Econ 302: Microeconomics II - Strategic Behavior

Problem Set # 4 – June 7 2016

1. T/F/U?

a) If property rights are assigned to the recipient of a negative (positive) externality, then the source of the externality may want to bribe the recipient to allow more (less) of the externality being produced.

True. See the smoking example in the lecture (Sally had to compensate Harry for being allowed to smoke if he had the right to deny smoking). If the externality is positive, it's the other way around. If I have the right to determine how much gardening my neighbor has to do, for instance, I am likely to order too much of this activity (since I don't bear any of the cost). So my neighbor may want to bribe me for allowing less garden work.

b) If Sally's privately optimal amount of the public good is 5 units, and Harry's privately optimal amount of the public good is 10 units, then the Pareto optimal amount of the public good must be 15 units.

False. The privately optimal amount is where marginal benefit equals marginal cost; Pareto optimal amount of a public good is where the sum of marginal benefits equals marginal cost. This doesn't mean privately optimal quantities can be added up to yield efficient amounts. See, e.g., the ice-rink example in class as a counter-example. (Bonus: Can you come up with an example where the statement happens to hold?)

2. Harry and Sally live together. Harry derives a utility of $250 per month, measured in dollars, from cooking spicy meals. The monetary equivalent of Sally's disutlity from having to smell the cooking fumes is $ 350 per month. The problem causes Sally to contemplate moving out, in which case both Harry and Sally would have to be renting more expensively elsewhere for $ 200 more *per person*.

a) Argue carefully that the current situation (live together, Harry cooks his spicy meals) is not Pareto optimal. *The current situation is not Pareto optimal because there is another allocation that would make both Harry and Sally better off. For instance, they could keep on living together and Sally gives Harry 300 $ to compensate him for not cooking. That makes her better off (by $ 50) and makes him better off (by $ 50). Thus, there is anther allocation that is better than the current allocation. Therefore, the current allocation is not Pareto optimal.*

b) In light of the Coase theorem, what do you expect to happen? Explain!

What will happen according to the Coase theorem is the Pareto optimal allocation. The Pareto optimal allocation is that both live together and Harry does not cook.

This results in a utility of 0 for both actors (ignoring the current rent and any transfers). We already know from a) that if Harry were to start cooking, then Harry would gain 250 utility but Sally would lose 350. Thus, Harry cannot compensate Sally for the negative externality from him cooking. The only other option is to move out. If this occurs than Sally gets - \$ 200 from the increased rent and Harry gets \$ 50 = 250 - 200. Again, there is no way that Harry can compensate Sally for the loss in rent. Thus, we cannot make someone better off without hurting the other.

What is the transfer? Assuming Harry has the right to cook (as in the question) Sally will have to pay him anything in between \$ 50 (=his net gain in utility from moving out) and \$ 200 (= her opportunity cost from moving out) as a compensation for not cooking and staying put.

3. An airport (A) is located next to a piece of land owned by a developer (D), and the noise from the planes reduces the value of land. If A has x planes flying per day and D builds y houses, total profit of A is $\pi_A = 48x - x^2$ and total profit for D is $\pi_D = 60y - y^2 - xy$, both measured in dollars.

a) How many airplanes should fly, and how many houses should be built from an efficiency point of view? (Hint: since both utilities are denominated in dollars, you can add them up to get total surplus). *Maximizing total surplus $\pi_A + \pi_D$ gives $x^* = 12$ and $y^* = 24$. Total surplus is* 1008.

b) 'Free to choose – no bargaining'. Suppose first A chooses x to maximize its profits, ignoring the negative external effect on D. Calculate the profits maximizing number of planes x per day. Knowing x, D chooses y to maximize π_D. Calculate the optimal number of houses y for D. What is total surplus (profit)? *Maximizing π_A with respect to x gives $x = 24$ planes per day, so D's profit is $\pi_D = 36y - y^2$, which is maximized at $y = 18$. Total surplus is* 900*, of which A makes* 576 *and D makes* 324*. Too many planes fly and too few houses are built. This is because A doesn't take the negative externality of noise pollution into account, so x is too large relative to the Pareto optimum. As a result, D sets y too small: given* 24 *planes fly (rather than* 12*), the marginal benefit of a house lower than what it should be.*

