# Econ 302: Microeconomics II - Strategic Behavior 

Problem Set \# 1 - May 162016
Note:
Questions without no star are of easy to medium difficulty. Everybody should be able to give the correct answer, or at least have a pretty good idea about what the solution could be. Questions with a star (*) are tricky and may require some thought and additional time to solve. The questions with two stars ( $* *$ ) are very hard and I certainly wouldn't expect you to necessarily give the right answer, but you should still try them!

1. True/False/Uncertain? Explain your answer.
a) If people have identical preferences, there are no benefits from trade.

False. If people don't have the same endowments, there may be gains from trade even if they have the same preferences. Say everybody equally likes apples and oranges. But some people only have apples, while others only have oranges. Everyone will benefit from trade.
b) The competitive equilibrium quantity and price depends on the initial distribution of goods/resources.

Uncertain. It depends on whether people's preferences display wealth/income effects. In general, the equilibrium quantity consumed/produced as well as the price depends on initial endowments. In the absence of wealth effects, however, people's willingness to pay for a good is independent of their wealth/income (as in the example I gave in class) and the competitive equilibrium is unique. Of course, the quantity traded always depends on initial endowments. Just compare a situation where the people buy the good in equilibrium to a situation where the same people already own the good at the outset. There will be no trade in the latter case.
2.* Airline Companies often sell more seats than are actually available on a flight. In the occasional event that all passengers do show up, this practice of overbooking naturally means that some people with tickets are "bumped" to a later flight. What criteria might be used to decide which passengers will get on the flight they booked and which do not? Discuss alternatives and evaluate them from an efficiency point of view.

Possible criteria for excluding confirmed passengers might include personal characteristics, the time of check-in, and random assignment. Neither of these is likely to please the people affected. To promote efficiency, it would be best if only the airline would dump passengers in order of their lowest willingness to pay/least dis-utility
for being excluded. That way, people on the plane are those for whom the flight is worth most. The problem is that the airline has no way to tell which people suffer less from not being on the flight than others. Note that asking people for their valuations will not solve the problem because nobody will be willing to admit a low valuation. So asymmetric information a priori prevents an efficient solution here. One way out of this dilemma is the practice of paying people not to take a flight. This can help because only those who stay are the ones for whom flying is worth more than the payment offered, while those who sell their claim to a seat are those for whom it is worth less. Adjusting the payment until the right number of people voluntarily give up their seats - in effect holding an auction - guarantees that efficiency is achieved. Note that a fixed payment risks to few or too many volunteers, and in the latter case it does not ensure that the ones who are dumped value the flight least.
3. There are still 5 tickets available for the upcoming Canucks playoff game and 8 fans who would really like to go. Their reservation prices for a ticket are

| person | $A$ | $B$ | $C$ | $D$ | $E$ | $F$ | $G$ | $H$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| reservation price in $\$$ | 15 | 10 | 8 | 5 | 4 | 3.5 | 2 | 1 |

Assume that the (marginal) cost of a selling/distributing a ticket is 1.
a) What is the efficient amount of tickets to be allocated and to whom?

All 5 ticket should be sold to those with the highest willingness to pay, i.e., person $A-E$,
b) The Canucks are thinking about how best to distribute these tickets. There are 4 options. For each case, explain what you expect to happen and whether the outcome will be efficient.
i) Sell the tickets through price-taking agents on a competitive market

Drawing a diagram with the supply and the demand curve, it's easy to see that the competitive equilibrium price must be in the range of \$3.5-\$4. All 5 tickets are sold to persons $A-E$. The outcome is efficient. The total profit is $\$ 12.5-\$ 15$, depending on the equilibrium price.
ii) Sell the tickets through a monopolist (ticketmaster)

Ticketmaster would want to set the price so as to maximize its profit $(p-1) \times$ tickets sold. Calculating this expression for each possible price, one finds that $p=8$ is the profit-maximizing price. At that price, 3 tickets are sold to persons $A-C$. The total profit is $\$ 21$ and thus higher than in the competitive equilibrium. The outcome is inefficient.
iii) Sell the tickets at a price of $\$ 3$ each.

If the price is set at \$ 3, there will be excess demand for tickets. The allocation problem must then be solved somehow, e.g., by random rationing. The total profit under this scheme is $\$ 10$. The outcome will be inefficient in general.

