Part 3: Game Theory II
Mixed Strategies

Mixed Strategy, Pure Strategy Nash Equilibrium, Mixed Strategy Nash Equilibrium, Constant Sum Games

June 2016

Choosing Actions Randomly

- reconsider 'Matching Pennies'

Row \ Column	Head	Tail
Head	$1, -1$	$-1, 1$
Tail	$-1, 1$	$1, -1$

Matching Pennies

- constant sum game (game of pure conflict)
- important for players not to play in a predictable way

$\rightarrow$ **no** Nash equilibrium **in pure strategies**

$\rightarrow$ players randomize over their actions, they play **mixed strategies**

Mixed Strategy Nash equilibrium

- a mixed strategy is one where a player plays (some of) the available pure strategies with certain probabilities
- concept best understood in repeated games, where each player's aim is to keep the other guessing

  Examples: Rock–Scissors–Paper game, penalty kicks, tennis point, bait cars, tax audits, drug testing etc.

- a mixed strategy Nash equilibrium is a Nash equilibrium where at least one player randomizes over his or her actions
- some games only have mixed strategy NE (Matching Pennies); others have NE's in both pure and mixed strategies (Chicken)

Best Response Analysis

- suppose p = probability Column plays Heads
  $\rightarrow 1 - p$ = probability Column plays Tails
- suppose q = probability Row plays Heads
  $\rightarrow 1 - q$ = probability Row plays Tails
- to find best-responses, compute players **expected payoffs**

		Column: Heads (prob p)	Column: Tails (prob $1-p$)	Row's exp payoff
Row	Heads (prob q)	$1, -1$	$-1, 1$	$2p - 1$
	Tails (prob $1-q$)	$-1, 1$	$1, -1$	$1 - 2p$
	Column's exp. payoff	$1 - 2q$	$-1 + 2q$	

Best Response Analysis (cont'd)

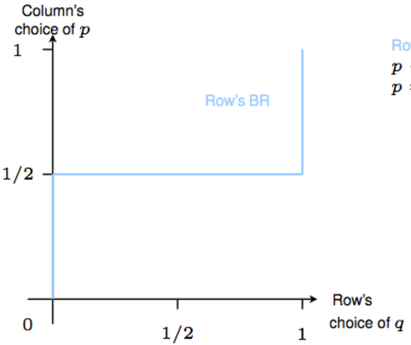
- if $2p - 1 > 0$ Row is better off playing Heads
  if $2p - 1 < 0$ Row is better off playing Tails
- if $2p - 1 = 0$, Row is equally well off playing Heads, Tails, or flipping a coin

$\rightarrow$ Row is willing to randomize **only if** Column plays Heads with $p = 1/2$; otherwise, Row plays either Heads or Tails

randomization requires equality of expected payoffs

- Nash equilibrium: Row's choice of q is best response to Column's choice of p and vice versa
- Note: pure strategies can be seen as special cases of mixed strategies where $p \in \{0, 1\}$ and $q \in \{0, 1\}$

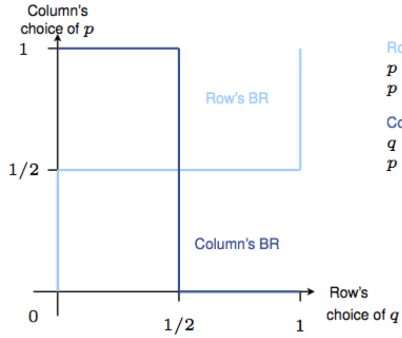
Graphic Illustration



Reaction Functions in 'Matching Pennies' Game

Row's best-response $q(p)$:
$p < 1/2 \rightarrow q = 0, p > 1/2 \rightarrow q = 1$
$p = 1/2 \rightarrow q \in (0, 1)$

Graphic Illustration



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Column's best-response $p(q)$:
$q < 1/2 \rightarrow p = 1, q > 1/2 \rightarrow p = 0$
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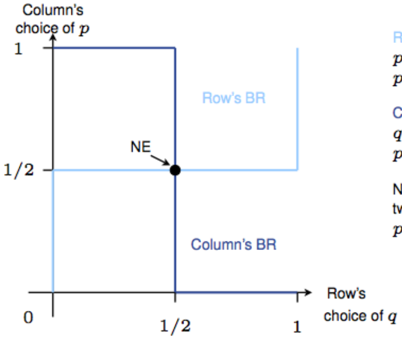
Graphic Illustration



