneman and Tversky, Levy and Levy (2002) question the validity of the design used for these experiments. In particular, the choices given to subjects involved only negative or positive outcomes when mixed outcomes provide a better description of actual investment situations. When mixed outcomes are admitted, Levy and Levy provide evidence against the S-shaped value function that is a core proposition of prospect theory.

Despite the potential limitations, prospect theory has assumed an important position in the emerging subject of behavioral finance, in general, and BPT, in particular. As Shefrin and Statman (2000, p. 128) observe: "Mean-variance investors choose portfolios by considering mean and variance. In contrast, BPT investors choose portfolios by considering expected wealth, desire for security and potential, aspiration levels, and probabilities of achieving aspiration levels." While the two fund separation theorem of modern Finance implies that rational investors will hold combinations of the riskless asset and the market portfolio, a key feature of optimal BPT portfolios is the resemblance to combinations of bonds and lottery tickets. Facing the same investment opportunities, Shefrin and Statman demonstrate that the BPT efficient frontier does not coincide with the mean-variance efficient frontier. Recognizing that the BPT portfolios being identified contain security combinations that are not transparently comparable to commonly observed "real world securities," Shefrin and Statman (2000, p. 149) observe:

Treasury bills are right for investors with very low aspiration levels, while equity participation notes are right for investors with higher aspiration levels. Investors with even higher aspiration levels choose stocks and those with yet higher aspiration levels choose out-of-the-money call options and lottery tickets. Stocks, call options, and lottery tickets feature many states with zero payoffs, but they also feature states with payoffs that meet high, even exceedingly high, aspiration levels.

The flexibility of BPT permits this approach to provide an adequate solution to the interior decorator fallacy. Yet, following Fisher and Statman (1999), this is a general result in behavioral finance, resulting from inherent modification of conventional risk concepts used in modern Finance.

The Shefrin and Statman (2000) BPT results are obtained by exploiting alternative approaches to von Neumann-Morgenstern expected utility theory. In particular, BPT has

decided similarities to the *safety-first portfolio theory* initially proposed by Roy (1952). Together with the approaches of maximizing the geometric mean and stochastic dominance, safety first provides a consistent method of making decisions in the face of random outcomes.³⁴ In comparison with using the expected utility approach, safety-first has a number of desirable and not so desirable properties. For example, safety-first avoids the Friedman-Savage puzzle where individuals buy both insurance and lottery tickets. This phenomenon presents complications for expected utility theory where concave utility functions defined over terminal wealth are employed. Such concavity requires a uniform attitude toward risk that leads to the Friedman-Savage puzzle. No such theoretical problem arises if safety-first is used. Similarly, the relatively straightforward decision problem associated with safety first avoids the complicated calculations that can arise when solving expected utility optimization problems.

The basic idea of safety-first is appealing. Instead of doing an expected utility calculation, individuals are assumed to use a less complicated decision-making model that aims to avoid unacceptable outcomes. To see this, let s denote the lower bound on terminal wealth, then the optimization problem for the safety-first investor is:

$$\min_{\{w_i\}} \text{Prob} \{W(T) < s\} \rightarrow \min_{\{w_i\}} \text{Prob} \left\{ \frac{W(T)}{W(0)} < \frac{s}{W(0)} \right\}$$

By dividing through by initial wealth it is possible to reexpress this specification in terms of returns. Where s is identified as a minimum subsistence level, then the problem is said to involve minimizing the probability of ruin. Assuming returns (or terminal wealth) are normally distributed then it follows that the optimal solution will be that portfolio which is the most standard deviations above the subsistence level. To see this consider three portfolios $\{A, B, C\}$ with expected returns $= \{7\%, 12\%, 16\%\}$ and standard deviations $= \{4\%, 5\%, 10\%\}$. Given $\{s/W(0)\} = R_s = 4\%$, then the number of standard deviations above 5% for $\{A, B, C\} = \{.75\sigma, 1.6\sigma, 1.5\sigma\}$. It follows that portfolio B is optimal for the safety first investor because it is the portfolio that has the greatest separation, measured in terms of standard deviation, from the subsistence return level.