c) 'Strict Prohibition'. Suppose local authorities make it illegal to land or start planes at the airport. How many houses will D built, and what is total surplus? *Since $x = 0$ by law, D's profit is $\pi_D = 60y - y^2$, which is maximized at $y = 30$. Total surplus (= sum of profits) is 900, all of which accrues to D (A makes no profit, of course). The solution is not efficient. Too many houses are built and too few planes fly. The latter is by law. The former is because A's marginal benefit of building a house is lower than the social marginal benefit (which would take the effect of noise pollution into account, given that* *the Pareto optimal number of planes fly).*

d) 'Lawyer's Paradise'. Suppose a law is passed that makes A liable to all damage to D's property values. If x planes fly and y houses are built, D is awarded damages that equal the a actual external cost, xy. A's profit is thus $\pi_A = 48x - x^2 - xy$ while D makes $\pi_D = 60y - y^2$ (including the damage payments). How many houses $\hat{x}$ will D choose to built? And how many planes $\hat{y}$ will A fly, knowing that D chose $\hat{x}$? What is total surplus? *Maximizing D's profits gives $\hat{y} = 30$. Maximizing A's profits net of damages gives $\hat{x} = 9$ planes. Total surplus is 981 (D makes 900 and A makes 81). Again, the solution is not efficient. Too many houses are built and too few airplanes are flown. This is because D doesn't take into account the negative externality he exerts on A (building houses increases the marginal cost of flying planes due to of the damage payments.*

f)** 'Bargaining'. Suppose that A and D remain independent, but D can negotiate with A about reducing noise pollution. Let $\tilde{x}$ be the agreed upon number of flights. Give Coasian solution to the problem. *One mutually beneficial agreement is the following. D offers A to pay for the lost profit from reducing the number of flights. Suppose the new number of flights agreed upon is $\tilde{x}$, then D will pay $T = \pi_A(x = 24) - \pi_A(\tilde{x}) = 576 - (48\tilde{x} - \tilde{x}^2)$. Obviously, A will be just as well off with the agreement than before (by construction). D's profit is now $\pi_D(y, \tilde{x}) - T = \pi_D(y, \tilde{x}) + \pi_A(\tilde{x}) - 576$. Maximizing this profit with respect to y and $\tilde{x}$ gives (of course) the efficient quantities $y = 24$ and $\tilde{x} = 12$. Total surplus is maximized at 1008. A gets $\pi(\tilde{x}) + T = 576$. D gets 432.*

g)*'Government Intervention'. The government levies a tax t on A for each airplane that landed/started at the airport. How high should the government set t to restore efficiency? Can the tax revenue be distributed to make both A and D better off relative to b) c) and d)? *The tax should be set equal to the marginal external cost, evaluated at the efficient solution. The marginal cost of flying one airplane is constant and equal to y. In the Pareto efficient solution $y^* = 24$ houses are built, so $t = 24$. This restores efficiency, since A's profit becomes $\pi_A = 48x - x^2 - 24x$, which is maximized at $x = 12$. D's profit is not directly affected by the tax; however, since now only 12 planes fly, we get $\pi_D = 60y - y^2 - 12y$, so $y = 24$, which is Pareto efficient. Total tax revenue is $24 \times 12 = 288$, and profits are $\pi_A = 144$ and $\pi_D = 576$, respectively. Total surplus is 1008. The tax revenue can always be distributed to make everybody better off, as is easily seen.*

4. Cowflop, Wisconsin, has 1000 identical inhabitants. Their utility is

$$u(x_i, G) = x_i + G^{\frac{1}{2}},$$

where x_i is the amount of milk consumed by individual i, measured in gallons, and g is the number of fireworks exploded in the town's Forth of July extravaganza (private use of fireworks is outlawed). Fireworks cost twenty gallons of milk per unit.

