TYS does not equal SS

$$P_B = \sum_{t=1}^{T} \frac{C}{(1 + y)^t} + \frac{M}{1 + y)^T} \qquad P_B{}^* = \sum_{t=1}^{T*} \frac{C}{(1 + y*)^t} + \frac{M}{1 + y*)^T}$$

$$P_{CB} = \sum_{t=1}^{T} \frac{C}{(1 + y_C)^t} + \frac{M}{1 + y_C)^T} = \sum_{t=1}^{T} \frac{C}{(1 + z_t + ss)^t} + \frac{M}{1 + z_t + ss)^T}$$

$$P_{CB} = \sum_{t=1}^{T} \frac{C}{(1 + y_C)^t} + \frac{M}{1 + y_C)^T} \qquad P_{CB}{}^* = \sum_{t=1}^{T^*} \frac{C}{(1 + y_C{}^*)^t} + \frac{M}{1 + y_C{}^*)^T}$$

y = riskless government yield

y_C = corporate bond yield

z_t = implied zero coupon rate (spot interest rate) at time t

ss = the static spread – a fixed number of basis points added to each implied zero coupon rate

TYS = traditional yield spread corporate minus government, same maturity= $y_C - y$

ss = TYS only when the term structure of interest rates (and the yield curve) is flat

Other forms of TYS: $y_C - y_C^*$ difference in corporate yield, different term to maturity
$y - y^*$ difference in government yields, different term to maturity

Why are TYS and ss not equal when yield curves slope up?

Example: Two year zero

$$\frac{1}{(1 + y_C)^2} = \left(\frac{1}{1 + z_1 + ss}\right)\left(\frac{1}{1 + z_2 + ss}\right)$$

$$\frac{1}{(1 + y)^2} = \left(\frac{1}{1 + z_1}\right)\left(\frac{1}{1 + z_2}\right) \qquad where \qquad z_1 < y < z_2$$

Let $y = .05$ –> $z_2 = .06$ $z_1 = .040095$ $(1.05)^{-2} =$ 0.907029 = 1/(1.06*1.040095))

If $y_C = .06$ then TYS = .01 $(1.06)^{-2}$ = (1/(1 + .06 + 01))(1/(1 + .040095 + .01))

–> ss and TYS are the same for zero coupon bonds

TYS for coupon PAR bonds (P = 100)

$$P_B = (100*y)\left(\frac{1}{y} - \frac{1}{y\ (1\ +y)^T}\right) + \frac{100}{(1\ +\ y)^T} = \sum_{t=1}^{T} \frac{100*y}{(1\ +\ z_t)^t} + \frac{100}{(1\ +\ z_T)^T}$$

$$P_{CB} = (100*y_C)\left(\frac{1}{y_C} - \frac{1}{y_C\ (1\ +y_C)^T}\right) + \frac{100}{(1\ +\ y_C)^T} = \sum_{t=1}^{T} \frac{100*y_C}{(1\ +\ z_t\ +\ ss)^t} + \frac{100}{(1\ +\ z_T\ +\ ss)^T}$$

Solving $y = .05$ gives $z_2 = .0505$ $z_1 = .0303$
$y_C = .06$ then TYS = .01 $ss = .010099$

$$100 = \frac{.05*100}{1.05} + \frac{105}{(1.05)^2} = \frac{.05*100}{1.0303} + \frac{105}{(1.505)^2}$$

$$100 = \frac{.06*100}{1.06} + \frac{106}{(1.06)^2} = \frac{.06*100}{1+(.0303+.010099)} + \frac{106}{(1\ +\ (.0505+.010099))^2}$$