Give all 5 of them away for free to the Smith family. The Smith's are persons $A, C$, $G$ and toddler $Z$ who can't yet go to a game

If the tickets are given away for free to some people, perhaps based on personal characteristics, profit is obviously zero and the outcome will be inefficient. However, if resale is allowed, the Smiths may turn around and resell their tickets. Resale may well re-establish efficiency, depending on how it is organized. If the Smith's sell their tickets to the highest bidders, for instance (they hold an auction), then the outcome will again be efficient. Note that resale would also work in case iii).
c)* Suppose the spouse of person $A$ plans to celebrate the couple's upcoming wedding anniversary the night of the game. If $A$ actually went to the game (rather than have a romantic dinner), the spouse would suffer a dis-utility of 12 (in \$). Are your answers in questions a) and b)i) affected and if so, how?

As long as person $A$ does not take the spouses' dis-utility into account when making the decision to purchase the ticket, the competitive equilibrium is not affected by this change. The price is still $p \in[3.5,4]$ and 5 tickets are sold, with person $A$ buying one of them. However, the equilibrium is no longer efficient. To prove this, we have to show that there is another allocation where some people is better off and nobody is worse off. Here is one: we take the ticket away from person $A$ and give it to person $F$. This move decreases $A$ 's utility by $\$ 15$, while increasing the spouse's utility by $\$ 12$ and $F$ 's utility by $\$ 3.5$. Nobody elses' utility changes: the price $A$ paid for the ticket still goes to the Canucks (so their profit of $p-1$ remains unchanged) and all other fans are not affected. To compensate $A$ for his utility loss, we need the spouse to pay 11.7 and person $F$ to pay 3.4 dollars. The spouse is better off because $A$ stays home, and her utility increases by $12-11.7=.3$. Person $F$ is better off his/her utility increases by $3.5-3.4=.1$. Person $A$ is better off, too. He doesn't get to go to the game but is compensated more than sufficient for not doing so. A's utility increases by $11.7+3.4-15=.1$. Finally, recall that all other market participants are not affected by this scheme, so they are equally well off.
d) $* *$ In the original problem, suppose that there are three additional tickets for sale (i.e., there are now 8 tickets in total), all of which are fake however. Fans cannot distinguish the fake tickets from the real thing and may discover that their tickets are worthless (have zero value) the night of the game, at which point whoever sold
them will have disappeared. Sellers do know whether their ticket is fake or not. The (marginal) cost of a fake ticket is zero. Are your answers in questions a) and b)i) affected and if so, how?

The introduction of fake tickets doesn't change the set of Pareto efficient allocations. All genuine tickets should still end up in the hands of persons $A-E$. The fake tickets should not exchange hands. The competitive equilibrium is affected, however. There are two things going on. First, more tickets may be on sale. Second, since fans don't know whether a ticket is fake, and since fake tickets have no value, the (expected) valuation for any ticket they buy drops. Both effects put a downward pressure on prices. The original equilibrium is certainly no longer valid, simply because the market no longer clears at $p \in[3.5,4]$. Indeed, for any $p \geq 1$, all 8 tickets are on offer. So can all of them be sold? If each fan has the same chance of buying a fake ticket, the valuation is proportionally reduced by 3/8. This gives for the reservation prices:

| person | $A$ | $B$ | $C$ | $D$ | $E$ | $F$ | $G$ | $H$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| reservation price in $\$$ | 9.4 | 6.2 | 5 | 3.1 | 2.5 | 2.2 | 1.25 | .6 |

Notice that for all 8 tickets to be sold in equilibrium, we would now require a price below person $H$ 's reservation price, i.e. $p \leq .6$. But the marginal cost of genuine tickets is 1, so all sellers with genuine tickets would drop out of the market. Fans would realize this and nobody would be willing to pay anything for a ticket anymore. The market has in effect disappeared. This phenomenon is called adverse selection: bad quality drive out good quality.

Additional Problem(s) for Review:
In Gas Pump, a small town in South Dakota, live 20 car owners. 10 people own a Beetle and 10 people own a Hummer. Every Beetle owner has a demand function for gasoline $D_{B}(p)=20-4 p$ for $p \leq 5$ and $D_{B}(p)=0$ for $p>5$, and every Hummer owner has a demand function $D_{D}(p)=16-2 p$ for $p \leq 8$ and $D_{D}(p)=0$ if $p>8$. There are many competitive gas stations in Gas Pump; the industry supply curve is $S(p)=-40+40 p$ for $p \geq 1$ and $S(p)=0$ otherwise (all prices are $\$$ per gallon and all quantities are gallons/week).
a) Draw a diagram with the (inverse) demand curves representing the total demand of gasoline by Beetle and Hummer owners, respectively, as well as the (inverse) aggregate demand function for gasoline. Draw the (inverse) supply function in your diagram. Give an analytic expression for the aggregate demand function.


