Part 4: Game Theory II
Sequential Games

Games in Extensive Form, Backward Induction,
Subgame Perfect Equilibrium, Commitment

June 2016

Introduction

Sequential Games

- games in **matrix** (normal) form can only represent situations where people move simultaneously

  $\rightarrow$ sequential nature of decision making is suppressed

  $\rightarrow$ concept of 'time' plays no role

- but many situations involve player choosing actions sequentially (over time), rather than simultaneously

$\Rightarrow$ need games in **extensive** form = sequential games

- example

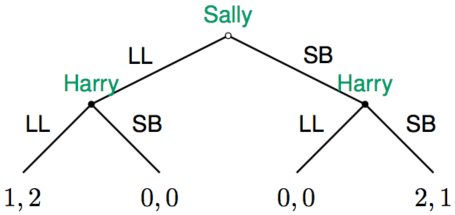
		Harry	
		Local Latte	Starbucks
Sally	Local Latte	$1, 2$	$0, 0$
	Starbucks	$0, 0$	$2, 1$

Battle of the Sexes (BS)

Battle of the Sexes Reconsidered

- suppose Sally moves first (and leaves Harry a text-message where he can find her)
- Harry moves second (after reading Sally's message)

⇒ extensive form game (game tree):



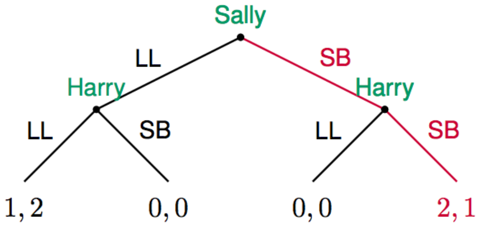
- game still has two Nash equilibria: (LL,LL) and (SB,SB)
- but (LL,LL) is no longer plausible...

Sequential Games

- a sequential game involves:
  - a list of **players**
  - for each player, a set of **actions** at each stage
  - each player's **information** at each stage
  - each player's preferences over all possible combination of actions = **payoffs**
- a **strategy** is a plan of action for <u>each</u> stage/history of game
- (mostly) confine ourselves to sequential games in which
  - players have perfect information (history of play at each stage is known)
  - each player moves only once

Solving the Sequential BS

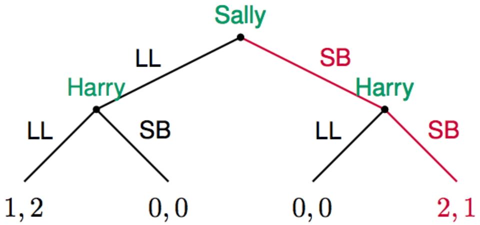
- the sequential BS has an obvious solution



- Sally chooses Starbucks (SB) and Harry follows → (SB,SB)
- the other Nash equilibrium (LL,LL) is no longer plausible... **Sally knows that Harry's best response is to follow her**

Solving the Sequential BS

- the sequential BS has an obvious solution

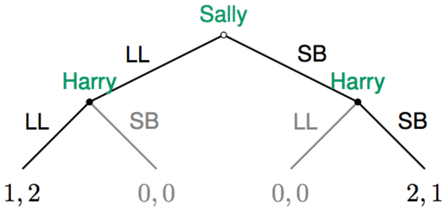


- Harry **cannot commit** to go to Local Latte - if Sally chooses Starbucks, his **best response** is to follow
- idea that player must choose their best responses **for each history** (at each stage) is called subgame perfection

Subgame Perfection

Subgame Perfect Nash Equilibrium

- a Subgame Perfect (Nash) Equilibrium is a Nash equilibrium with the property that all players play best responses after each history of the game
- we can solve for a Subgame Perfect Equilibrium (SPE) by using the backward induction method (=rollback)



- taking Harry's best responses (LL if LL) and (SB if SB) into account, the best strategy for Sally is SB → (SB,SB) is unique SPE

