## Econ 302: Microeconomics II - Strategic Behavior

Problem Set \# 2 - May 24

1. True/False/Uncertain? Explain your answer briefly.
a) If a monopolist knows that its customers have identical preferences, it can raise its profit by using a two-part tariff instead of a linear price.
True. The monopolist could stick with whatever linear price it is charging, and charge a small fixed fee on top of that. Since at any linear price, the consumer surplus is positive, the fixed fee (sufficiently small) will never cause any loss in customers. Profits increase.
b) Tax incidence is the burden of the tax as measured by the change in the resources available to economic agents as a result of the tax. In a monopoly market, the incidence for consumers of a specific per-unit tax on the monopolist's product cannot exceed $100 \%$, i.e. the price increase cannot be larger than the tax itself. (Hint: consider a constant elasticity demand curve $\left.p(q)=q^{-\frac{1}{\epsilon}}\right)$.
False. Consider a monopolist with constant marginal cost $c$ and a constant elasticity demand curve $p(q)=q^{-\frac{1}{\epsilon}}$. The marginal revenue is equal to $\left(1-\frac{1}{\epsilon}\right) q^{-\frac{1}{\epsilon}}$. Let $t$ be the per unit tax, which increases the marginal cost of the monopolist to $c+t$. In the profit maximizing optimum, marginal revenue equals marginal cost:

$$
\left(1-\frac{1}{\epsilon}\right) q^{-\frac{1}{\epsilon}}=c+t
$$

Solving this equation for $q$ gives $q^{M}=\left[(c+t) /\left(1-\frac{1}{\epsilon}\right)\right]^{-\epsilon}$ as the profit maximizing quantity. Plugging $q^{M}$ into the demand curve to get the profit-maximizing prices yields:

$$
p^{M}=\frac{c+t}{1-\frac{1}{\epsilon}} .
$$

How the price changes as a result of a small change in the tax is given by the first derivative $\partial p^{M} / \partial t$. Since $\epsilon>1$ (why?), we obtain:

$$
\frac{\partial p^{M}}{\partial t}=\frac{1}{1-\frac{1}{\epsilon}}>1
$$

Thus, the incidence of the tax that falls onto consumers exceeds $100 \%$.
2. Vancouver based Dr. Jean Carruthers discovered Botox but never patented her invention. Instead, the patent was claimed by the US firm Allergan, who subsequently went on to make Botox treatments the most popular cosmetic procedure in the world. The production cost for a vial of Botox is $\$ 25$ dollars. It is sold for about $\$ 400$ dollars to doctors.
a) Assuming the firm is setting it's price to maximize short run profit, determine the elasticity of demand for Botox (bonus question: "why did I add the qualifier "shortrun"?). Using the formula derived in class, we get

$$
\epsilon=\frac{p}{p-M C}=\frac{400}{400-25} \approx 1.067
$$

Thus, the demand is only slightly elastic.
b)* Assume a linear demand curve. If sales were 1 million vials in 2002, determine Allergan's inverse demand function, marginal revenue, profit, consumer surplus, and deadweight loss from monopoly pricing. How can this loss to society be justified? Assuming a linear demand function of the form $p=a-b Q$, elasticity is $\epsilon=\frac{1}{b} \frac{p}{Q}$. Solving for $b$ in $-400 / 375=-\frac{1}{b} \frac{400}{1}$ (where $Q=1$ million vials), gives $b=-375$. Solving $p=400=a-375$ gives $a=775$. Hence, Allergan faces an inverse demand function

$$
p=775-375 Q .
$$

The corresponding marginal revenue curve is $M R=775-750 Q$. The optimal monopoly quantity of 1 million vials is where

