Econ 302: Microeconomics II - Strategic Behavior
Problem Set \# 3 - May 31

1. Identify the type of price discrimination (direct, indirect) if any. Explain your answer.
a) "Ladies' night" at nightclubs. Direct price discrimination. However, the motive behind this type of price discrimination is likely not to extract additional consumer surplus (although one might argue that females have a more elastic demand for drinks etc. than males) in the classic sense. Rather, nightclubs want to maintain gender balance, presumably because missing women make the club less attractive for men.
b) Car insurance companies charging car drivers different rates depending on driver's age and make of car. Probably not price discrimination. Rate differences more likely to reflect differences in insurance cost (accident rates, likelihood of car being stolen, driving behavior), i.e. product is not homogeneous.
c) Financial aid for lower-income students at colleges and universities. Direct price discrimination (proof of income status usually is required). Again, note that the objective here is not to extract consumer surplus but rather to redistribute income (and help lower-income students get a college education.
d) Mail-in rebates for consumer products. Indirect price discrimination. Consumers who take the time and the effort to mail in the rebate form should have a lower income/higher elasticity of demand than customers who won't bother with the rebate.
e) Boxing Day sales Indirect price discrimination. Consumers who rather avoid the Boxing Day rush and prefer buying in an orderly manner with attentive sales people and no queues have a higher willingness to pay/less elastic demand than the bargain hunters.
f) Dinner prices versus lunch prices. Probably no price discrimination. Reflects higher cost of serving dinner patrons (eat longer).
2. A monopolist has identified two markets. The first has a demand represented by $q=56-p$, the second has a demand identified by $q=24-p$. The cost function is $C=10+\frac{1}{2} q^{2}$.
a) What is the profit maximizing price and output for the monopolist? You can assume that market segmentation is feasible. The firm chooses $q_{1}$ and $q_{2}$ to maximize profits

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\left(56-q_{1}\right) q_{1}+\left(24-q_{2}\right) q_{2}-10-\frac{1}{2}\left(q_{1}+q_{2}\right)^{2} .
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Taking derivatives and solving the FOC's gives gives $q_{1}=18$ and $q_{2}=2$. Note that not taking the total marginal cost for the aggregate quantity into account will give the wrong answer.
b)* Suppose now the monopolist's cost increases to $C=10+\frac{3}{2} q^{2}$. How is your answer in a) affected? Discuss what happened. Maximizing profit now gives FOC's that will imply a negative quantity for $q_{2}$. So the solution must be a 'corner solution' and we have $q_{2}=0$ and $q_{1}=56 / 5$. What happened is that costs have gone up sufficiently so as to make serving the second market no longer profitable. Note that the marginal cost of a unit of output for the second market is $M C=3\left(q_{1}+q_{2}\right)$. Hence, if $q_{1}=56 / 5$ is produced for the first market, the marginal cost of the first unit for the second market (MC evaluated at $q_{1}=56 / 5$ and $q_{2}=0$ ) is $168 / 5$, which already exceeds marginal revenue for the second market (which equals 24 for the first unit).
3.*The Mall Street Journal is considering to offer a new service which will send news articles to readers by email. Their market research indicates that there are two types of potential users, impecunious students and high-level executives. Let $q$ be the number of articles a user requests per year. Each executive has a demand function $P^{E}(q)=100-q$ and each student has a demand function $P^{S}(q)=80-q$ (prices are measured in cents). The Journal has a zero marginal cost of sending articles by email. Assume the number of students equals the number of executives.
a) Suppose the Journal can identify which users are students and which are executives. It offers each type of user a different all or nothing deal: each student can by access to 80 articles per year at a price $p_{S}$, or none at all. Each executive can buy 100 articles per year at a price $p_{E}$, or none at all. Determine the maximum price each type of users is willing to pay for this service (note that this is not a linear pricing scheme). Determine profits and consumer surplus. Is this outcome Pareto efficient?
The maximum students are willing to pay is $p_{S}=32$ dollars. This is equal to consumer surplus, i.e., the area under a student's demand curve. The maximum executives are willing to pay is $p_{E}=50$ dollars. Again, this is equal to consumer surplus at the respective quantity. Profit is $\$ 32$ per student and $\$ 50$ per executive. Note that the monopolist is extracting the entire surplus from both customers, given the quantities. Moreover, the
quantities are such that marginal willingness to pay (price) $=$ marginal cost, so the outcome is Pareto efficient (this is simply perfect price discrimination).
b) Now suppose the Journal can't tell the two groups apart. What would happen if it were continue to offer the two packages (price/quanity combinations) as calculated in a)? What profit would the Journal make? Is the outcome efficient? An executive would go for the student package since her utility (consumer surplus) from buying 80 articles for a price of 32 dollars is $\frac{1}{2} 80 \times 80+80 \times 20-3200=1600$ while her utility from buying 100 articles at 50 dollars is zero. Going for the student package thus represents a net gain of $\$ 16$. Everybody would now purchase the student package. Consumers surplus is zero for students, 16 for executives. Profit is 32 dollars per customer, which is less than in a). The outcome is not efficient since executives' WTP for one more article when they buy only 80 articles exceeds marginal cost.
c) Assuming the Journal wants to continue offering two packages, one that allows up to 80 articles and one that allows up to 100 articles, but at prices different from $p_{S}$ and $p_{E}$ as calculated in a). Determine the maximum price the Journal can charge for each package, assuming it wants the students to buy the former and the executives to buy the latter. Determine profits and consumer surplus. Is the outcome efficient? Since the journal has to prevent the executives from pretending to be students, lets assume it continues to charge the maximum price to students (that this is in fact optimal can be verified later). So the 80 articles package costs 32 dollars. An executive who buys 100 articles at some price $p$ will then not switch to the student package if her utility is lower in the latter case, i.e., if

