Part 4: Game Theory II
Sequential Games

Games in Extensive Form, Backward Induction, Subgame Perfect Equilibrium, Commitment

June 2016
Introduction
Sequential Games

- games in **matrix** (normal) form can only represent situations where people move simultaneously
  - sequential nature of decision making is suppressed
  - concept of ‘time’ plays no role
- but many situations involve player choosing actions sequentially (over time), rather than simultaneously
  ⇒ need games in **extensive** form = sequential games
- example

\[
\begin{array}{c|cc}
& \text{Local Latte} & \text{Starbucks} \\
\hline
\text{Local Latte} & 1, 2 & 0, 0 \\
\text{Starbucks} & 0, 0 & 2, 1 \\
\end{array}
\]

Battle of the Sexes (BS)
suppose Sally moves first (and leaves Harry a text-message where he can find her)

Harry moves second (after reading Sally’s message)

⇒ extensive form game (game tree):

- game still has two Nash equilibria: (LL,LL) and (SB,SB)
- but (LL,LL) is no longer plausible...
Sequential Games

- a sequential game involves:
  - a list of **players**
  - for each player, a set of **actions** at each stage
  - each player’s **information** at each stage
  - each player’s preferences over all possible combination of actions = **payoffs**
- a **strategy** is a plan of action for each stage/history of game
- (mostly) confine ourselves to sequential games in which
  - players have perfect information (history of play at each stage is known)
  - each player moves only once
Solving the Sequential BS

- the sequential BS has an obvious solution

Sally chooses Starbucks (SB) and Harry follows → (SB,SB)

- Sally knows that Harry’s best response is to follow her

- the other Nash equilibrium (LL,LL) is no longer plausible... Sally knows that Harry’s best response is to follow her
Solving the Sequential BS

- the sequential BS has an obvious solution

- Harry cannot commit to go to Local Latte - if Sally chooses Starbucks, his best response is to follow

- idea that player must choose their best responses for each history (at each stage) is called subgame perfection
Subgame Perfection
Subgame Perfect Nash Equilibrium

- A Subgame Perfect (Nash) Equilibrium is a Nash equilibrium with the property that all players play best responses after each history of the game.

- We can solve for a Subgame Perfect Equilibrium (SPE) by using the backward induction method (=rollback).

- Taking Harry's best responses (LL if LL) and (SB if SB) into account, the best strategy for Sally is SB → (SB,SB) is unique SPE.
An Entry Game

- a potential entrant chooses whether to enter a market controlled by a monopolist
- if the entrant enters, the monopolist can either begin a price war, or share the market
- the game in normal form:

<table>
<thead>
<tr>
<th></th>
<th>Incumbent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share</td>
<td>5, 5</td>
</tr>
<tr>
<td>Fight</td>
<td>−1, 1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Entrant</th>
<th>Incumbent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enter</td>
<td>0, 10</td>
</tr>
<tr>
<td>Don’t</td>
<td>5, 5</td>
</tr>
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- the game has two NE’s: (Enter, Share) and (Don’t, Fight)
An Entry Game (Cont’d)

- the game in extensive form

- game still has two NE’s (Enter, Share if Enter) and (Stay out, Fight if Enter).

- solving by backward induction: if the entrant chooses Enter, the incumbent’s best response is Share

- given the incumbent’s best response (Share if Enter), the entrant chooses (Enter)
An Entry Game (Cont’d)

- the game in extensive form

![Game Tree]

- backward inductions gives unique SPE (Enter, Share if Enter)
- the incumbent cannot credibly commit to fight if the entrant enters → the incumbent cannot deter entry
Commitment

- the Battle of the Sexes and the Entry game are both examples of situations where there is a **first-mover advantage**

- in these situations, the second-mover is at a disadvantage:
  - he/she would like to commit to an action which prompts the first-mover to adopt a certain strategy
  - however, he/she **cannot commit to play any strategy other than what is sequentially optimal** (best-response),
  - the second-mover may be able to improve his/her **commitment**, e.g., through hostages, bonds, contracts, limiting capacity
  - there are other situations, however, where it is the second mover who has an advantage
Another Example: Avoiding Rocky

- Rocky recently met a pretty girl, and wants to see her again (she can’t stand him)

- simultaneous-move game in normal form:

<table>
<thead>
<tr>
<th></th>
<th>Party 1</th>
<th>Party 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Party 1</td>
<td>5,15</td>
<td>20,10</td>
</tr>
<tr>
<td>Party 2</td>
<td>15,5</td>
<td>0,20</td>
</tr>
</tbody>
</table>

Avoiding Rocky

- game has no pure strategy Nash equilibria

- game has one mixed strategy Nash equilibrium in which Rocky and Girl meet with probability $7/12$
Avoiding Rocky (Cont’d)

- the sequential game in extensive form

- if moves are sequential, Rocky can condition his strategy on what the Girl does
  \[\rightarrow\] Rocky has a second-mover advantage

- unique SPE is the Girl chooses Party 1 and Rocky chooses (Party 1 if Party 1) and (Party 2 if Party 2) \[\rightarrow\] Rocky cannot be avoided
Is Backward Induction Reasonable?

- may work in simple games with few players and moves
- more difficult in complex games, e.g., Chess
- may not always predict actual behavior (error, altruism, fairness and intention considerations)
- example: the centipede game

![Centipede Game Diagram]

- unique SPE has A play ‘Take’ in first round but actual play in experiments involves at least some cooperation