a) Calculate the (absolute value of the) marginal rate of substitution between fireworks and milk for each citizen. Does it depend on individual milk consumption? Find the Pareto optimal amount of fireworks for Cowflop. *The absolute value of the marginal rate of substitution between fireworks and milk can be calculated as*

$$-\frac{\Delta x_i}{\Delta G} = \frac{\partial u/\partial G}{\partial u/\partial x_i} = \frac{\frac{1}{2}\frac{1}{\sqrt{G}}}{1} = \frac{1}{2}\frac{1}{\sqrt{G}},$$

which is also the marginal benefit from G (measured in milk units). By inspection, it does not depend on individual milk consumption x_i*. The Pareto optimal amount of fireworks is determined by summing up the marginal rates of substitution between milk and fireworks (the marginal benefits from fireworks in terms of milk) and equating this to the marginal rate of transformation between milk and fireworks (the marginal cost of fireworks in terms of milk):*

$$\sum_{i=0}^{i=1000}\left(-\frac{\Delta x_i}{\Delta G}\right) = 1000\frac{1}{2}\frac{1}{\sqrt{G}} = 20 \iff G^* = 625.$$

Further questions for review:

1. The Kinder Morgan Oil company proposes to expand their existing pipeline from the Alberta Oilsands to Burnaby. The plans would lead to a doubling of tanker traffic in the Burrard Inlet, to about a tanker a day. The estimated cost of an oil spill in the Burrard Inlet are $ 40 Billion. Both the city of Burnaby and the City of Vancouver are opposed to the plan. The Vancouver city council passed a motion this month demanding that Kinder Morgan pipeline company carry full liability to cover the costs of an oil spill in our Vancouver Harbour. Which of the solutions to the problem of externalities we discussed in class – if any – does this proposal represent?

Like Pigouvian taxation, liability rules are designed to make people to internalize the external costs of their activities. Here, the policy requires **direct** *compensation of those harmed: under a liability rule a party who takes an action that harms others must compensate the affected parties for their losses. Liability rules induce decision-makers to internalize all external costs. In some cases, liability rules are preferable because the government needs less information to effectively use a liability rule than it would for an emissions standard or a Pigouvian Tax. The effect, however, is the same: if the person responsible for the externality has to pay damages that equal the actual harm caused, the external costs are fully internalized and an efficient outcome prevails.*

2.* The government of Smogville wants to enact new pollution control policies and you are called upon to advise them. The initial level of pollution is $P_0 = 60$ units, that the cost of abatement are $C(Q_A) = \frac{3}{2}Q_A^2$, and that the benefits of abatement are $B(Q_A) = 120Q_A - Q_A^2$ where Q_A is the level of unabated (remaining) pollution.

a) Find the Pareto optimal level of abatement and the resulting amount of unabated pollution. *The optimal level of abatemenn maximizes net benefits of abatement* $B(Q_A) - C(Q_A) = 120Q_A - Q_A^2 - \frac{3}{2}Q_A^2$. *Taking derivatives and setting the marginal net benefit equal to zero gives* $Q_A^* = 24$.

b) At what rate should an effluent tax (=Pigou tax on the activity causing the pollution) be set in order to achieve the optimal level of abatement? *To achieve the optimal level of abatement, the effluent tax needs to equal marginal abatement cost at the optimal level of abatement. Marginal costs are* $dC(Q_A)/dQ_A = 3Q_A$, *so the tax should be* $t = 3 \times 24 = 72$ *\$.*

c) Why isn't it a good idea to eliminate the remaining (unabated) pollution? *By definition, the optimal level of abatement is where the marginal benefit of abatement equals the marginal cost. Any abatement beyond that level necessarily had the marginal cost exceed the marginal benefit. Hence, total cost of abatement must be larger than total benefit. This can also be shown by aggregating over the marginal cost: total cost for the remaining abatement are* $C(60) - C(24) = 7560$ *which is larger than the additional benefit of* $B(60) - B(24) = 1296$.

3.* Suppose four families share a common stretch of beach and they are considering a program of improvements, including a stairway and a play structure for children. If they spend an amount of x in total for the improvements, the (gross) value is $v_1(x) = v_2(x) = 5x - \frac{1}{2}x^2$ for families # 1 and #2, $v_3(x) = 10x - \frac{3}{4}x^2$ for family #3, and $v_4(x) = x - \frac{3}{4}x^2$ for family #4 (everything is measured in 100's of dollars). Let t_i be family #i's contribution to the cost of the measure and assume each family has the same income $\bar{y} = 10$.

