Part 3: Game Theory I
Nash Equilibrium: Applications

Oligopoly, Cournot Competition, Bertrand Competition,
Free Riding Behavior, Tragedy of the Commons

June 2016

Illustrating Nash Equilibrium

- many models use notion of Nash equilibrium to study economic, political or biological phenomena
- often, these games involve continuous actions
- examples:
  - firms choosing a business strategy in an imperfectly competitive market (price, output, investment in R&D)
  - candidates in an election choosing platforms (policies)
  - animals fighting over prey choosing time at which to retreat
  - bidders in auction choosing bid

Oligopoly

- a market or an industry is an oligopoly if it is dominated by **a small number of sellers (oligopolists)** who each have a non-negligible effect on prices
- oligopoly is a market form in between perfect competition and monopoly
- various economic models study oligopoly:
  - Cournot model (quantities, homogeneous good)
  - Bertrand model (prices, homogeneous good)
  - price competition with differentiated products
  - Hotelling model of product differentiation
  - and many more...

Cournot Competition

The Cournot Model

- two firms $i = 1, 2$ produce a **homogeneous product**
- firm i's output $q_i \geq 0$, constant marginal costs c
- total industry output $Q = q_1 + q_2$
  inverse demand function $p(Q) = a - b(q_1 + q_2)$
- firms **simultaneously** choose own output q_i, **taking** the rival firm's output q_j **as given** $\rightarrow$ strategies are q_i's
- firm i maximizes

$$\pi_i = (a - bQ)q_i - c_i q_i \qquad \text{s.t. } q_j \text{ is given}$$

- FOC $\quad \frac{\partial \pi_i}{\partial q_i} = 0 \rightarrow$ best response functions $q_i = q_i^{br}(q_j)$
- (q_1^*, q_2^*) Nash equilibrium if $q_1^* = q_1^{br}(q_2^*)$ and $q_2^* = q_2^{br}(q_1^*)$

Analysis

- maximizing profit yields

$$\max_{q_1}\ \pi_1 = (a - b(q_1 + q_2))q_1 - cq_1 \quad \Rightarrow\ q_1^{br}(q_2) = \frac{a-c}{2b} - \frac{1}{2}q_2$$
$$\max_{q_2}\ \pi 2 = (a - b(q_1 + q_2))q_2 - cq_2 \quad \Rightarrow\ q_2^{br}(q_1) = \frac{a-c}{2b} - \frac{1}{2}q_1$$

- solving for the NE $q_1^* = q_1^{br}(q_2^*)$ and $q_2^* = q_2^{br}(q_1^*)$ gives

$$q_1^* = q_2^* = \frac{a-c}{3b} \quad \text{and} \quad \pi_1^* = \pi_2^* = \frac{(a-c)^2}{9b}$$

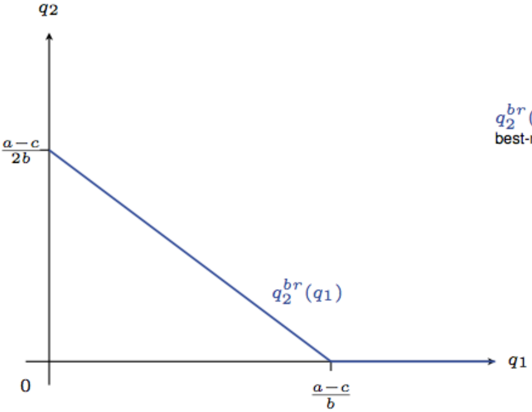
- compare outcome to monopoly

$$q_1^* + q_2^* > q_m = \frac{a-c}{2b},\ p^* < p_m \quad \text{and} \quad \pi_1^* + \pi_2^* < \pi_m = \frac{(a-c)^2}{4b}$$

- symmetric market with n firms:

$$q_i^* = \frac{a-c}{b(n+1)},\ p^* = \frac{a+nc}{(n+1)}, \quad \text{and} \quad \pi_i^* = \frac{(a-c)^2}{b(n+1)^2}$$

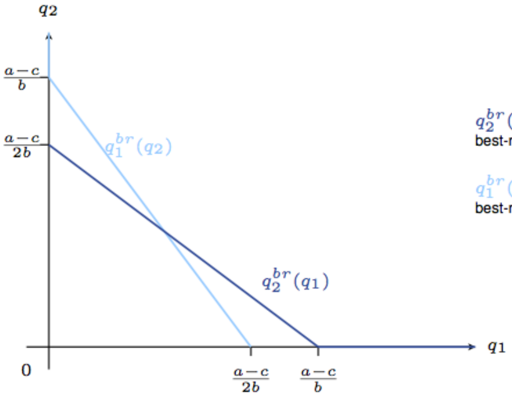
Graphic Illustration



$q_2^{br}(q_1) = \frac{a-c}{2b} - \frac{1}{2}q_1$
best-response function of firm 2

