Part 4: Market Failure II - Asymmetric Information - Uncertainty

Expected Utility, Risk Aversion, Risk Neutrality, Risk Pooling, Insurance

July 2016

Expected Utility

Evaluating Uncertain Outcomes

- how do people evaluate choices that yield uncertain outcomes (lottery,gamble)?
- example: lottery L that pays $x_1 = 100$ \$ with probability p and \$ 200 with probability $1 - p$

  expected monetary value of the lottery is

$$E(L) = p100 + (1 - p)200$$

- but: expected value does not say anything about utility...

$\rightarrow$ **expected utility function** v

- expected utility of lottery L is

$$E\big[v(L)\big] = pv(100) + (1 - p)v(200)$$

Expected Utility

- consider a lottery L which has uncertain outcomes $x_1, \ldots, x_n$ that occur with probabilities $p_1, \ldots, p_n$
- the expected utility function (Von Neuman Morgenstern utility) assigns a utility value $v(x_i)$ to each possible outcome x_i
- the utility value of the lottery is then the average (expected) value of v

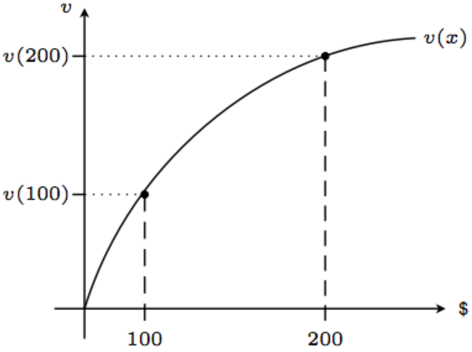
$$u(L) = E\big[v(L)\big] = p_1 v(x_1) + p_2 v(x_2) + \ldots + p_n v(x_n)$$

- characteristics:
  - $v(x)$ is cardinal, not ordinal
  - shape of $v(x)$ determines attitude toward risk

Risk Aversion

Characterizing Risk Aversion

- suppose $v(x)$ is strictly concave: $v'(x) > 0$, $v''(x) < 0$
- example: lottery that pays $x_1 = 100$ and $x_2 = 200$ with equal probability
- graphically:



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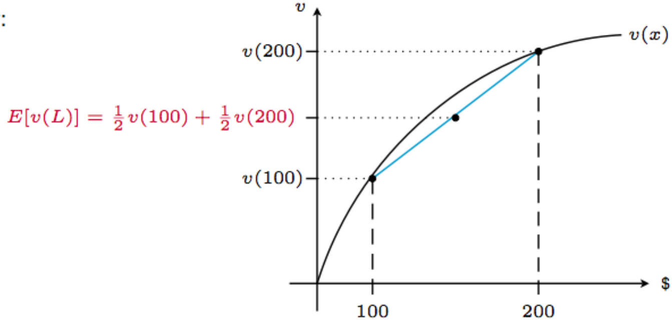

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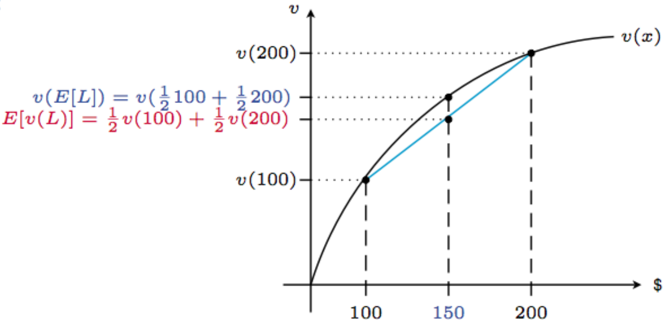


Characterizing Risk Aversion

- expected utility from lottery is $E[v(L)] = \frac{1}{2}v(100) + \frac{1}{2}v(200)$
- utility from expected monetary value of lottery is $v(E[L]) = v(\frac{1}{2}100 + \frac{1}{2}200)$
- we see $v(E[L]) > E[v(L)]$ due to concavity of v

the consumer prefers to receive the expected value of the lottery for sure to the lottery itself $\rightarrow$ the consumer is **risk averse**

Risk Aversion (Cont'd)

- consider a lottery L which has uncertain outcomes $x_1, \ldots, x_n$ that occur with probabilities $p_1, \ldots, p_n$
- an individual is risk averse if

$$v\big[E(L)\big] = v(p_1 x_1 + \ldots + p_n x_n) > p_1 u(x_1) + \ldots + p_n u(x_n) = E\big[v(L)\big]$$

note: risk aversion $\Leftrightarrow$ $v(x)$ is strictly concave $(v'' < 0)$

- an individual is risk neutral if

$$v\big[E(L)\big] = v(p_1 x_1 + \ldots + p_n x_n) = p_1 u(x_1) + \ldots + p_n u(x_n) = E\big[v(L)\big]$$

note: risk neutrality $\Leftrightarrow$ $v(x)$ is linear $(v'' = 0)$

- the degree of risk aversion can be measured by the Arrow-Pratt measure of (absolute) risk aversion

$$\rho(x) = -\frac{v''(x)}{v'(x)}$$

Risk Premium and Certainty Equivalent

- the certainty equivalent (CE) of a lottery is the amount of money (for sure) that makes the individual indifferent to the lottery; it is implicitly defined by

$$v(CE) = p_1 v(x_1) + \ldots p_n v(x_n)$$

note: risk aversion $\Leftrightarrow ; CE < E(L)$, risk neutrality $\Leftrightarrow \; CE = E(L)$