$$
M R=775-750 Q=25=M C .
$$

The consumer surplus at this quantity is $\$ 187.5$ million. Allergan's producer surplus is $\$ 375$ million. Allergans profit is its producer surplus minus the fixed cost (if any). If Allergan would price at marginal cost instead, it would sell 2 million vials and consumer surplus would be $1 / 2 \times 750 \times 2=\$ 750$ million. Compared to the competitive market, consumers thus loose $\$ 562.5$ million, part of which goes to Allergan. The rest is the deadweight loss (unrealized gains from trade) of \$ 187.5 million. The loss cannot be justified because the innovation (Botox) already existed, so one cannot argue that without the monopoly profit, Allergan would never have invented Botox.
c) Suppose the government sets a per-unit tax of $\$ 75$ on a vial of Botox, paid by consumers. What are the welfare effects? Give an intuitive explanation for your finding. The inverse demand function is $p=775-375 Q$. Imposing a per-unit tax of $\$ 75$ will be equivalent to shifting the demand curve to $p=700-375 Q$. With $M C=25$, profit-maximizing quantity and price are $Q=0.9$ and $p=437.5$. The new deadweight loss will be $1 / 2 \times(437.5-25)(2-0.9)=226.875$ million, which is more than without the tax. The intuition is not that the tax increases the deadweight loss because it distorts the initial allocation (note that the initial allocation wasn't efficient to begin with, so this line of argument doesn't apply). Rather, the tax
increases the welfare loss because it distorts the initial allocation further in the direction of inefficiency, specifically, by reducing the already sub-optimally low quantity even further. The point of this argument can easily be seen if one considers a subsidy instead (which would give the opposite result).
3. The monopolist RippOff-TV holds a monopoly in the local market for cable television services, which consists of 100 residential customers (households). If RippOff-TV offers $q$ channels to households on a monthly basis, the price it can charge per channel is $p=8-\frac{2}{3} q$ from each household. The marginal cost of providing a channel is zero.
a) Calculate the profit-maximizing linear price and the number of channels offered. Is this outcome Pareto efficient? Maximizing profit $\pi=\left(8-\frac{2}{3} q\right) q$ with respect to $q$ gives the first-order conditions:

$$
8-\frac{4}{3} q=0 \quad \Rightarrow \quad q^{m}=6, \quad \text { and } \quad p^{m}=4
$$

The outcome is not efficient. For example, selling an channel at a price $p=3$ would make the monopolist better off (since its costs are zero) and all customers better off (since their willingness to pay for that channel is $8-\frac{2}{3} 7=\frac{10}{3}>3$ ). Thus, a Pareto improvement is possible and the original allocation cannot be Pareto optimal.
b) A clever manager at RippOff-TV suggests to switch to two-part tariff pricing. Determine the profit-maximizing monthly fixed fee $F$ and the optimal per-channel price $p$.

We know that because consumer surplus can be extracted through the fixed fee in a two-part tariff pricing scheme, the monopolist wants to set the per-unit price to maximize consumer surplus - cost, which occurs at the efficient quantity where demand equals marginal cost. Thus, the profit maximizing per-unit price in a two part-tariff is equal to marginal cost. Hence, RippOff-TV should charge $p=0$ per channel (unlimited channels), resulting in a de-facto demand of 12 channels. The (consumer) surplus generated is $S=8 \times 12 \times \frac{1}{2}=48$. Thus, the optimal fixed monthly fee is $F=48$.
c) Comparing your answers in a) and b), conclude that profits increase but customers are worse-off under the two-part tariff. Give a brief intuition. Carefully argue whether or not the outcome in b) is Pareto efficient.
RippOff-TV is better off because its profit increased from $4 \times 6=24$ to 48 . Consumers are obviously worse off because their consumer surplus dropped from
$4 \times 6 \times \frac{1}{2}=12$ to zero. Intuitively, the two-part tariff allows the monopolist to extract all surplus, resulting in higher profits but less (=zero) consumer surplus. Although the new allocation in b) is not a Pareto improvement from a), it is nevertheless Pareto efficient. Since the willingness to pay for an additional channel is zero at $q=12$, there is no positive price (i.e., a price covering the cost) at which consumers would be willing to buy an additional channel, so that at least one party would be worse off if $q=13$. Similarly, reducing the number of channels to $q=11$ makes at least one party worse off because the loss to consumers cannot be compensated for by the cost savings of the monopolist (which are zero).
4. Sit-N-Sleep Inc. is the only seller of sofa beds in a remote town. Its total cost for producing $q$ sofa beds is $50,000+\frac{20}{3} q^{1.5}$ The demand for sofa beds in town is $Q=$ $2,400-2 p$.
a) Sit-N-Sleep maximizes profits. How many sofa beds will it produce, and at what price will they sell? Setting marginal revenue equal to marginal cost gives

$$
1,200-q=10 \sqrt{q} \quad \Leftrightarrow \quad \sqrt{q}=30,
$$

or $q^{*}=900$. The monopoly price is $p=750$.
b) What is the price elasticity of demand at the quantity you found in part a)? Verify that you get the same answer, no matter whether you directly use the definition of elasticity or the markup formula derived in the lecture. The elasticity is $\epsilon=-\frac{1}{1 / 2} \frac{p^{*}}{q^{*}}$, which gives $\epsilon \approx-1.67$. The markup formula gives the same, as is easily verified.
c)* What quantity of sofa beds maximizes social surplus? The government tries to achieve this quantity through a per unit subsidy. How much would this subsidy be? Briefly comment on its size. The efficient quantity is where price equals marginal cost, or

$$
1,200-\frac{1}{2} q=10 \sqrt{q} \quad \Leftrightarrow \quad q^{e f f}=1,600
$$

Let s be the per-unit subsidy, so the cost function is now $50,000+\frac{20}{3} q^{1.5}-s q$. Setting marginal revenue equal to marginal cost gives at the desired quantity of qeff $=1,600$ gives

$$
1,200-1,600=10 \sqrt{1,600}-s \quad \Leftrightarrow \quad s=800 .
$$

Because marginal revenue is negative where $p=M C$, the subsidy would have to be twice as large as marginal cost at the target quantity. That's a lot of tax payer money, but if the tax revenue could be raised without any further distortions (e.g., a head tax), the resulting allocation would be efficient (although it would not result in a Pareto improvement).