Quantity Setting Oligopoly: Best Response Functions

Graphic Illustration



Quantity Setting Oligopoly: Best Response Functions

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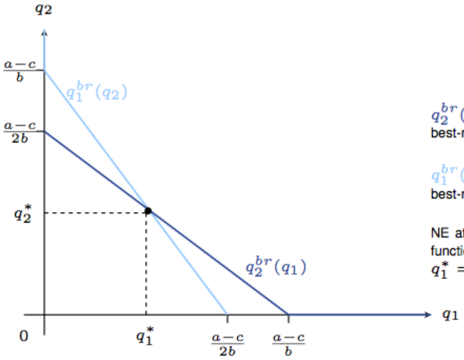
best-response function of firm 2

$q_1^{br}(q_2) = \frac{a-c}{2b} - \frac{1}{2}q_2$
best-response function of firm 1

Graphic Illustration



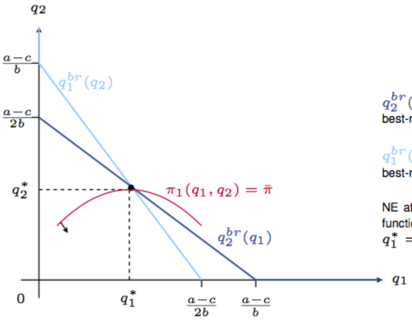
Quantity Setting Oligopoly: Cournot Equilibrium

$q_2^{br}(q_1) = \frac{a-c}{2b} - \frac{1}{2}q_1$
best-response function of firm 2

$q_1^{br}(q_2) = \frac{a-c}{2b} - \frac{1}{2}q_2$
best-response function of firm 1

NE at intersection of two best response functions
$q_1^* = q_2^* = \frac{a-c}{3b}$

Graphic Illustration



Quantity Setting Oligopoly: Cournot Equilibrium

$q_2^{br}(q_1) = \frac{a-c}{2b} - \frac{1}{2}q_1$
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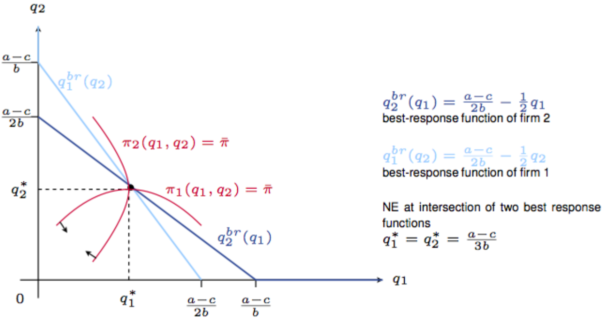
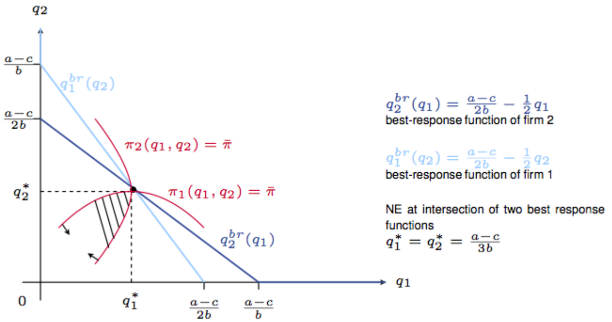


Quantity Setting Oligopoly: Cournot Equilibrium

- firm 1's isoprofit curve $\pi_1(q_1, q_2) =$ constant is tangent to $q_2 = q_2^*$ line

Graphic Illustration



Quantity Setting Oligopoly: Cournot Equilibrium

- firm 1's isoprofit curve $\pi_1(q_1, q_2) =$ constant is tangent to $q_2 = q_2^*$ line
- firm 2's isoprofit curve $\pi_2(q_1, q_2) =$ constant is tangent to $q_1 = q_1^*$ line
- there are (q_1, q_2) combinations (the shaded area) that would make both firms better off $\rightarrow$ the NE is not efficient (Pareto optimal) from the firm's point of view

Summary

In a Cournot oligopoly

- each firm's profit is decreasing in the other firm's quantity
  in producing output, firms impose negative externalities on rivals

