Part 3: Game Theory I Basic Concepts, Dominance Solvability

Simultaneous Move Games, Payoff Matrix, Dominant Strategy, Iterated Elimination of Dominated Strategies

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Simultaneous Move Games, Payoff Matrix, DomPart 3: Game Theory I Basic Concepts, Dominal

Basic Concepts

Why do we study games?

A game is a way to model strategic behavior – people recognize that their own behavior affect the choices of others and that the outcome depends on any one person's choice, not just one's own

- Strategic behavior is important in situations where
 - A small number of individuals interact (negotiations, auctions)
 - There is imperfect competition
 - Externalities and public goods are present
- Game theory not limited to economics
 - e.g. biology, sociology, political science, dating, sports, ...

An Example: The Prisoners' Dilemma

- The simplest game is a game with two players where players choose actions simultaneously
- Example of a 2x2 matrix (normal form or strategic form) game:



The Prisoners' Dilemma

Matrix entries = payoffs

First Entry = Row's payoff, second entry = Column's payoff

What is a game?

- A simultaneous-move game involves:
 - A list of players
 - For each player, a set of actions = strategies
 - For each player, preferences over each possible strategy combination = payoffs
- When solving a game we assume player are rational:
 - They choose their action to maximize their payoff
 - They form beliefs about what others will do
 - Those beliefs are *correct* in equilibrium

• The PD has an obvious solution (equilibrium)

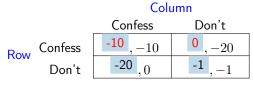
Column

Row Confess Don't

Confess	Don't
-10, -10	0, -20
-20,0	-1, -1

The Prisoners' Dilemma

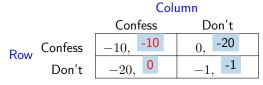
• The PD has an obvious solution (equilibrium)



The Prisoners' Dilemma

• No matter what Column chooses, Row does better by confessing

• The PD has an obvious solution (equilibrium)



The Prisoners' Dilemma

• The same is true for Column

• The PD has an obvious solution (equilibrium)

 $\begin{array}{c|c} & & & \\ \hline \text{Column} \\ \hline \text{Confess} & & & \\ \hline \text{Confess} & & & \\ \hline \text{-10,-10} & & & \\ 0,-20 \\ \hline \text{Don't} & & -20,0 & & -1,-1 \end{array}$

The Prisoners' Dilemma

- Confessing is a dominant strategy for each player: it is the best choice **regardless** of what the other player does
- Confessing is a dominated strategy: it is worse than some other strategy **regardless** of the other player
- Both players confessing is an equilibrium of the game (even though it is worse for both than if neither confesses)

Applications of the PD

- Many other situations have structures similar to the Prisoners' Dilemma
- ex. 1: working on a joint project (private provision of a public good)

Column

		Column		
		Goof off	Work Hard	
Row	Goof off	1,1	3,0	
ROW	Work Hard	0, 3	2, 2	

Private Provision of a Public Good

Applications of the PD

- Many other situations have structures similar to the Prisoners' Dilemma
- ex. 1: working on a joint project (private provision of a public good)

		Column		
		Goof off	Work Hard	
Row	Goof off	1,1	3,0	
	Work Hard	0, 3	2,2	

Private Provision of a Public Good

• Eliminating strictly dominated actions gives (Goof off, Goof off) as equilibrium

Applications of the PD Cont'd

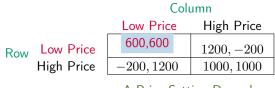
• ex. 2: Two firms produce the same good and can choose prices (duopoly)

		Column		
		Low Price	High Price	
Row	Low Price	600,600	1200, -200	
T(Ovv	High Price	-200, 1200	1000, 1000	

A Price Setting Duopoly

Applications of the PD Cont'd

• ex. 2: two firms produce the same good and can choose prices (duopoly)

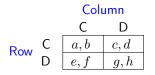


A Price Setting Duopoly

• Eliminating strictly dominated actions gives (Low Price, Low Price) as equilibrium

Applications of the PD Cont'd

• In general, the PD encompasses all situations in which players can 'Cooperate' (C) or 'Not cooperate/Defect' (D):



The Prisoners' Dilemma

• Where payoffs are:

e > a > g > cd > b > h > f

- Defecting is a dominant strategy for each player
- The equilibrium outcome is (D, D)

Dominance Solvability

A Dominance Solvable Game

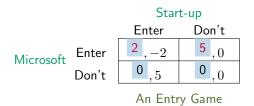
• Another example: Microsoft vs Start-up in a market for new online service

		Start-up	
		Enter	Don't
Mircosoft	Enter	2, -2	5,0
	Don't	0,5	0,0

An Entry Game

A Dominance Solvable Game

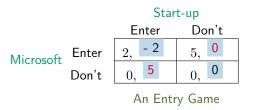
• Another example: Microsoft vs Start-up in a market for new online service