Under the critical assumption of normally distributed returns, it is possible to convert this safety-first optimization problem to a more familiar form:

$$\min_{\{w_i\}} \text{Prob} \left\{ \frac{W(T)}{W(0)} < \frac{s}{W(0)} \right\} \rightarrow \min_{\{w_i\}} \frac{R_s - R_p}{\sigma_p} = \max_{\{w_i\}} \frac{R_p - R_s}{\sigma_p}$$

If R_s is set equal to the riskless rate r then it is apparent that the safety-first optimization problem is the same as maximizing the Sharpe ratio, the basic approach underlying the CAPM and two fund separation results. Geometrically, the safety-first investor can define preference lines in mean-standard deviation space that identify sets of portfolios for which the investor will be indifferent (see end of chapter questions). The efficient frontier for the safety-first investor will be that line with maximum slope. Introducing the investment opportunity set defines the available portfolios that can be obtained.

It follows that the optimal safety-first portfolio is identified by the portfolio preference line that is just tangent to the investment opportunity set. When $R_s = r$ this is qualitatively the same solution as in the mean-variance optimization model, with the qualification that all assets are risky in the safety-first decision problem.

From this basic framework, a number of extensions have been pursued. For example, the safety-first optimization problem assumes that all assets are risky. Hence, though the mean-variance and safety-first portfolios are equal when $R_s = r$, the absence of a riskless asset means that two-fund separation does not apply in a safety first framework. Extending the model to include riskless lending and borrowing raises undesirable possibilities such as infinite borrowing as well as complete investment in the riskless asset (see end of chapter questions). Another complication arises when the assumption of normally distributed returns is dropped. In this case, Elton and Gruber (1984, pp. 224–5) argue that Chebyshev's inequality can be used to demonstrate that the results can be generalized to distributions with finite first and second moments. However, Shefrin and Statman (2000, p. 138) demonstrate that this result is incorrect. Telser (1955), Arzac (1974) and Arzac and Bawa (1977) reformulate the safety-first decision problem as:

$$\max_{\{w_i\}} R_p \quad \text{subject to:} \quad \text{Prob} \{R_p < R_s\} \leq \alpha$$

where α is the probability of ruin. In Telser (1955), α is fixed while in Arzac and Bawa (1977) the value is allowed to vary. This approach presents a number difficulties, such as the possible absence of a feasible solution.

In section 10.3, it was argued that Keynes (1936, esp. ch. 12) is a sophisticated precursor of a number of views that have been popularized in behavioral finance. This begs the question: to what extent can BPT be used as a surrogate for a *Keynesian investment strategy*? Keynes (1936) gives a central role to uncertainty and the use of conventions to make decisions in the face of uncertainty. Yet, the type of uncertainty visualized by Keynes differs from the random variables that motivate BPT. Consistent with the approach used in modern Finance, uncertainty in BPT appears as measurable risk. Variables such as the return on the portfolio have a well defined probability distribution, complete with expected values and standard deviations. For Keynes (1936, p. 148): "It would be foolish, in forming our expectations, to attach great weight to matters which are very uncertain." Keynes is more concerned about the "state of confidence" that is determined by the interaction of the confidence that various traders have in the expectations that are the basis of investment decisions. It is the instability in the state of confidence that is a significant source macroeconomic disturbance.

Keynes (1936, ch. 12) proposes two general types of investment strategies. On the one hand, there is the approach to: "Investment based on genuine long-term expectation." Though Keynes does not provide much fleshing out of this approach, it is reasonable to connect this approach with value investing. Yet, Keynes (1936, p. 157) felt this approach "is so difficult to-day as to be scarcely practicable" and "evidence from experience" indicates this approach is not likely to be "most profitable." Those seeking to follow this approach are advised not to use borrowed money, i.e., do not purchase stocks on margin or otherwise leverage invested capital. Keynes argued that the "most profitable" investment strategy would be one that Graham and Dodd referred to as the

anticipation approach, where the objective is to trade based on estimates of the “waves of optimistic and pessimistic sentiment” and “anticipating what average opinion expects average opinion to be.” This approach has a short-term trading horizon of three months and less. Implicitly, Keynes appears to be assuming that this approach will involve trading on margin. However, the role that leveraging plays in the profitability of this approach is not precisely specified.