$\Rightarrow$ in equilibrium, the firms produce **too much** output relative to joint profit maximization (monopoly); they **fail to maximize their joint profit**

- firms have an incentive to collude/cooperate
- firms also have an incentive to cheat on any collusive/cooperative agreement (any agreement other than a NE is not self-enforcing)

Bertrand Competition

The Bertrand Model

Named after Joseph Louis François Bertrand (1822-1900)

- two firms $i = 1, 2$ produce a **homogeneous product**
- firm i's price $p_i \geq 0$, constant marginal cost c_i
- linear demand function

$$D_i(p_i) = \begin{cases} a - p_i & \text{if } p_i < p_j \\ (a - p_i)/2 & \text{if } p_i = p_j \\ 0 & \text{if } p_i > p_j \end{cases}$$

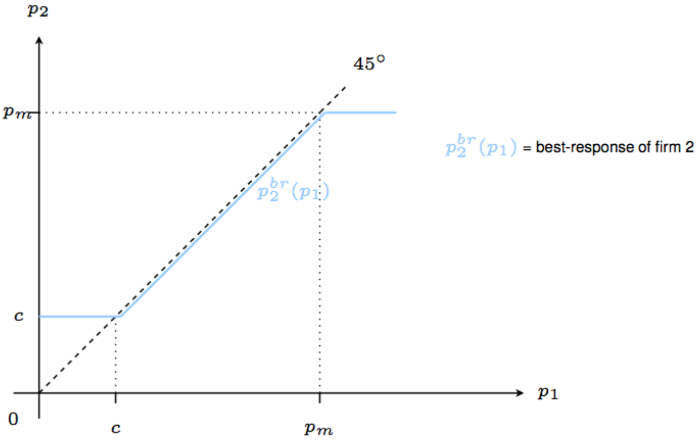
- firms **simultaneously** choose prices p_i, **taking** the rival firm's price p_j **as given** $\rightarrow$ strategies are p_i's
- firm i maximizes $\pi_i = (p_i - c)D_i(p_i)$ taking p_j as given
- (p_1^*, p_2^*) Nash equilibrium if $p_1^* = p_1^{br}(p_2^*)$ and $p_2^* = p_2^{br}(p_1^*)$

Analysis

- by pricing just below the rival firm, each firm can obtain the full market demand $D(p) \rightarrow$ strong incentive to 'undercut' one's rival
- claim: in the unique NE, $p_1^* = p_2^* = c$ (marginal cost pricing) = same outcome as perfect competition!
- to see this, look at firm 1's best response to p_2
  - if $p_2 \leq c$, firm 1 can set $p_1 = c$ (makes no profit anyway)
  - if $c < p_2 \leq p_m$, firm 1 should set $p_1 = p_2 - \epsilon$
  - if $p_2 > p_m$, firm 1 should set $p_1 = p_m$
- analogous for firm 2

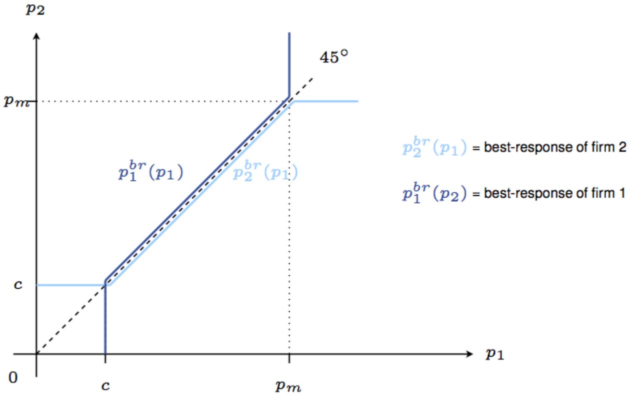
$\Rightarrow$ best responses intersect only at $p_1 = p_2 = c$

Graphic Illustration



Price Setting Oligopoly: Reaction Functions

Graphic Illustration



Price Setting Oligopoly: Reaction Functions

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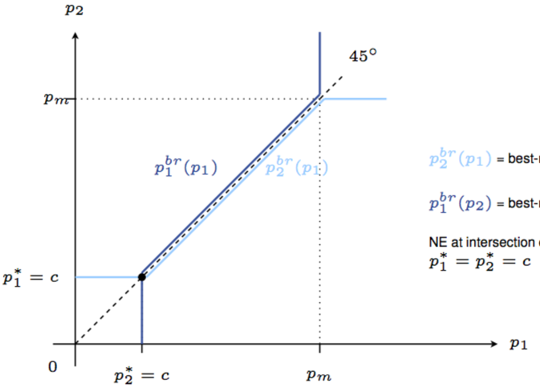