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50-p \geq 48-32=16
$$

The maximum price for the 100 article package is thus $p=50-16=34$ dollars. Note that there is no point charging less for the 'student package' (as profits would only go down for both types of consumers) so 32 dollars was in fact optimal. Consumer surplus is zero for students, 16 for executives, as in b). Profit is 32 per student and 34 dollars per executive, which is less than in a) but more than in b). The outcome is efficient.
d)** Show that the Journal can increase profits even further by restricting the number of articles that students can buy to some number lower than 80 , and adjusting the prices accordingly (Hint: this means that prices for executives may change, too). Give an intuition for this finding. The answer to this question requires a bit of experimentation. You can try different quantities for students and see whether they work or not. Here is one possibility that works. Suppose the Journal reduces the 'student package' to 60 articles per year. This means the price to students will have to go down. Calculating the consumer surplus gives $p=30$ as the maximum price that can be charged for this package. For the
executives, the new maximal price can be calculated by solving

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50-p \geq 12 \quad \Rightarrow
$$

where the left hand side is the net consumer surplus for executives who buy the 60 article student package. So the 100 dollar package can now command a price of $p=38$ dollars. Consumer surplus is still zero for students, it is down to 12 for executives, and profits are up by $\$ 4$ per executive. The outcome is inefficient (students are getting too few articles). What is going on? By reducing the number of articles meant to be purchased by students (away from what is efficient), the Journal makes the student package less attractive to the executives. That means it can charge a higher price for the package meant to be purchase by them, thus extracting more consumer surplus.

Further questions for review:

1. Consider a monopolistic mobile phone service provider serving two population segments, $A$ and $B$. Each individual in $A$ has monthly demand $q=100-p$. Each individual in $B$ has monthly demand $q=240-2 p$, where $q$ are the monthly minutes and p is the per minute price (in cents). There are 4000 customers in $A$ and 1000 customers in $B$. The marginal cost of service is constant and equal to 20 cents.
a) Calculate the optimal two part tariffs (monthly service charge + per minute price) for the different cohorts, assuming the monopolist can separate the markets.

The profit maximizing two-part tariff sets the per-unit price equal to marginal cost and extracts the entire consumer surplus with the fixed fee. Hence, the optimal per minute charge is 20 cents, which gives a consumer surplus of $\frac{1}{2} 80 x 80=3200$ and $\frac{1}{2} x 200 x 100=10000$, respectively. The optimal monthly fee is thus $\$ 32$ for type $-A$ customers, and \$ 100 for type $=B$ customers.
b) Now suppose the monopolist cannot discriminate between $A$ and $B$. If the firm was going to offer a plan that both toes would join, which monthly charge and per unit price would the firm choose? If the firm was going to offer a plan that only one type of customer would join, which monthly charge and per unit price would the firm choose? Compare the profitability of the two options and explain.