## Further questions for review

1. Canadian competition law prohibits mergers that unduly lessen competition. However, mergers that fall into this category but are shown to result in cost efficiencies are permitted nevertheless. Suppose for simplicity a merger results in a monopoly. Provide an example (graphic analysis) of the circumstances under which this 'efficiency defense' makes economic sense.

The efficiency defence only makes sense if the post-merger allocation at a minimum generates more gains from trade. Here is an analytical example. Let the inverse demand function in a market be given by $P(q)=25-\frac{1}{2} q$. The pre-merger cost function is given by $C(q)=15 q$. If the pre-merger situation is competitive, the equilibrium price would be $p=15$, the equilibrium quantity would be $q=20$ and total surplus in this market (=consumer surplus + no profits) would be $T S=100$.
Now assume the post-merger cost function is given by $C(q)=9 q$. If there is a monopoly post-merger, the price would rise to $p^{m}=17$, the quantity would drop to $q^{m}=16$, making consumers worse off. Due to the cost savings, however, total surplus (=consumer surplus + monopoly profit) increases to $64+128=192$.
If the cost savings are even more significant, consumers would also benefit. As is easily seen, this would happen for any marginal cost $<5$.
2. A monopolist faces a demand function of $Q(p)=16 / p^{2}$ and has constant marginal cost equal to $M C=1$.
a) Calculate the price elasticity of demand in this market. Does it depend on the quantity demanded/the price? Comment. We have $\epsilon=-\frac{\partial Q}{\partial p} \frac{p}{Q}=\frac{32}{p^{3}} \frac{p}{1^{2}}=2$. It is constant everywhere, i.e., it does not depend on price or quantity. Regardless of how high or low the price is, a 1 percent price reduction will always result in a 2 percent increase in demand.
b) Determine the profit-maximizing price and output in this market. Since we already $k n o w \epsilon$, it is easiest to use the inverse elasticity rule here. This gives $p_{m}=2 \times M C \Rightarrow$ $p_{m}=2$. The corresponding quantity is $Q_{m}=4$.
c) $* *$ Assume the demand curve is instead $Q(p)=16 / p$. How does your answer in b) change? Explain. For this demand function, the price elasticity of demand is $\epsilon=1$ everywhere. Again using the formula derived in class, marginal revenue is zero for this case, irrespective of what the quantity (or the price) is. The formula derived in class makes no sense in this case: we cannot find a quantity such that the Lerner Index equals the inverse elasticity of demand (formally, the firm's FOC for profit
maximization never hold. One can check easily that total revenue is 16 in this case regardless of how much is produced. Since any unit is costly, the firm should produce the smallest quantity possible.
3. Dombardier is the only firm that produces snowmobiles in Canada. The inverse demand function is $P(q)=100-2 q$, where $q$ is the number of snowmobiles sold per year. Average cost are constant and equal to $\$ 60$ per unit.
a) Calculate the equilibrium price and output.

Because $A C$ is constant, $A C=M C$. Setting marginal revenue equal to 60 gives $p_{m}=80$ and $q=10$. Profits are $\pi=200$.
b) Dombardier engages in R\& D. It hopes to discover a new material that will cut average cost dramatically to $\$ 8$ per unit. How much would Domardier willing to spend on $R \& D$ to discover this innovation?

If $M C=8$, the new equilibrium price and quantity are $p_{m}=54$ and $q_{m}=23$. Profits are $\pi=1058$. The firm would be willing to spend the present value of $1058-200=858$ each year for as long as it expects to have a monopoly on this good.
c) * Suppose the industry is competitive rather than being a monopoly. Compared to the monopoly in the pre-innovation situation, is society better off or worse off? Make your answer precise. Would your answer change if you are looking at the post-innovation situation?