$p_2^{br}(p_1)$ = best-response of firm 2

$p_1^{br}(p_2)$ = best-response of firm 1

NE at intersection of two best-response functions
$p_1^* = p_2^* = c$

Price Setting Oligopoly: Bertrand Equilibrium

Summary

In a Bertrand oligopoly:

- each firm's profit is increasing in the other firm's price
  in raising their price, firms impose positive externalities on rivals

$\Rightarrow$ in equilibrium, the firms set the price **too low** relative to joint profit maximization

Similarities to Cournot:

- price below monopoly price
- firms fail to maximize their joint profit $\rightarrow$ incentive to collude/cooperate
- still: incentive to cheat on any collusive/cooperative agreement
  (any agreement other than a NE is not self-enforcing)

Summary

Differences to Cournot:

- Bertrand predicts two firms is enough to generate marginal cost pricing
- in practice: differentiated products $\rightarrow p > MC$
- if capacity and output can be easily changed, Bertrand fits situation better; otherwise Cournot

The Free Rider Problem

Games of Collective Action

- **games of collective action** = situations where the benefit to group depends on the actions (efforts) of all members
- in collective action games, individuals have a tendency to free ride on the contribution of others; they contribute nothing/too little themselves but still reap the benefit
- primary example: contributing to a public good
- in general, the **free-rider problem** means that
  - there are too few 'volunteers'
  - there is too little group effort

⇒ benefit from collective action too low → Pareto inefficient outcomes (market failure)

- the problem gets worse as the group becomes larger

Ex: Private Provision of a Public Good

- two roomates, Harry (H) and Sally (S)
- public good = cleanness of apartment G, utilities are

$$u_S = 40\sqrt{G} + \text{money} \qquad u_H = 20\sqrt{G} + \text{money}$$

- a cleaning lady will take G hours to produce G and costs \$ 10/hour $\rightarrow$ $C(G) = 10G$ and $MC(G) = 10$
- Pareto optimal amount is at $MB_S + MB_B = MC$,

$$20\frac{1}{\sqrt{G}} + 10\frac{1}{\sqrt{G}} = 10 \quad \Rightarrow \quad G^{eff} = 9$$

- if $i = H, S$ contributes \$$x_i$, total cleaning budget is $x_H + x_S$ and cleanness is $G = \frac{1}{10}(x_H + x_L)$

Contributing to a Public Good (contd.)

- how much will each person contribute?
- answer depends on how much each expects the roommate to contribute

  $\rightarrow$ strategic game

  $\rightarrow$ solve using NE concept
- if S expects H to contribute x_H, her optimal contribution solves

$$\max_{x_S} \ u_S = 40\sqrt{\frac{1}{10}(x_S + x_H)} - x_S \quad \text{s.t. } \ x_H \ \text{ given, } \ x_S \geq 0$$

- FOC gives

  $x_S^{br}(x_H) = 40 - x_H$ for $x_H \leq 40$ (otherwise $x_S^{br}(x_H) = 0$)

Contributing to a Public Good (contd.)

- if H expects S to contribute x_S, his optimal contribution solves

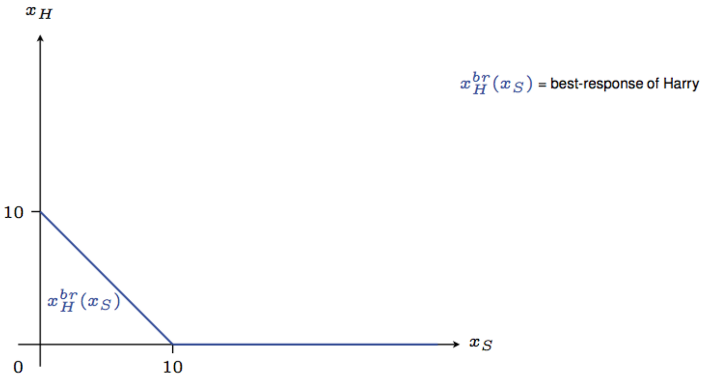
$$\max_{x_H} u_H = 20\sqrt{\frac{1}{10}(x_S + x_H)} - x_H \quad \text{s.t. } x_H \text{ given, } x_H \geq 0$$

- FOC gives

$$x_H^{br}(x_S) = 10 - x_S \text{ for } x_S \leq 10 \text{ (otherwise } x_H^{br}(x_S) = 0)$$