The anticipation approach encompasses a range of investment strategies, including the bulk of the trading intensive approaches found in technical analysis. Though Keynes did not spell out precisely how “average opinion about average opinion” is to be estimated, it is clear that he did not intellectually favor these types of investment strategies. However, based apparently on anecdotal evidence, Keynes did believe that such strategies were likely to be the most profitable. Keynes was intellectually disposed to a general value investing approach, even though such an approach could present real challenges to the investor. Though it is comforting to feel that the substantial progress in securities markets that has taken place since Keynes (1936) has turned the tables on the value investor making these investors the dominant market players, there is still considerable evidence that “waves of optimistic and pessimistic sentiment” still drive security price determination. For example, it is difficult to avoid this conclusion based on an examination of technology stock pricing during the 1998–2000 period. The upshot is that there is no identifiable Keynesian investment strategy. Both value investors and technical analysts could, arguably, find some support in Keynes (1936).

? Questions

1 In section 10.2, it was observed that the buy-and-hold approach to investment is sub-optimal if rebalancing is permitted along the time path. This result arises because changes in asset prices along the time path cause deviations from the optimal weights determined at the beginning of the investment horizon. What is the assumed investment situation that is associated with this result? For example: what is the initial capital? what is the rebalancing frequency? what are the cash flows along the path? (Hint: see Constantinides 1979.) Rebalancing to maintain the optimal weights will result in higher expected utility at the terminal date for the investor. In a discrete time model, what theoretical conditions are required for this result to apply? What further conditions are required for the result to extend to cases where the length of the rebalancing interval is arbitrary or diverse? (Hint: see Levy and Samuelson 1992.)

2 In section 10.2, three measures of performance evaluation directly related to the CAPM were introduced: the Sharpe ratio (SR); Treynor's measure (TM); and, Jensen's alpha (J). Letting the subscript p denote the portfolio of interest and M the market portfolio, derive the following results:

$$TM_p = \frac{E[R_p] - r}{\beta_p} = \frac{\alpha_p}{\beta_p} + TM_M = \frac{J_p}{\beta_p} + TM_M$$

$$SR_p = \frac{E[R_p] - r}{\sigma_p} = \frac{\alpha_p}{\sigma_p} + \rho SR_M = \frac{J_p}{\sigma_p} + \rho SR_M$$

where ρ is the correlation between the return on the market and the return on the portfolio of interest. (Hint: see Bodie, Kane and Marcus 1999.)

3 In section 10.4, an example involving the expected log utility function and a binomial process was used to illustrate the Samuelson (1963) conditions. The binomial process set the probability of up and down moves to be equal at 50% with the upside gain equal to $1/3$ and the downside loss equal to $1/4$. What is the mean and variance of the associated wealth process and return process? How is this process iid? Referring to the discussion of the random walk in the previous paragraph, re-express this binomial process as a discrete random walk: $X(t) = X(t-1) + Z(t)$. (Hint: In order for $Z(t)$ to be a mean zero process, this will be a random walk for the expected return process.) What are the mean and variance of the sum of the $\{Z(t)\}$?

4 In section 10.4, it was observed Kritzman (2000) argues that mean-reversion in stock returns is insufficient to produce time diversification if the investor has a constant relative risk aversion utility function. Barberis (2000) claims the opposite. Using the binomial techniques developed in section 5.1 for mean-reverting interest rate processes, evaluate directly whether the expected utility will increase over an increasing time horizon for a mean-reverting stock return process. Assume that the investor has: a log utility function; and, a power utility function. Compare the results for these two cases. Do your results confirm the conclusions of Ingersoll (1987, p. 257) that the log utility function is completely "myopic" while the power utility function is only myopic if security returns are statistically independent?

5 In section 10.4, the safety-first decision model led to the specification of lines in expected return-standard deviation space that defined portfolios of equal preference. Explain how these lines are defined (Hint: see Elton and Gruber 1984, pp. 222-4). This solution depends on there being only risky assets in the choice problem. If riskless borrowing and lending is permitted, demonstrate that this raises the possibility of infinite lending and borrowing or complete investment in the riskless asset (Hint: see Elton and Gruber 1984, pp. 238-9). Finally, explain the limitations of using Chebyshev's inequality (see section 1.3) to generalize the safety-first approach to distributions other than the normal (Hint: see Shefrin and Statman 2000, p. 139).

