Part 3: Game Theory I
Basic Concepts, Dominance Solvability

Simultaneous Move Games, Payoff Matrix, Dominant Strategy,
Iterated Elimination of Dominated Strategies

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Basic Concepts

Why do we study games?

A game is a way to model strategic behavior – people recognize that their own behavior affect the choices of others and that the outcome depends on any one person's choice, not just one's own

- Strategic behavior is important in situations where
  - A small number of individuals interact (negotiations, auctions)
  - There is imperfect competition
  - Externalities and public goods are present
- Game theory not limited to economics

  e.g. biology, sociology, political science, dating, sports, ...

An Example: The Prisoners' Dilemma

- The simplest game is a game with two players where players choose actions simultaneously
- Example of a 2x2 matrix (normal form or strategic form) game:

		Column	
		Confess	Don't
Row	Confess	$-10, -10$	$0, -20$
	Don't	$-20, 0$	$-1, -1$

The Prisoners' Dilemma

Matrix entries = payoffs

First Entry = Row's payoff, second entry = Column's payoff

What is a game?

- A simultaneous-move game involves:
  - A list of **players**
  - For each player, a set of actions = **strategies**
  - For each player, preferences over each possible strategy combination = **payoffs**
- When solving a game we assume player are rational:
  - They choose their action to maximize their payoff
  - They form beliefs about what others will do
  - Those beliefs are *correct* in equilibrium

Solving the Prisoners' Dilemma

- The PD has an obvious solution (equilibrium)

Column

Row		Confess	Don't
	Confess	$-10, -10$	$0, -20$
	Don't	$-20, 0$	$-1, -1$

The Prisoners' Dilemma

Solving the Prisoners' Dilemma

- The PD has an obvious solution (equilibrium)

Column

		Confess	Don't
Row	Confess	-10, -10	0, -20
	Don't	-20, 0	-1, -1

The Prisoners' Dilemma

- *No matter what Column chooses, Row does better by confessing*

Solving the Prisoners' Dilemma

- The PD has an obvious solution (equilibrium)

Column

		Confess	Don't
Row	Confess	-10, -10	0, -20
	Don't	-20, 0	-1, -1

The Prisoners' Dilemma

- The same is true for Column

Solving the Prisoners' Dilemma

- The PD has an obvious solution (equilibrium)

		Column: Confess	Column: Don't
Row	Confess	-10,-10	$0, -20$
	Don't	$-20, 0$	$-1, -1$

The Prisoners' Dilemma

- Confessing is a dominant strategy for each player: it is the best choice **regardless** of what the other player does
- Confessing is a dominated strategy: it is worse than some other strategy **regardless** of the other player
- Both players confessing is an equilibrium of the game (even though it is worse for both than if neither confesses)

Applications of the PD

- Many other situations have structures similar to the Prisoners' Dilemma
- ex. 1: working on a joint project (private provision of a public good)

Column

		Goof off	Work Hard
Row	Goof off	$1, 1$	$3, 0$
	Work Hard	$0, 3$	$2, 2$

Private Provision of a Public Good

Applications of the PD

- Many other situations have structures similar to the Prisoners' Dilemma
- ex. 1: working on a joint project (private provision of a public good)

Column

Row		Goof off	Work Hard
	Goof off	1,1	$3, 0$
	Work Hard	$0, 3$	$2, 2$

Private Provision of a Public Good

- Eliminating strictly dominated actions gives (Goof off, Goof off) as equilibrium

Applications of the PD Cont'd

- ex. 2: Two firms produce the same good and can choose prices (duopoly)

		Column: Low Price	Column: High Price
Row	Low Price	$600, 600$	$1200, -200$
Row	High Price	$-200, 1200$	$1000, 1000$

A Price Setting Duopoly

Applications of the PD Cont'd

- ex. 2: two firms produce the same good and can choose prices (duopoly)

		Column: Low Price	Column: High Price
Row	Low Price	600,600	$1200, -200$
	High Price	$-200, 1200$	$1000, 1000$

A Price Setting Duopoly

- Eliminating strictly dominated actions gives (Low Price, Low Price) as equilibrium

Applications of the PD Cont'd

- In general, the PD encompasses all situations in which players can '**Cooperate**' (C) or '**Not cooperate/Defect**' (D):

Column

		C	D
Row	C	a, b	c, d
	D	e, f	g, h

The Prisoners' Dilemma

- Where payoffs are:

$$e > a > g > c$$
$$d > b > h > f$$

- Defecting is a dominant strategy for each player
- The equilibrium outcome is (D, D)

Dominance Solvability

A Dominance Solvable Game

- Another example: Microsoft vs Start-up in a market for new online service

		Start-up	
		Enter	Don't
Mircosoft	Enter	$2, -2$	$5, 0$
	Don't	$0, 5$	$0, 0$

An Entry Game

A Dominance Solvable Game

- Another example: Microsoft vs Start-up in a market for new online service

Start-up

		Enter	Don't
Microsoft	Enter	2 , -2	5 , 0
	Don't	0 , 5	0 , 0

An Entry Game

- Microsoft has a dominant strategy: Enter

A Dominance Solvable Game

- Another example: Microsoft vs Start-up in a market for new online service

		Start-up	
		Enter	Don't
Microsoft	Enter	2, - 2	5, 0
	Don't	0, 5	0, 0

An Entry Game

- Microsoft has a dominant strategy: Enter
- Start-up has no dominant strategy: if MS enters, Start-up should stay out; if MS stays out, Start-up should enter