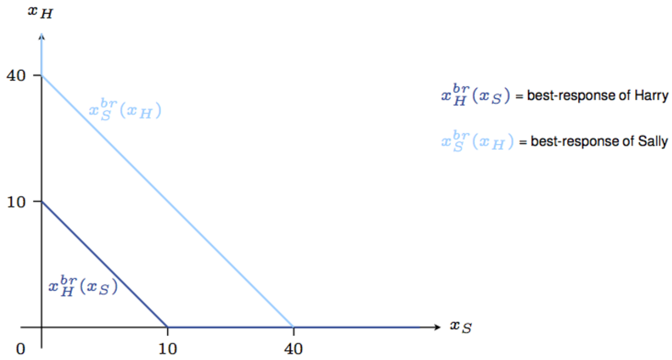
- (x_S^*, x_H^*) Nash equilibrium if $x_S^* = x_S^{br}(x_H^*)$ and $x_H^* = x_H^{br}(x_S^*)$
- NE is at $x_S^* = 40$ and $x_H^* = 0 \Rightarrow G^* = 4 < 9 = G^{eff}$

Graphic Illustration



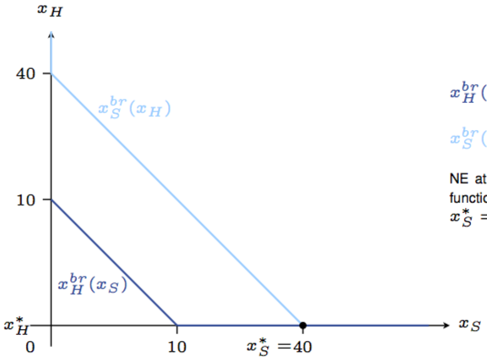
Private Provision of Public Goods: Best-Response Functions

Graphic Illustration



Private Provision of Public Goods: Best Response Functions

Graphic Illustration



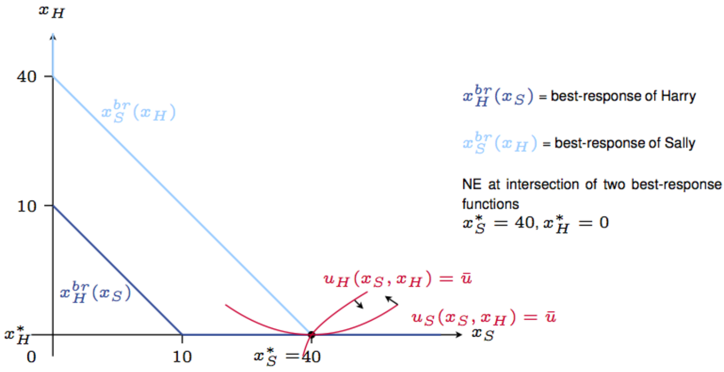
$x_H^{br}(x_S)$ = best-response of Harry

$x_S^{br}(x_H)$ = best-response of Sally

NE at intersection of two best-response functions
$x_S^* = 40, x_H^* = 0$

Private Provision of Public Goods: Nash Equilibrium

Graphic Illustration



Private Provision of Public Goods: Nash Equilibrium

- Sally's indifference curve $u_S(x_S, x_H) =$ constant is tangent to $x_H = x_H^*$ line
- Harry's indifference curve $u_H(x_S, x_H) =$ constant is **not** tangent to $x_S = x_S^*$ line
- there are (x_S, x_H) combinations that would make both better off
  $\rightarrow$ the NE is not efficient (Pareto optimal) from collective point of view

Contributing to a Public Good (contd.)

In private provision of public goods:

- each person's utility is increasing in the other person's contribution
  $\rightarrow$ positive externality (not internalized)
- each person's optimal contribution is decreasing in the other person's contribution
  $\rightarrow$ free riding behavior

$\Rightarrow$ in equilibrium, people contribute **too little** relative to joint surplus (welfare) maximization

Contributing to a Public Good (contd.)

Other instances where similar problem occur:

- tragedy of the commons = overuse of common resource
- public infrastructure (roads, parks)
- natural resources (oceans, air, water)

other games of collective action need not share same problems
(e.g. adopting a common standard)

Solving Collective Action Problems

Free riding and other problems in collective action games can often be mitigated or solved by:

- detection and punishment/rewards
- sanctions, customs, and social norms (in repeated interaction)
- government provision
- government regulation (taxes, subsidies)
- Coasian bargaining