a) Calculate the Pareto efficient level x^* of expenditure on beach improvements. How much should each family optimally contribute? *The Pareto efficient level of x maximizes total surplus. Since all utilities are measured in dollars here, we can add them up and subtract the cost to get total surplus:*

$$v_1(x) + v_2(x) + v_3(x) + v_4(x) - x = 20x - \frac{5}{2}x^2,$$

which is maximized at $x^* = 4$. *So the Pareto efficient expenditure is* 400 *dollars. There is no 'optimal' contribution for any family (this was a trick question). As long as the sum of the t_i's covers the cost, i.e., as long as* $\sum_i t_i = t_1 + t_2 + t_3 + t_4 = x^*$ *and* $t_i \leq 10$, *we will be able to finance* x^*. *Who ends up with how much money left in their pockets is irrelevant.* ***Any allocation of money*** *is Pareto optimal here, given* x^* *is spent (but not every* x*; you are asked to verify this in c).*

b) Now suppose the cost of beach improvements are shared equally among the families. Show that, under this cost sharing rule, family # 4 would prefer not have any improvements at all ($x = 0$) rather than the efficient level (x^*). What is the largest improvement $\bar{x}$ that all families would agree upon if costs must be shared equally? *If costs are shared equally and x is spent, each family pays $t_i = \frac{1}{4}x$. If the efficient expenditures is made, $x = 4$, if nothing is built, $x = 0$. Comparing family # 4's utility in both cases gives*

$$v_4(x = 4) - t_4(x = 4) = 4 - \frac{3}{4}4^2 - 1 = -9 \; < \; 0 = v_4(x = 0) - t_4(x = 0),$$

so they prefer not to have any improvements. If one wants to have family #4 on board while maintaining equal cost sharing, the largest amount of expenditures that this family would agree to (others are not a problem) is

$$v_4(\bar{x}) - t_4(\bar{x}) = \bar{x} - \frac{3}{4}\bar{x}^2 - \frac{1}{4}\bar{x} = 0 \;\Rightarrow\; \bar{x} = 1.$$

c) Demonstrate *formally* that $\bar{x}$ is Pareto inefficient. *To show that $\bar{x}$ is not Pareto optimal, we have to find another allocation (i.e., an alternative level of expenditures and an alternative cost sharing pattern) that makes some families better off without hurting anyone. One possibility is to expend $x = 4$ (this must obviously better from our calculation in a)) but note that this level of expenditures would have to involve family # 4* ***receiving*** *money from the other families. Another possibility that does not require compensation payments for family #4 is to set $x = 4/3$, which makes family #4 just indifferent to $\bar{x}$ given it does not have to pay anything now, $t_4 = 0$. The remaining 3 families can split the bill equally so, $t_1 = t_2 = t_3 = 4/9$. It is easy to check that all three are better off that in $\bar{x}$ is spent and the bill is split among all four.*

4, * Answer question 4 if the utility of each citizen is

$$u(x_i, G) = \ln x_i + \ln G.$$

Again, we can calculate the (absolute value of the) marginal rate of substitution between fireworks and milk:

$$-\frac{\Delta x_i}{\Delta G} = \frac{\partial u/\partial G}{\partial u/\partial x_i} = \frac{\frac{1}{G}}{\frac{1}{x_i}} = \frac{x_i}{G}.$$

By inspection, the MRS now ***does*** *depend on individual milk consumption x_i (WHY?). Again, the Pareto optimal amount of fireworks is determined by summing up the marginal rates of substitution between milk and fireworks (the marginal benefits from fireworks in terms of milk) and equating this to the marginal rate of transformation between milk and fireworks (the marginal cost of fireworks in terms of milk).*

$$\sum_{i=0}^{i=1000} \left(-\frac{\Delta x_i}{\Delta G}\right) = \sum_i \frac{x_i}{G} = \frac{\sum_i x_i}{G} = 20 \iff G^* = \frac{1}{20}\sum_i x_i. \qquad (*)$$

The Pareto optimal amount of G now depends on aggregate milk consumption. For instance, if the total amount of milk available in Cowflop is $Y = 10000$ gallons, then the aggregate resource constraint for Cowflop is

$$\sum_i x_i + 20G = 100000 \qquad \Longleftrightarrow \qquad G = (10000 - \sum_i)/20.$$

Plugging this expression into (*) gives $\sum_i x_i = 5000$ and $G^* = 250$.