• Microsoft has a dominant strategy: Enter

A Dominance Solvable Game

• Another example: Microsoft vs Start-up in a market for new online service



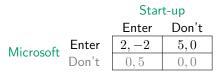
- Microsoft has a dominant strategy: Enter
- Start-up has no dominant strategy: if MS enters, Start-up should stay out; if MS stays out, Start-up should enter

• we can solve the game by iterated elimination of dominated strategies

$\begin{array}{c|c} & \text{Start-up} \\ & \text{Enter} & \text{Don't} \\ \hline \text{Mircosoft} & \begin{array}{c} \text{Enter} & 2, -2 & 5, 0 \\ \text{Don't} & 0, 5 & 0, 0 \end{array} \end{array}$

An Entry Game

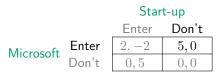
• we can solve the game by iterated elimination of dominated strategies



An Entry Game

 eliminate first MS's dominated strategy (Don't enter) (seems reasonable if Start-up knows MS's payoffs)

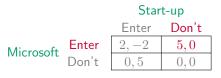
• we can solve the game by iterated elimination of dominated strategies



An Entry Game

- eliminate first MS's dominated strategy (Don't enter)
- with remaining game eliminate Start-up's dominated strategy (Enter)

• we can solve the game by iterated elimination of dominated strategies



An Entry Game

- eliminate first MS's dominated strategy (Don't enter)
- with remaining game eliminate Start-up's dominated strategy (Enter)
- ullet only (Enter, Don't enter) is left ightarrow equilibrium outcome

Another example

	Column		
	Left	Center	Right
Тор	-5, -1	2,2	3, 3
Row Middle	1, -3	1,2	1, 1
Bottom	0, 10	0,0	0, -10

A Dominance Solvable Game

• Another example

	Column		
	Left Center Right		
Тор	-5, -1	2,2	3, 3
Row Middle	1, -3	1,2	1, 1
Bottom	0,10	0,0	0, -10

A Dominance Solvable Game

 $\bullet\,$ 'Middle' dominates 'Bottom' for Row $\rightarrow\,drop\,$ 'Bottom'

• Another example

		Column		
		Left Center Right		
	Тор	-5, -1	2,2	3, 3
Row	Middle	1, -3	1,2	1, 1
	Bottom	0, 10	0, 0	0, -10

A Dominance Solvable Game

- $\bullet\,$ 'Middle' dominates 'Bottom' for Row $\rightarrow\,drop\,$ 'Bottom'
- $\bullet\,$ 'Left' is dominated for Column in remaining game $\rightarrow\,$ drop 'Left'

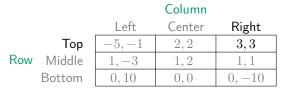
• Another example

		Column		
		Left Center Right		
	Тор	-5, -1	2,2	3, 3
Row	Middle	1, -3	1, 2	1, 1
	Bottom	0, 10	0,0	0, -10

A Dominance Solvable Game

- $\bullet\,$ 'Middle' dominates 'Bottom' for Row $\rightarrow\,drop\,$ 'Bottom'
- $\bullet\,$ 'Left' is dominated for Column in remaining game $\rightarrow\,$ drop 'Left'
- $\bullet\,$ 'Top' dominates 'Middle' for Row in remaining game $\rightarrow\,$ drop 'Middle'

• Another example



A Dominance Solvable Game

- $\bullet\,$ 'Middle' dominates 'Bottom' for Row \rightarrow drop 'Bottom'
- $\bullet\,$ 'Left' is now dominated for Column $\rightarrow\,drop$ 'Left'
- ullet 'Top' dominates 'Middle' for Row in remaining game ightarrow drop 'Middle'
- $\bullet\,$ 'Right' is better than 'Center' for Column $\rightarrow\,$ drop 'Center'
- Equilibrium outcome is (Top, Right)

• Another example

		Column		
		Left	Center	Right
	Тор	-5, -1	2,2	3,3
Row	Middle	1, -3	1,2	1, 1
	Bottom	0, 10	0,0	0, -10

A Dominance Solvable Game

- Outcome (Top, Right) reasonable if players know each others' payoffs; plus: no player would want to change their behavior given the behavior of others
- But iterated elimination of dominated strategies does not always give an outcome ...

A Coordination Game

A coordination game

		Sally		
		Renaissance Starbucks		
Harry	Renaissance	2, 1	0, 0	
rially	Starbucks	0,0	1, 2	

Battle of the Sexes

- Both players prefer to cooperate (meet at same location) but disagree on about best outcome (location)
 - \rightarrow No dominated strategies
- But: if Sally chooses Starbucks, Harry should also choose Starbucks (and vice versa) \rightarrow reasonable to presume that they will meet