Mixed Strategy NE in 'Matching Pennies' Game

Row's best-response $q(p)$:
$p < 1/2 \rightarrow q = 0, p > 1/2 \rightarrow q = 1$
$p = 1/2 \rightarrow q \in (0, 1)$

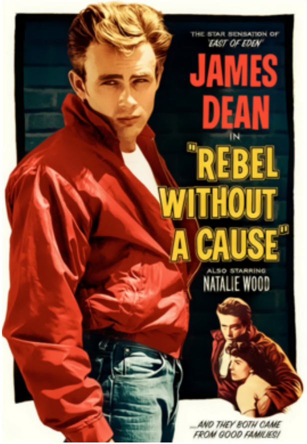
Column's best-response $p(q)$:
$q < 1/2 \rightarrow p = 1, q > 1/2 \rightarrow p = 0$
$p = 1/2 \rightarrow p \in (0, 1)$

NE at intersection of
two best response functions
$p^* = q^* = 1/2$

- in constant sum games, keeping your opponent indifferent means that he/she cannot recognize and exploit systematic patters in your behavior
- keeping your opponent indifferent is equivalent to keeping yourself indifferent
- same principle applies to non-constant sum games (where players' interests are not totally opposed to each other)

Another Example: Chicken

Game was made famous in the 1955 James Dean movie ‘Rebel Without a Cause’, where it was called the “chicky game”. Dean and his opponent drive toward a cliff. The first to jump out before his car went over is the chicken. In the mid 1960, the game chicken was used as an analogy to the nuclear arms race between the U.S. and the U.S.S.R



Another Example: Chicken

- two teenagers drive toward each other at a high rate of speed, the driver that swerves first is deemed a chicken and loses face with the rest of the crowd

Dean

		Swerve	Don't
James	Swerve	$0, 0$	$-1, 1$
	Don't	$1, -1$	$-4, -4$

Chicken

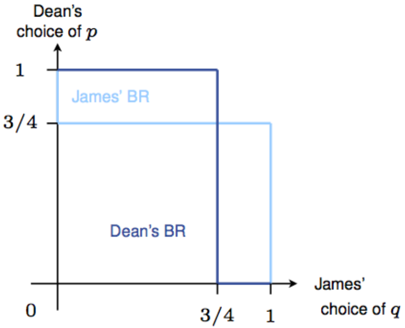
- (Swerve,Don't) and (Don't, Swerve) are NE in pure strategies, game also has a mixed strategy NE!
- let p = prob Dean swerves, q = prob James swerves
- in mixed strategy NE,
  $p^* = q^* = 3/4 \rightarrow 1/16$ = prob of a crash

Best Response Analysis of Chicken

- compute players **expected payoffs**

		Dean Swerve (prob p)	Dont' (prob $1-p$)	James' exp payoff
James	Swerve (prob q)	$0, 0$	$-1, 1$	$p-1$
	Don't (prob $1-q$)	$1, -1$	$-4, -4$	$5p-4$
	Dean's exp. payoff	$q-1$	$5q-4$	

Chicken: Graphical Analysis

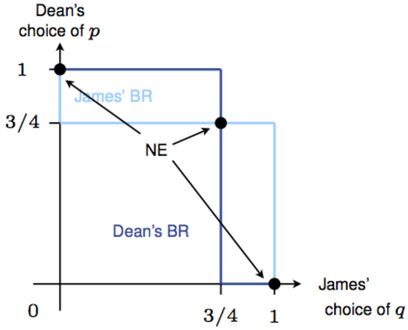


Best Responses in 'Chicken'

James' best-response $q(p)$:
$p < 3/4 \to q = 1, p > 3/4 \to q = 0$
$p = 3/4 \to q \in (0, 1)$

Column's best-response $p(q)$:
$q < 3/4 \to p = 1, q > 3/4 \to p = 0$
$q = 3/4 \to p \in (0, 1)$

Chicken: Graphical Analysis



Nash Equilibria in 'Chicken'

James' best-response $q(p)$:
$p < 3/4 \rightarrow q = 1, p > 3/4 \rightarrow q = 0$
$p = 3/4 \rightarrow q \in (0, 1)$

Column's best-response $p(q)$:
$q < 3/4 \rightarrow p = 1, q > 3/4 \rightarrow p = 0$
$q = 3/4 \rightarrow p \in (0, 1)$

NE at intersection of
two best response functions
$p^* = 0, q^* = 1, p^* = 1, q^* = 0,$
$p^* = q^* = 3/4$