Part 3: Game Theory II
Repeated Games

Finitely Repeated Game, Infinitely Repeated Game/Supergame, Grim-Strategy, Punishment and Cooperation, Folk Theorem

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Repeated Games

- if a game is played repeatedly, players may behave differently than if the game is played only once

  example: cleaning a shared apartment

- a repeated game is a given **stage game** repeated several times
  - **finitely** many times → game has fixed length
  - **infinitely** or **unknown** number of times → game has no predetermined length = supergame

- many games are supergames
  - interactions in the market place (between firms, between customers and firms)
  - interactions in the workplace and the private sphere (among co-workers, family members, friends)

Finitely Repeated Games

An Example

- stage game: two firms produce same good and set prices

		Firm 2	
		Low Price	High Price
Firm 1	Low Price	$5, 5$	$25, 0$
	High Price	$0, 25$	$15, 15$

A Price Setting Duopoly

- stage game has unique Nash equilibrium/equilibrium in dominant strategies is (Low Price,Low Price)
- what if stage game is played two (or more) times?

  repetition $\rightarrow$ players can use history-dependent strategies

  ...maybe cooperation feasible?
- if stage game has **unique** Nash equilibrium, can use backward induction method!

Example (Cont'd)

- suppose pricing game is played twice, in $t = 1$ and in $t = 2$
- at last stage, backward induction (subgame perfection) means that the unique Nash equilibrium will be played
  $\rightarrow$ (Low Price, Low Price) in $t = 2$ for **each** of history of game

$\Rightarrow$ players' actions in $t = 1$ do **not** influence their behavior in $t = 2$

$\Rightarrow$ unique Nash equilibrium in $t = 1$ is also to play (Low Price, Low Price)

in a finitely repeated game, if the stage game has only one Nash equilibrium,
the unique subgame perfect equilibrium is to play the Nash equilibrium in every period

- other equilibria may exist if there is more than one Nash equilibrium in the stage game

Infinitely Repeated Games (Supergames)

The Pricing Game Revisited

- now suppose pricing game is played over and over $t = 1, 2, \ldots$
- playing (Low Price, Low Price) in each period still a SPE

  but it is no longer the only subgame perfect equilibrium
- consider the following strategy = grim trigger strategy:
  - set High Price until you see your rival sets Low Price (=trigger)
  - from then on, set Low Price forever (=grim strategy)
- if Firm 1 uses a grim trigger strategy, what is Firm 2's best response?
- note: once Firm 2 sets Low Price in some period t, should set Low Price in all subsequent periods $t + 1, t + 2, \ldots$

... but does it ever pay to play Low Price in some period?

Example Cont'd

- suppose Firm 2 plays High Price forever $\rightarrow$ if Firm 1 plays trigger strategy, should play (High Price) forever
- present value from doing so for Firm 2 is, where $\delta \in [0,1]$ is the discount factor

$$\begin{aligned} V_2(\text{High Price}) &= 15 + \delta 15 + \delta^2 15 + \delta^3 15 + \ldots \\ &= (1 + \delta + \delta^2 + \ldots)15 \\ &= \frac{1}{1-\delta} 15 \end{aligned}$$

Example Cont'd

- now suppose Firm 2 plays Low Price → Firm 2 will retaliate (trigger strategy!) with Low Price forever, starting next period
- present value from doing so for Firm 2 is

$$V_2(\text{Low Price}) = 25 + \delta 5 + \delta^2 5 + \delta^3 5 + \dots$$
$$= 25 + \delta(1 + \delta + \dots)5 = 25 + \frac{\delta}{1-\delta}5$$

- for Firm 2, **setting High Price forever is better than deviating and setting Low Price** – which then triggers the (Low Price,Low Price) forever – if

$$V_2(\text{High Price}) > V_2(\text{Low Price}) \quad \Leftrightarrow \quad \frac{1}{1-\delta}15 > 25 + \frac{\delta}{1-\delta}5$$
$$\Leftrightarrow \quad \delta > \frac{1}{2}$$

Example Cont'd

- argument is symmetric for Firm 1
- similar reasoning also works if players play Tit-for-Tat strategies and/or if game ends only with some probability

if players are sufficiently patient, the cooperative outcome (High Price, High Price) in every period is an SPE of the infinitely repeated game

Cooperation through Repetition

- sequential nature of relationship means that players can adopt **contingent** strategies $\rightarrow$ punish bad behavior/reward good behavior
- if game is finitely repeated, **backward induction rules out punishments and rewards that are not sequentially rational**
- in repeated relationships without predetermined length, however, there is no last period

$\Rightarrow$ strategies such as "grim-trigger" or "tit-for-tat" are sequentially rational and can sustain cooperation/collusion if players are sufficiently patient/have low discount rate ($\Rightarrow$ Folk Theorem)

$\Rightarrow$ repeated play + low discount factor means **future matters**

  - future punishments (for defecting) are severe
  - future rewards (for cooperating) are valued high