NOTES

- 1 Whitman (1999, ch. 11) also discusses the "sloppy" terminology used in the financial community. Some important words that are identified as having imprecise meanings are: investor, speculator, value, price, company, risk, margin of safety, capital, earnings and cash flow.
- 2 Kritzman and Page (2002) is a recent example of studies that dispute the relative importance of asset allocation versus security selection: "Contrary to the widely held view, it turns out that choosing stocks within the equity component of a portfolio is substantially more important than choosing a portfolio's exposure among stocks, bonds and cash." Recognizing that there are methodological difficulties in making a comparison between gains to asset allocation and security selection, Kritzman and Page find the gains from security selection relative to asset allocation vary across countries, ranging from just over two times to almost four times.
- 3 Another example of potentially confusing semantics in this area occurs with the use of "strategic asset allocation" to refer to the normal, long-term asset mix and "tactical asset allocation" to refer to the market timing decision. The definition used here follows Ragsdale and Rao (1994).
- 4 In the jargon of the trade, such investors are referred to as "widows and orphans." This terminology makes reference to an earlier time where social support systems, such as Social Security, were not available and bequests and inheritances were an often essential source of financial support. Because the husband was, typically, the primary breadwinner, upon the death of her husband, a widow would often be dependent on the income from a bequest or inheritance to pay for day-to-day expenses. As such, the potential for disruption of cash flow associated with, say, a downturn in equity values and dividend payouts, would be an important consideration in the investment strategy decision.
- 5 In addition to Siegel, the "Dogs of the Dow" strategy is touted by L. Kudlow on the regularly scheduled CNBC show *Kudlow and Kramer*.
- 6 Visscher and Fillbeck (2003, p. 100) provide much the same history, dating the *Washington Post* story to 1988 and observing that Slatter worked for Prescott, Ball and Turben of Cleveland, Ohio.
- 7 A number of investment strategies that incorporate dividend yield have been proposed in modern Finance. For example, the PEGY models (see section 8.1) aim to incorporate *P/E* ratios, growth rates and dividend yields to estimate the modern Finance version of the "intrinsic value" for a stock.
- 8 The fund was converted to a limited partnership in 1952.
- 9 The collapse of LTCM has been exhaustively examined in a number of sources, e.g., Dunbar (2000). Many of the regulatory aspects of the collapse are examined in PWGFM (1999). Poitras (2002, pp. 64–7) provides a brief overview.
- 10 Links to a number of hedge fund sources can be found at www.sfu.ca/~poitras/links.htm. The President's Working Group on Financial Markets (PWGFM) that examined the hedge fund industry in the wake of the collapse of LTCM has led to a process of discussion and examination of the hedge fund industry. The most recent round in the process that was initiated by the PWGFM *Report* (1999) is a SEC sponsored Roundtable on Hedge Funds held in May 2003. The submissions to this Roundtable (accessible through the SEC website www.sec.gov) are a useful source for information on the current state of hedge fund regulation.
- 11 A number of links to hedge fund websites can be found in the Security Analysis section on the author's links page at www.sfu.ca/~poitras/links.htm. Included in the links are the MARhedge site, Hedgeworld at Deutschebank, Van Hedge Fund Advisors and Magnum Funds.
- 12 The "two fund separation" terminology can be found in various sources. For example, Levy and Samuelson (1992, p. 1530) observe: "The Sharpe-Lintner CAPM can be derived by assuming either a quadratic utility function or

normally distributed returns. In a multiperiod framework, the quadratic utility assumption also leads to the two-fund Separation Theorem, and hence implies the CAPM."