The inverse demand function $D(p)$ is

$$
P(q)= \begin{cases}8-\frac{1}{20} q & \text { for } 0 \leq q<60 \\ 6-\frac{1}{60} q & \text { for } 60 \leq q<360 \\ 0 & \text { otherwise }\end{cases}
$$

b) Determine the competitive equilibrium price of gasoline. How many gallons are sold in equilibrium? The price is $p=4.120$ gallons are sold.
c) Tensions in the Middle East drive the cost of crude oil up. The new industry supply curve is $S(p)=-140+40 p$ for $p \geq 3$ and $S(p)=0$ otherwise. What is the new equilibrium price and quantity? The new price is $p=5.60$ gallons are sold.
d) Compare the equilibria in b) and $c$ ). Show that everybody (consumers and gas station owners) are worse off in c). Does this mean that the equilibrium in c) is not Pareto optimal? Explain! The price has increased which makes all consumers worse off. Producers are worse off because the producer surplus falls from 240 to 150 . This does not mean that equilibrium in c) is not Pareto efficient. Pareto efficiency is always defined relative to a given situation (a given group of people with given valuations and costs). Everybody may be better off in the old equilibrium, but the new equilibrium is still efficient.
2. Suppose that David and Harry consume oranges and apples. David views the two as perfect complements. Harry views them as perfect substitutes. David has 2 orange and 4 apples. Harry has 5 oranges and 1 apple. Determine the competitive equilibrium allocation(s) and the (relative) equilibrium price. Determine the
set of Pareto optimal allocations. How do these allocations depend on the total endowments of apples and oranges, and their initial distribution?

Because Harry views the two as perfect substitutes, he is always indifferent (not made worse off) with any one-for-one trade. Because David views the two as perfect complements, his utility level is determined by the number of pairs of fruit that he has. The unique (why?) equilibrium allocation is if the relative price of apples to oranges is one, and David trades two oranges for two apples, thus ending up with three of each. Harry then has 4 oranges and 2 apples. This depends on the initial allocation (assume for example, Harry had only 4 oranges, and no apple, then he would end up with 2 oranges and 2 apples after trade). The set of Pareto optimal allocation is any distribution where Harry has equal amounts of each fruit,i.e., (5 oranges, 5 apples), (4 oranges, 4 apples) and so on, and David has whatever is left over. This does not depend on the initial allocation.
3. Lolita, an intelligent and charming Holstein cow, consumes only two goods, cow feed and hay. Her preferences are represented by the utility function $U(x, y)=$ $x-x^{2} / 2+y$, where $x$ is her consumption of cow feed and $y$ is her consumption of hay. Lolita has been instructed in the mysteries of budgets and optimization and always maximizes her utility subject to her budget constraint. Lolita has an income of $\$ \mathrm{~m}$ that she is allowed to spend as she wishes on cow feed and hay. The price of hay is always $\$ 1$, and the price of cow feed will be denoted by $p \leq 1$
a) Write Lolita's inverse demand function for cow feed.

When $y$ is the numeraire and the price of $x$ is $p$, someone with quasilinear utility $u=f(x)+y$ has a willingness to pay for $x$ which equals $f^{\prime}(x)$, so the inverse demand function is $p=f^{\prime}(x)=1-x$. Of course, you can also just go through the standard utility optimization program; you'll arrive at the same conclusion.
b) If the price of cow feed is $p$ and her income $m$,how much hay does Lolita choose? What is the utility level that she enjoys at this price and this income?

The money that she doesn?t spend on feed is used to buy hay, so $y=m-p(1-p)$. Plugging the optimal quantities of $x$ and $y$ into the utility, we get $u=n=(1-p)^{2} / 2$
c) Suppose that Lolita's daily income is $\$ 3$ and that the price of feed is $\$ 0.50$. What bundle does she buy? What bundle would she buy if the price of cow feed rose to $\$ 1$ ?

Lolita would buy $(1 / 2,11 / 4)$. If the price of cow feed rises, she'll buy $(0,3)$.
d) How much money would Lolita be willing to pay to avoid having the price of cow feed rise to $\$ 1$ (equivalent variation)? Suppose the price of cow feed rose to
\$1. How much extra money would you have to pay Lolita to make her as well-off as she was at the old prices (compensating variation)? Compare equivalent and compensating variation, and explain. Both equivalent and compensating variation are the same and equal to $1 / 8$. Lolita's utility is quasilinear. In this case, the compensating and equivalent variation are the same since indifferences curves are parallel, so the distance between any two indifference curves is the same no matter where it is measured, Calculate Lolita's change in consumer surplus. Same answer, 1/8. Lolita's utility is quasilinear, so changes in consumer surplus (e.g. due to a price change) can be measured in monetary terms and are identical to asking how much money the consumer would be willing to forfeit (respectively, needed to be given) to avoid (respectively, compensate for) the change.