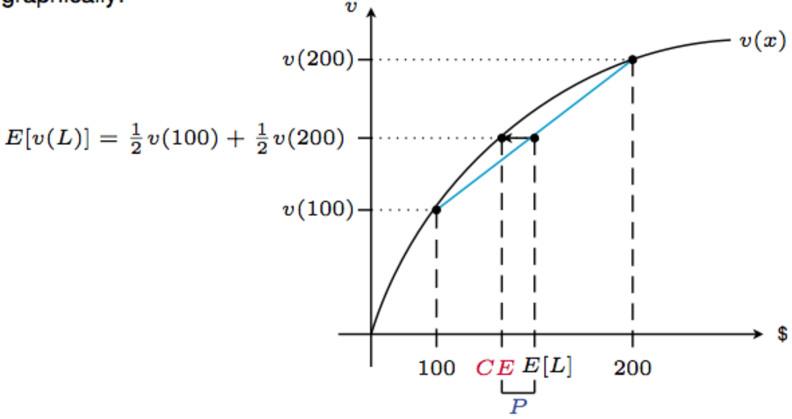
- the risk premium (P) associated with a lottery L is the difference between the certainty equivalent and the expected value of the lottery

$$P = E(L) - CE \quad \text{or} \quad v(E[L] - P) = p_1 u(x_1) + \ldots p_n u(x_n)$$

note: risk aversion $\Leftrightarrow \; P = E(L) - CE > 0$; risk neutrality $\Leftrightarrow \; P = 0$

Risk Premium and Certainty Equivalent

graphically:



Insurance

The Demand for Insurance

- consumer's income in good state is y (with prob α), in bad state is $y - L$ where L = loss (with prob $1 - \alpha$)
- insurance: coverage q at price π = premium per $ coverage → income in good state is $y_G = y - \pi q$ and in bad state is $y_B = y - L + q - \pi q$
- given π, individual chooses q so as to maximize expected utility

$$\max_q \qquad \alpha\, v\big(y - \pi q\big) + (1 - \alpha) v\big(y - L + q - \pi q\big)$$

- FOC:

$$\frac{v'(y - \pi q)}{v'(y - L + q - \pi q)} = \frac{1 - \alpha}{\alpha} \frac{1 - \pi}{\pi}. \qquad (*)$$

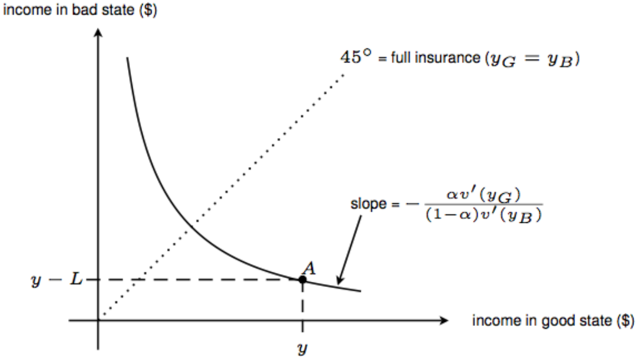
The Demand for Insurance

- suppose insurance market competitive: price (premium) give zero expected profits $\rightarrow \pi = 1 - \alpha \rightarrow$ **actuarially fair insurance**
- from (*), marginal utilities in both states of world are equalized
  $\rightarrow u'' < 0$ implies L = q $\Rightarrow$ **full insurance**

If insurance contracts are actuarially fair, risk averse individuals will **fully insure** themselves against all (income) risk

Graphic Illustration

graphically:



- income without insurance = point A

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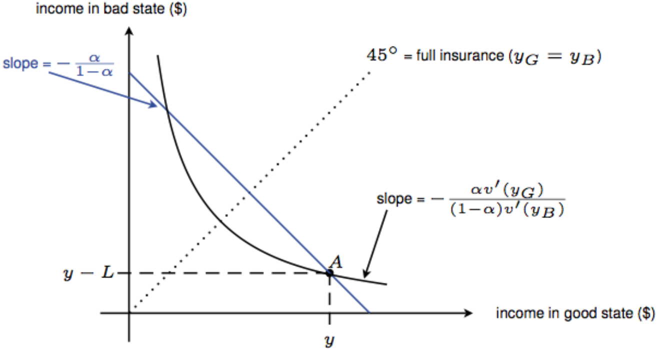

- income without insurance = point A
- budget line if insurance company offers coverage q at per \$ price $\pi = (1 - \alpha)$

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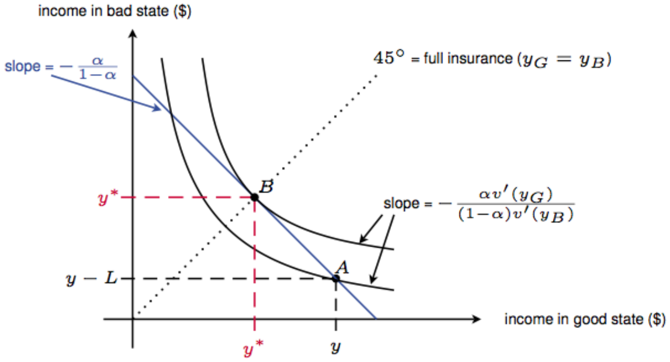

- income without insurance = point A
- budget line if insurance company offers coverage q at per $ price $\pi = (1 - \alpha)$
- point that maximizes expected utility is at full insurance = point B, where $q = L$ and $y_G = y_B = y^* = y - \alpha L$