- 13 Among others, Levy and Samuelson (1992) provide four sets of sufficient conditions for multiperiod generalization of the single-period Sharpe-Lintner CAPM where investors are permitted to have diverse holding periods and rebalancing frequencies. In the multiperiod context, if portfolio rebalancing is not permitted then the CAPM and two fund separation do not hold, even in the restrictive case of quadratic utility.
- 14 Portfolio performance measurement is an important aspect of modern Finance. Yet, as demonstrated in Campbell et al. (1997) and Cochrane (2001), adequate treatment of this topic takes the discussion decidedly in the direction of sophisticated financial econometrics. Unfortunately, this requires the development of techniques and notions that are outside the general framework of this book. As a consequence, this topic is only given a cursory treatment which is incongruent with the relevance of this topic to investment strategy, in general, and modern Finance, in particular. Included in the useful recent studies on this topic are Brown et al. (1992), Elton et al. (1996) and Carpenter and Lynch (1999) which consider the problem of *survivorship bias* in studies of fund (or common stock) performance. This bias can arise in empirical studies of mutual fund performance because poorly performing funds disappear and are typically deleted from the sample because the price history does not cover the full sample (and other reasons). The result is that the samples used for fund performance studies may be biased toward overrepresentation of the better performing funds. Other recent studies of portfolio performance measurement include Pastor and Stambaugh (2002), Carhart (1997) and Ferson and Schadt (1996).
- 15 This assumes that nominal returns are the variable of interest. If, as is common in modern Finance, the real return is the variable of interest, even a default-free, zero coupon fixed income security with maturity date equal to the investment horizon will be risky. Even though the nominal return will be certain, there is still purchasing power risk associated with inflation. This complication can be handled by using an inflation-indexed default-free fixed income security as the riskless asset.
- 16 An alternative approach to defining cash is provided by the corporate financial statements where there is a "cash and cash equivalents" item. There is also the "cash flow statement."
- 17 Strictly speaking, the return in the Sharpe ratio requires that the holding period for the return equals the investment horizon.
- 18 The need to reveal the size and change in equity positions at Berkshire Hathaway is another feature that distinguishes Warren Buffett from the small individual investor. As reflected in numerous sources, Buffett has consistently and persistently complained about having to reveal his equity positions. Among other negative aspects, this creates difficulties for unwinding the positions due to "front-running" when Berkshire-Hathaway is adding or adjusting common stock positions. However, compared to other investors that also have to report equity positions, Buffett does have the benefit of two classes (A and B) of Berkshire Hathaway common stock (BRK.A and BRK.B) that simplifies the process of making substantial changes in portfolio composition. Finally, relative to the dividend yield strategies of section 10.1, Berkshire-Hathaway does not engage in stock splits or cash dividend payouts. As such, Berkshire Hathaway is a useful example of a company that does not pay dividends but would qualify as an excellent investment for a wide range of investors able to make the sizable investment required to purchase an individual share.
- 19 Not all studies support the view that the advice of financial planners is inconsistent with, say, modern portfolio theory. For example, in an empirical study using a sample of TIAA-CREF participants, Bodie and Crane (1997) conclude: "Individual asset allocations are consistent with the advice of expert practitioners and with the prescriptions of economic theory." Included in the type of advice dispensed by expert practitioners is the following (Bodie and

- Crane 1997, p. 14): "The fraction of assets invested in equities should decline as an investor's age advances. A popular rule-of-thumb regarding the age-equity relationship is that the percentage of one's portfolio to invest in equities should be 100 minus one's age. So, a person thirty years old should invest 70 percent in equities, and a person 70 years old should invest 30 percent in equities."
- 20 Bernstein (1992, p. 63) credits Richard Brealey with the introduction of this terminology.
 - 21 This lack of a generalized framework is not for lack of trying. The extent of the efforts is reflected in Viceira (2001, p. 435): "This paper extends the previous literature in three directions. First, it incorporates retirement into a dynamic model of optimal consumption and portfolio choice with uninsurable labor income risk. Second, it explores the ability of stocks to hedge consumption from unexpected falls in labor income when labor income is correlated, but not possibly perfectly correlated, with stock returns. Third, it derives an approximate analytical solution of the model... This is particularly useful to understand the effects of an uncompensated increase in labor income risk on savings and portfolio decisions."
 - 22 This is not meant to imply that the "portfolio management" aspect of modern Finance lacks empirical contributions. Quite the contrary, consistent with the methodology of logical positivism, there are numerous empirical explorations of portfolio management theories. However, despite decades of empirical analysis, the difficulties of estimating expected returns, *ex post*, from non-experimental data, for use in practical *ex ante* applications of the theory is still elusive, e.g., Elton (1999). As such, the contributions of the portfolio management theory of modern Finance lie primarily in the theoretical sphere.
 - 23 Included in the class of constant relative risk aversion (CRRA) utility functions are the log-wealth, square root wealth and power utility functions. Despite sharing the CRRA property, these functions do not typically generate the same solutions. For example, the log-wealth utility function often produces an intertemporal solution that is the same as the single-period solution, a result that is only achieved by power utility in special cases.
 - 24 The Friedman-Savage puzzle is concerned with the shape of the expected utility function. In particular, assuming risk aversion requires that the function $EU[W]$ be everywhere concave (downwards). Similarly, the expected utility function for a risk-lover is everywhere convex. The puzzle posed in Friedman and Savage (1948) concerns the shape of the expected utility function for an individual that both gambles (risk lover) and buys insurance (risk averter). Because a large number of individuals do engage in both sorts of activities, Friedman and Savage argue that the expected utility function is concave at low incomes, convex at middle incomes and concave again at high incomes. The puzzle revolves around the assumption, say in the mean-variance optimization framework, that individuals are risk-averse, which implies an everywhere concave expected utility function.
 - 25 In words, the notation $Z(t) \sim N[0, 1]$ means that the continuous (or possibly discrete) random variable $Z(t)$ is distributed normally with mean equal to zero and variance equal to one.
 - 26 Another variant of a time diversification strategy would be to invest in higher beta stocks for long investment horizons and lower beta stocks for short investment horizons. For individual investors able to select the investment horizon, then time diversification suggests a preference for high beta stocks with long investment horizons. When Fama and French (1992) examined such strategies, no statistically significant difference was found between the performance of the high and low beta portfolios.
 - 27 The connection between mean reversion and time diversification is not original to Siegel (1998) and Barberis (2000). As indicated in section 5.2, the result follows from the solutions for the expected return and variance of a mean-reverting process. Lee (1990) is an earlier study that demonstrates mean reversion will lead to time diversification because the non-iid character of the time paths means that the variances of risky assets will converge to values that are smaller than in the iid case.