If the industry is competitive, we have $p_{c}=60$ and $q_{c}=20$ under the old technology. Clearly, consumers are better off. Compared to the post-innovation situation, however, consumers are worse off. Innovation leads the monopolist to charges a lower price than the competitive industry, assuming the competitive industry does not innovate. But why should the competitive industry not innovate? One answer to this question is that profits are zero anyway, so there is no incentive to innovate...but wait! If one firm innovated, wouldn't it get the entire market at $p=54$ (the monopoly price)? Yes, and it would reap monopoly profits provided the other firms cannot copy the new technology. So innovation does happen even in a competitive industry as long as the innovator can realize some profit from the innovation (e.g. there is a patent policy in place.)
d) * Suppose the industry is competitive initially, but the government has a patent policy that guarantees the innovator all the returns from the innovation for certain time period. What are the consequences of such a policy?

As mentioned above, the consequences are that competitive firms also have an incentive to innovate. In fact, since they make zero profits before the innovation and monopoly profits afterwards, their gain from the innovation is even larger than the gain for a monopolist (assuming that the innovator gets to exploit its monopoly over the same period of time and has the same discount rate in both cases). But notice that the race (among the competitive firms) to be the first to discover the innovation may not be socially desirable: because only one firm wins the race, $R \mathcal{E} D$ costs are duplicated and resources are wasted.
e) What alternatives to patents does a government have to spur innovation? What advantages and dis-advantages would you expect those alternatives to have (as compared to patents)? The government could subsidize innovations instead. One example would be solar power technology. It could also pass legislation imposing new (future) standards that essentially require innovation to either meet the standards or make them profitable for producers. This was, for instance, done with restrictions on CO2 emissions in vehicles. There are many other possibilities, of course.
4. The Internet auction giant eBay is currently facing a class-action lawsuit in which its customers accuse eBay of "illegal bundling" by essentially leaving PayPal (which is owned by eBay) as the only 'economically viable option' of conducting transactions on the eBay market. Why would this kind of bundling be an anti-trust issue? After all, one could view the PayPal service as an input into eBays production function, and all eBay is doing is choosing a specific input. For example, anti-trust law does not require Honda to allow all car-radio manufacturers to install their devices in its vehicles. Comment.

Tying or bundling occurs when a company makes the purchase of one product or service (the tying good or service) conditional on the purchase of a second good or service (the tied good or service). The difference between Honda and eBay is that eBay is a de facto monopoly. In this case, tying one product (car, eBay) to another (radio, PayPal) can be anticompetitive. The arrangement will harm competitors who sell the second (tied) good or service, i.e., competitors of Paypal. Indeed, many PayPal competitors such as Citibank c2it, Yahoo PayDirect, and BidPay have left the market. It will also hurt consumers, who are forced to purchase a good or service they do not necessarily want or that is inferior. This is especially true when the good is tied to a product that many consumers consider critical. A classic example of this was Microsoft's strategy of tying its Internet Explorer software to the Microsoft Windows operating system.
5. Here is a test of how good your intuition is. Take a look at the list of price elasticities on the class slides, which can be found here and give an intuitive explanation for the following observations.

1. Long-run price elasticities are typically higher than short-run price elasticities

There are fewer substitutes for a good in the short run than in the long run. For example, if the price of oil increases people with regulars cars still need to buy gasoline. However, over time, a high price of gasoline will induce people to replace older cars with new cars powered by alternative energy sources, e.g, electric cars. Another related factor is that even if the price of a good goes up, consumers may keep buying a good out of habit, possibly expecting the price to fall again. However, when they realize the price rise is permanent, they will expend more energy and time in looking for alternatives, or they may be more willing to adjust their behavior (drive less, take public transportation in the case of gasoline).
2. The price elasticity for cars in general is lower than the elasticity of Chevrolet automobiles.

There are many close substitutes to Chevrolet automobiles even in the short run, namely similar cars from other car manufacturers. There are fewer substitutes for 'cars' as a general mode of transportation.
3. The demand for salt is very inelastic. 4. The demand for gasoline is also quite inelastic.

Both goods can be considered "essential" goods - necessities. This is true for salt and, at least in the short run, for gasoline as well. Also, in the case of salt, the product is very inexpensive and constitutes only a tiny fraction in consumers' budgets. Consumers tend to be relatively indifferent about a price increase for those goods.
5. The list does not contain newer consumer electronics such as, for example, the Apple iPad. Do you expect the measured price elasticity of demand for iPads to be elastic or inelastic? Why?

There is a number of close substitutes to the iPad, tablet models from other manufacturers, and some alternatives produced by Apple itself. This would suggest a relatively elastic demand. Also, recall that a monopolist operates on the elastic portion of the demand curve. If Apple has sufficient market power in the market for tablets, and is pricing optimally, then if we measure the price elasticity by observing Apple's existing pricing strategy and the consumer response to variations in the iPad price, we should expect an elastic response - otherwise, Apple would not be maximizing profits.