An Entry Game

- a potential entrant chooses whether to enter a market controlled by a monopolist
- if the entrant enters, the monopolist can either begin a price war, or share the market
- the game in normal form:

		Incumbent	
		Share	Fight
Entrant	Enter	$5, 5$	$-1, 1$
	Don't	$0, 10$	$0, 10$

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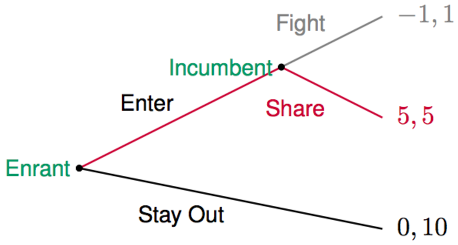
An Entry Game

- game has two NE's: (Enter,Share) and (Don't, Fight)

An Entry Game (Cont'd)

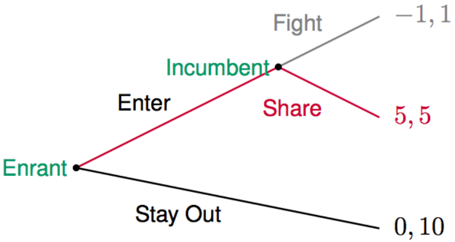
- the game in extensive form



- game still has two NE's (Enter, Share if Enter) and (Stay out, Fight if Enter).
- solving by backward induction: if the entrant chooses Enter, the incumbent's best response is Share
- given the incumbent's best response (Share if Enter), the entrant chooses (Enter)

An Entry Game (Cont'd)

- the game in extensive form



- backward inductions gives unique SPE (Enter, Share if Enter)
- the incumbent **cannot credibly commit** to fight if the entrant enters → the incumbent cannot deter entry

Commitment

- the Battle of the Sexes and the Entry game are both examples of situations where there is a **first-mover advantage**
- in these situations, the second-mover is at a disadvantage:
  - he/she would like to commit to an action which prompts the first-mover to adopt a certain strategy
  - however, he/she **cannot commit to play any strategy other than what is sequentially optimal (best-response)**,
- the second-mover may be able to improve his/her **commitment**, e.g., through hostages, bonds, contracts, limiting capacity
- there are other situations, however, where it is the second mover who has an advantage

Another Example: Avoiding Rocky

- Rocky recently met a pretty girl, and wants to see her again (she can't stand him)
- simultaneous-move game in normal form:

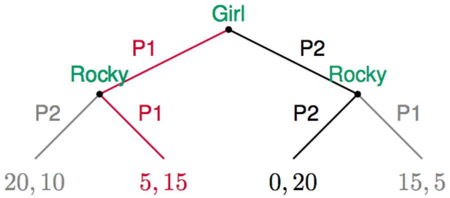
		Rocky: Party 1	Rocky: Party 2
Girl	Party 1	$5, 15$	$20, 10$
Girl	Party 2	$15, 5$	$0, 20$

Avoiding Rocky

- game has no pure strategy Nash equilibria
- game has one mixed strategy Nash equilibrium in which Rocky and Girl meet with probability $7/12$

Avoiding Rocky (Cont'd)

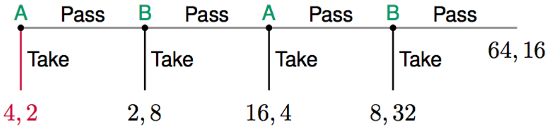
- the sequential game in extensive form



- if moves are sequential, Rocky can **condition his strategy** on what the Girl does
  → Rocky has a **second-mover advantage**
- unique SPE is the Girl chooses Party 1 and Rocky chooses (Party 1 if Party 1) and (Party 2 if Party 2) → Rocky cannot be avoided

Is Backward Induction Reasonable?

- may work in simple games with few players and moves
- more difficult in complex games, e.g., Chess
- may not always predict actual behavior (error, altruism, fairness and intention considerations)
- example: the centipede game



- unique SPE has A play ‘Take’ in first round but actual play in experiments involves at least some cooperation