- 28 Inflation also has a range of theoretical implications. For example, a fixed income security with no default risk and a term to maturity equal to the investment or rebalancing horizon has the properties of a riskless asset if the model uses nominal returns. However, if real returns are used, then the fixed income security will not be riskless. Though it is possible to argue for the use of inflation-indexed Treasuries as a candidate for the riskless asset when real returns are used, there are a number of practical difficulties that arise in this case.
- 29 An illustration of the optimization problem, solution procedure and optimal weights that can be derived in continuous time is provided in Ingersoll (1987, pp. 271–6). Discrete-time solutions are given in Ingersoll (1987, ch. 11). These optimal consumption-portfolio models are usually expressed in terms of an additively separable intertemporal utility function defined over consumption between the decision date $t = 0$ and the terminal date $t = T$ with the terminal value of wealth also entering the utility function, i.e., $U[C(0), C(1), \dots, C(T), W(T)]$. In addition to consumption and terminal wealth a separate bequest motive function, defined over terminal wealth, is typically also included.
- 30 Following Campbell and Viceira (2002, p. 89): “Any evidence that risk does not scale with horizon . . . is indirect evidence for the predictability of asset returns.” Hence, Siegel’s claim that mean-reversion in stock returns results in a lower level for the variance of cumulative returns compared to the random walk model implies that there is some empirical model that can be used to predict future returns.
- 31 In addition to the intertemporal optimal rebalancing case, mean reversion has empirical implications for time diversification in the buy-and-hold approach. The extent of mean reversion depends on the parameter values such as the size of the adjustment coefficient (see section 5.2). If the adjustment coefficient is relatively small, it is possible for a mean-reverting process to wander quite far from the long run mean. Hence, to exploit the potential reduction in the variance of cumulative returns provided by mean-reversion, a buy-and-hold investor would have to determine if the time path of realized returns had permitted the level of stock prices to rise to a level that was so high the potential gains in variance reduction would not likely be obtained within the investor’s investment horizon. Hence, while mean-reversion may be theoretically consistent with time diversification, this result may not be applicable to specific points in time for buy-and-hold investors. The pressure this puts on obtaining accurate empirical estimates is evident.
- 32 The resolution of the St Petersburg paradox is given in Feller (1957) where it is demonstrated that the paradox is a result of attempting to determine a fixed entry fee for a game that has a theoretically infinite number of trials (see section 1.3). The game will have a finite entry value if the number of possible trials is also finite.
- 33 In this case, the semantic use of “uncertainty” is consistent with the conventional usage in modern Finance where uncertainty is associated with random variables. In turn, these variables have well defined distributions resulting in a measurable form of uncertainty. This usage is not consistent with the use of “uncertainty” by, say, Frank Knight and Keynes (see section 1.2).
- 34 The stochastic dominance approach will not be examined in detail here, if only because the pairwise comparisons involved in employing stochastic dominance make this approach generally infeasible for portfolio management applications. Elton and Gruber (1984, pp. 229–36) provides a introductory overview of this approach.