Part 3: Game Theory I
Nash Equilibrium

Simultaneous Move Games, Nash Equilibrium, Best-Response Analysis

May 2016

Nash Equilibrium

Nash Equilibrium

- In a Nash Equilibrium (NE), each player chooses a strategy that maximizes their expected payoff **given** the strategies employed by other players

→ each player's equilibrium strategy is a best response to the equilibrium strategy of the other player

		Column: Confess	Column: Don't
Row	Confess	$-10, -10$	$0, -20$
	Don't	$-20, 0$	$-1, -1$

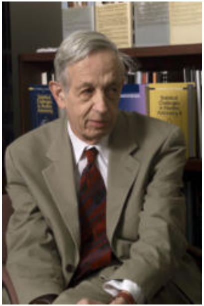
The Prisoners' Dilemma

- (Confess,Confess) satisfies the definition of a NE and is the only strategy combination that does so

Nash Equilibrium (contd.)

- 'Nash Equilibrium' after John Nash (1928-2015)

He won 1994 Nobel Prize in Economics (together with two other game theorists, Reinhard Selten and John Harsanyi). He is the subject of the Hollywood movie, A Beautiful Mind (4 Oscars), about his mathematical genius and his struggles with the paranoid type of schizophrenia. Nash died in a car accident on May 24 2015.



Nash Equilibrium (contd.)

- Justifications for the NE concept
  - if players play a NE, no one has an incentive to change their behavior or have second thoughts about their strategy
  - other potential outcomes do not have this property: if an outcome is not a NE, there is at least one player who wishes to reconsider his/her strategy
  - outcomes that are not NE involve mistakes from at least one player; thus sophisticated (rational) players must be able to (learn how to) play a NE
  - if a game is dominance solvable, the outcome is the unique NE
- NE equilibrium is **not:**
  - the jointly best outcome for all players
  - a situation where players always choose the same action
  - a unique outcome

Finding Nash Equilibria

- The most reliable way to find all NE is the cell-by-cell inspection method (finding a match in best-responses)
- A coordination game

Harry \ Sally	Renaissance	Starbucks
Renaissance	$2, 1$	$0, 0$
Starbucks	$0, 0$	$1, 2$

Battle of the Sexes

Finding Nash Equilibria

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Battle of the Sexes

- the battle of the sexes has two Nash equilibria
- in both NE, the parties coordinate their actions (meet)
- but: parties disagree as to which NE they should play!

Finding Nash Equilibria (cont'd)

- Another coordination game

		Sally: Renaissance	Sally: Starbucks
Harry	Renaissance	$1, 1$	$0, 0$
Harry	Starbucks	$0, 0$	$2, 2$

An Assurance Game

Finding Nash Equilibria (cont'd)

- Another coordination game

		Sally: Renaissance	Sally: Starbucks
Harry	Renaissance	$1, 1$	$0, 0$
	Starbucks	$0, 0$	$2, 2$

An Assurance Game

- the assurance game also has two NE
- other examples of coordination games: which side of the road to drive on, adopting a technology standard etc.
- coordination games have multiple Nash equilibria

Equilibrium Selection

- we can't predict a unique outcome if there are multiple Nash equilibria $\rightarrow$ equilibrium selection? (focal points)
- Games like the driving example above have illustrated the need for solution to coordination problems.
- Often we are confronted with circumstances where we must solve coordination problems without the ability to communicate with our partner.
- Many authors have suggested that particular equilibria are focal for one reason or another, for instance:
  - some equilibria may give higher payoffs, be naturally more salient
  - may be more fair, or may be safer

Nash Equilibria need not exist

- A strictly competitive game (game of pure conflict):

		Column Head	Column Tail
Row	Head	$1, -1$	$-1, 1$
	Tail	$-1, 1$	$1, -1$

Matching Pennies

- other examples of strictly competitive (constant sum) games: penalty kicks in soccer, point in a tennis match
- important for players not to play in a predictable way $\rightarrow$ players need to randomize over their actions; they play *mixed strategies* (to be discussed later)
- some games have no NE in pure strategies