Dominance Solvability (cont'd)

- we can solve the game by **iterated elimination of dominated strategies**

		Start-up: Enter	Start-up: Don't
Mircosoft	Enter	$2, -2$	$5, 0$
	Don't	$0, 5$	$0, 0$

An Entry Game

Dominance Solvability (cont'd)

- we can solve the game by **iterated elimination of dominated strategies**

		Start-up	
		Enter	Don't
Microsoft	Enter	$2, -2$	$5, 0$
	Don't	$0, 5$	$0, 0$

An Entry Game

- eliminate first MS's dominated strategy (Don't enter)
  (seems reasonable if Start-up knows MS's payoffs)

Dominance Solvability (cont'd)

- we can solve the game by **iterated elimination of dominated strategies**

		Start-up	
		Enter	Don't
Microsoft	Enter	$2, -2$	$5, 0$
	Don't	$0, 5$	$0, 0$

An Entry Game

- eliminate first MS's dominated strategy (Don't enter)
- with remaining game eliminate Start-up's dominated strategy (Enter)

Dominance Solvability (cont'd)

- we can solve the game by **iterated elimination of dominated strategies**

		Start-up: Enter	Start-up: Don't
Microsoft	Enter	$2, -2$	$5, 0$
	Don't	$0, 5$	$0, 0$

An Entry Game

- eliminate first MS's dominated strategy (Don't enter)
- with remaining game eliminate Start-up's dominated strategy (Enter)
- only (Enter, Don't enter) is left $\rightarrow$ equilibrium outcome

Dominance Solvability (cont'd)

- Another example

		Column		
		Left	Center	Right
Row	Top	$-5, -1$	$2, 2$	$3, 3$
	Middle	$1, -3$	$1, 2$	$1, 1$
	Bottom	$0, 10$	$0, 0$	$0, -10$

A Dominance Solvable Game

Dominance Solvability (cont'd)

- Another example

		Column		
		Left	Center	Right
	Top	$-5, -1$	$2, 2$	$3, 3$
Row	Middle	$1, -3$	$1, 2$	$1, 1$
	Bottom	$0, 10$	$0, 0$	$0, -10$

A Dominance Solvable Game

- 'Middle' dominates 'Bottom' for Row $\rightarrow$ drop 'Bottom'

Dominance Solvability (cont'd)

- Another example

		Left	Center	Right
		Column		
	Top	$-5, -1$	$2, 2$	$3, 3$
Row	Middle	$1, -3$	$1, 2$	$1, 1$
	Bottom	$0, 10$	$0, 0$	$0, -10$

A Dominance Solvable Game

- 'Middle' dominates 'Bottom' for Row → drop 'Bottom'
- 'Left' is dominated for Column in remaining game → drop 'Left'

Dominance Solvability (cont'd)

- Another example

Row \ Column	Left	Center	Right
Top	$-5, -1$	$2, 2$	$3, 3$
Middle	$1, -3$	$1, 2$	$1, 1$
Bottom	$0, 10$	$0, 0$	$0, -10$

A Dominance Solvable Game

- 'Middle' dominates 'Bottom' for Row → drop 'Bottom'
- 'Left' is dominated for Column in remaining game → drop 'Left'
- 'Top' dominates 'Middle' for Row in remaining game → drop 'Middle'

Dominance Solvability (cont'd)

- Another example

Row \ Column	Left	Center	Right
Top	$-5, -1$	$2, 2$	$3, 3$
Middle	$1, -3$	$1, 2$	$1, 1$
Bottom	$0, 10$	$0, 0$	$0, -10$

A Dominance Solvable Game

- 'Middle' dominates 'Bottom' for Row → drop 'Bottom'
- 'Left' is now dominated for Column → drop 'Left'
- 'Top' dominates 'Middle' for Row in remaining game → drop 'Middle'
- 'Right' is better than 'Center' for Column → drop 'Center'
- Equilibrium outcome is (Top, Right)

Dominance Solvability (cont'd)

- Another example

		Column		
		Left	Center	Right
	Top	$-5, -1$	$2, 2$	$3, 3$
Row	Middle	$1, -3$	$1, 2$	$1, 1$
	Bottom	$0, 10$	$0, 0$	$0, -10$

A Dominance Solvable Game

- Outcome (Top, Right) reasonable if players know each others' payoffs; plus: no player would want to change their behavior given the behavior of others
- But iterated elimination of dominated strategies does not always give an outcome ...

A Coordination Game

- A coordination game

		Sally	
		Renaissance	Starbucks
Harry	Renaissance	$2, 1$	$0, 0$
	Starbucks	$0, 0$	$1, 2$

Battle of the Sexes

- Both players prefer to cooperate (meet at same location) but disagree on about best outcome (location)

  $\rightarrow$ No dominated strategies

- But: if Sally chooses Starbucks, Harry should also choose Starbucks (and vice versa) $\rightarrow$ reasonable to presume that they will meet