If the firm was going to only serve one type, it would serve only type B customers and the optimal two part tarrif would be $\$ 100$ per month and $\$ 0.20$ a minute. If the firm wants to serve both types, the per-unit price would still be 20 cents, but the monthly fee would now be \$ 32. Only by setting the monthly fee equal to the minimum of the consumer surplus in the market.
c) Can the firm do better than any of the plans in b)? Explain.

Yes. By raising the per-unit price for type A customers, the firm can serve both market segments but extract more surplus from group B, The strategy is profitable because, at the margin, the firm does not make an additional (per-unit) profit by selling the last unit to a type-A customer; raising the price slightly is therefore costless and does not impede profits for that group. Profits for the other group, however, unambiguously increase as the move allows the firm to increase the fixed fee beyond type-A's surplus.
2. Intrawest manages ski lift operations at Whistler Blackcomb. It has two types of customers: locals (BC residents) and non-locals (American tourists). Each local has a demand function $p_{L}=10-q_{L}$, each non-local has a demand function $p_{N}=12-\frac{1}{2} q_{N}$, where $p_{L}$ (respectively, $p_{N}$ ) is the price of a lift ticket (day pass) for the ski resort, and $q_{L}$ (respectively, $q_{N}$ ) is the number of lift tickets sold per season to the locals and non-locals, respectively. Assume there are 100 locals and 100 non-locals. The cost to the resort for a lift ticket is constant and equal to 2 .
a) Suppose locals can be separated from non-locals by requiring the former to show their BC drivers' license. Determine the aggregate demand functions. Calculate the number of ski passes $q_{L}$ and $q_{N}$, and the (linear) price per pass $p_{L}$ and $p_{N}$ that maximize Intrawest payoffs. How much profit is Intrawest making?

The aggregate demand functions are $Q_{L}=1000-100 p_{L}$ and $Q_{N L}=2400-200 p_{N L}$. Maximizing profits for the non-locals and the locals separately gives $q_{L}=400, p_{L}=6$ and $q_{N L}=1000, p_{N L}=7$ for a profit of $\pi=6,600$.
b) A clever American has come up with a way to fake BC drivers licenses, and hands them out to his fellow Americans as soon as they arrive in Whistler. What will happen if Intrawest sticks to the prices $p_{L}$ and $p_{N}$ from question a)? Calculate profit in the new situation.

Because the price for locals is cheaper, all the tourists will get fake drivers licenses and pretend to be locals. If Intrawest sticks to the price $p_{L}=6$, this means it will still sell 400 tickets to the locals, but now all the non-locals buy the passes for $p_{L}=6$ as well, which gives a fake-locals demand of 1,200 passes. The total profit of Intrawest is $6,400<6,600$.
c) The CEO of Intrawest has an idea. Perhaps he can eliminate the tourists' incentives to use fake BC drivers licenses by introducing "Edge Cards" that restrict the number
of days which locals can use purchase lift tickets at a lower price $p_{L}$. Carefully explain whether or not this would work, and if so, how. (Hint: draw a picture)

This is indirect price discrimination. Because Intrawest can no longer directly separate locals from non-locals, the Edge Cards are designed to separate the two consumer groups by making the non-locals (voluntarily) choose the non-local price. This is done by artificially restricting the number of days and thus making the locals-package less attractive for non-locals (note that the locals aren't affected by this because their optimal demand stays at 4 days, so they don't care). It works because the consumer surplus of a non-local who now pretends to be a local - and thus buys the Edge card at the lower price of $p_{L}=6-$ is $C S_{N L}=42-24=18$ whereas the consumer surplus of a non-local who goes for the non-local price, and arbitrarily many days of skiing is $C S=25$. The latter is higher, hence, the tourists no longer pretend to be locals and voluntarily "reveal" themselves as non-locals.

